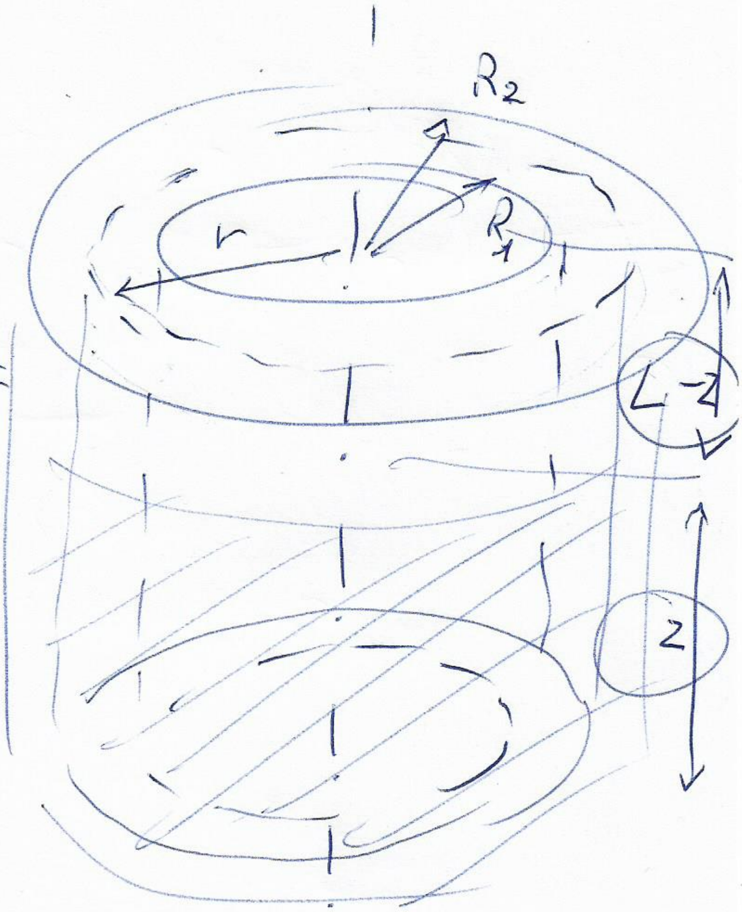
CENTRO DI CULTURA SCIENTIFICA
"A. VOLTA"



$$\vec{E}_o(\vec{r}) = E_{or}(r)\,\hat{e}_r$$

$$E_{or}(r)\,2\pi r L = \frac{1}{\varepsilon_0}\underbrace{\sigma_o\, 2\pi R_1}_{\lambda_o} L =$$

$$= \frac{1}{\varepsilon_0}\lambda_o L$$

$$E_{or}(r) = \frac{\lambda_o}{2\pi\varepsilon_0}\frac{1}{r}$$

$$\Delta V_o = V_1 - V_2 = \int_{R_1}^{R_2} E_{or}(r)\,dr = \frac{\lambda_o}{2\pi\varepsilon_0}\log(R_2/R_1)$$

$$\lambda_o = 2\pi\varepsilon_0 \Delta V_o / \log(R_2/R_1) = 24.92 \text{ nC/m}$$

$$Q_o = \lambda_o L = 2\pi\varepsilon_0 \Delta V_o L / \log(R_2/R_1) = 997 \text{ pC}$$

$$C_o = Q_o/\Delta V_o = 9.97 \text{ pF}$$

$$U_o = \frac{1}{2} Q_o \Delta V_o = \frac{1}{2}\frac{Q_o^2}{C_o} = 49.8 \text{ nJ}$$

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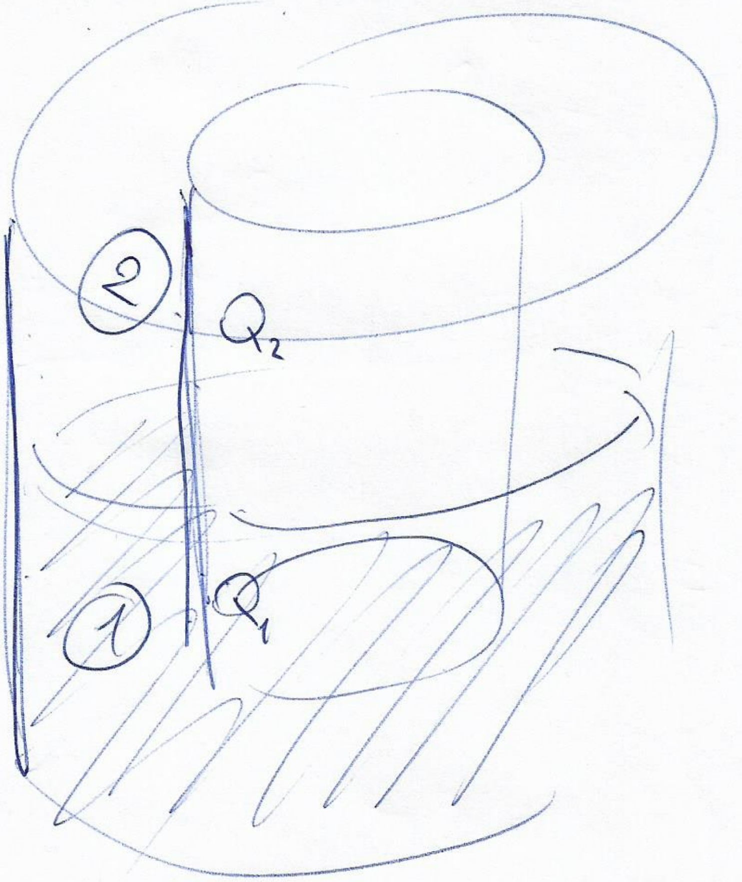
b) Φ ε_r



$$\begin{cases} E_{1r}(r) = E_{2r}(r) \leftarrow \\ Q_1 + Q_2 = Q_0 \leftarrow \end{cases}$$

$$\lambda_1 z + \lambda_2 (L - z) = \lambda_0 L$$

$$\boxed{\begin{aligned} E_{1r}(r) &= \frac{1}{2\pi\varepsilon_0\varepsilon_r}\frac{\lambda_1}{r} \\ E_{2r}(r) &= \frac{1}{2\pi\varepsilon_0}\frac{\lambda_2}{r} \end{aligned}} \longrightarrow \lambda_1 = \varepsilon_r \lambda_2$$

$$\downarrow$$

$$\frac{Q_1}{z} = \varepsilon_r \frac{Q_2}{L-z} \rightsquigarrow Q_2 = \frac{L-z}{\varepsilon_r z} Q_1$$

$$Q_1 + Q_2 = Q_1\left[1 + \frac{L-z}{\varepsilon_r z}\right] = \frac{(\varepsilon_r - 1)z + L}{\varepsilon_r z} Q_1 = Q_0$$

$$Q_1 = Q_0 \varepsilon_r z \Big/ \left[(\varepsilon_r - 1)z + L\right] \quad ; \quad Q_2 = Q_0 (L - z) \Big/ \left[(\varepsilon_r - 1)z + L\right]$$

$$\Delta V = \int_{R_1}^{R_2} E_{1r}(r)\,dr = \frac{\lambda_1}{2\pi\varepsilon_0\varepsilon_r}\log(R_2/R_1) = \frac{Q_0}{2\pi\varepsilon_0}\frac{\log(R_2/R_1)}{[(\varepsilon_r - 1)z + L]}$$

$\hookrightarrow L\Delta V_0$

$$\Delta V = \Delta V_0 \frac{L}{[(\varepsilon_r - 1)z + L]}$$

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$z = L/2 \qquad \varepsilon_r = 2$

$Q_1 = 665\ pC\ ;\ Q_2 = 332\ pC\ ;\ \Delta V = 66.6\ V$

$\lambda_1 = 33.2\ nC/m\ ;\ \lambda_2 = 16.6\ nC/m$

$$C_0\ (L) \nearrow C_0\,\frac{L-z}{L}\quad (2)$$

$$C_0\ (L) \searrow C_0\,\varepsilon_r\,\frac{z}{L}\quad (1)$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 + Q_2 = Q_0$$

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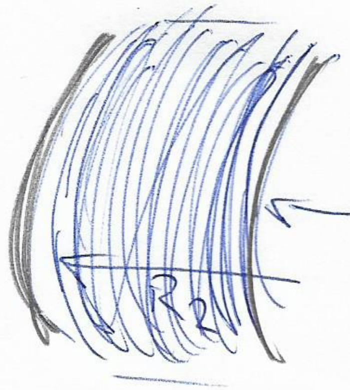
$$\rho_p = -\bar{\nabla}\cdot\bar{P} = \phi$$

$$\sigma_p(R_1) = \bar{P}(R_1)\cdot(-\hat{e}_r) = -P(R_1) = -\varepsilon_0(\varepsilon_r - 1)E_r(R_1)$$

$\hat{e}_r$



$$E_r(r) = \frac{1}{2\pi\varepsilon_0\varepsilon_r}\frac{\lambda_1}{r} = \frac{Q_0}{2\pi\varepsilon_0}\frac{1}{[(\varepsilon_r-1)z+L]}\frac{1}{r}$$

$$r = R_1$$

$$\sigma_p(R_1) = -\frac{Q_0}{2\pi}\frac{(\varepsilon_r-1)}{[(\varepsilon_r-1)z+L]}\frac{1}{R_1}$$

$$\sigma_p(R_2) = \bar{P}(R_2)\cdot\hat{e}_r = P(R_2) = \frac{Q_0}{2\pi}\frac{\varepsilon_r-1}{[(\varepsilon_r-1)z+L]}\frac{1}{R_2}$$

$$Q_{p1} = \sigma_p(R_1)\,2\pi R_1 z = -Q_0\frac{(\varepsilon_r-1)z}{[(\varepsilon_r-1)z+L]}$$

$$Q_{p2} = \sigma_p(R_2)\,2\pi R_2 z = +Q_0\frac{(\varepsilon_r-1)z}{[(\varepsilon_r-1)z+L]}$$

$z = L/2 \quad \varepsilon_r = 2$

$\sigma_{p1} = -330.7\ nC/m^2$; $\sigma_{p2} = 264.5\ nC/m^2$; $Q_{p2} = -Q_{1p} = 332.41\ pC$

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c) $$U' = \frac{1}{2} Q_0 \Delta V = \underbrace{\frac{1}{2} Q_0 \Delta V_0}_{U_0} \frac{L}{[(\varepsilon_r - 1)z + L]}$$

$$U' = \frac{2}{3} U_0 = 33.2 \text{ nJ}$$

d) $$F_z = -\frac{\partial U'}{\partial z} = \frac{U_0(\varepsilon_r - 1)/L}{[(\varepsilon_r - 1)z/L + 1]^2}$$

$\hat{e}_z \uparrow$ $$F_z = 553.8 \text{ nN}$$

e) $$\mathcal{L} = \int_0^z F_z(z')\,dz' = U_0(\varepsilon_r - 1)/L \int_0^z \frac{dz'}{[(\varepsilon_r - 1)z'/L + 1]^2} =$$

$$= -U_0 \frac{\varepsilon_r - 1}{L} \frac{L}{\varepsilon_r - 1} \left[\frac{1}{(\varepsilon_r - 1)z'/L + 1} \right]_0^z =$$

$$= U_0 \left[1 - \frac{1}{(\varepsilon_r - 1)z/L + 1} \right] = U_0 \frac{(\varepsilon_r - 1)z/L}{(\varepsilon_r - 1)\frac{z}{L} + 1}$$

work by $\bar{E}$ field

$$U'(z) + \mathcal{L}(z) = U_0$$

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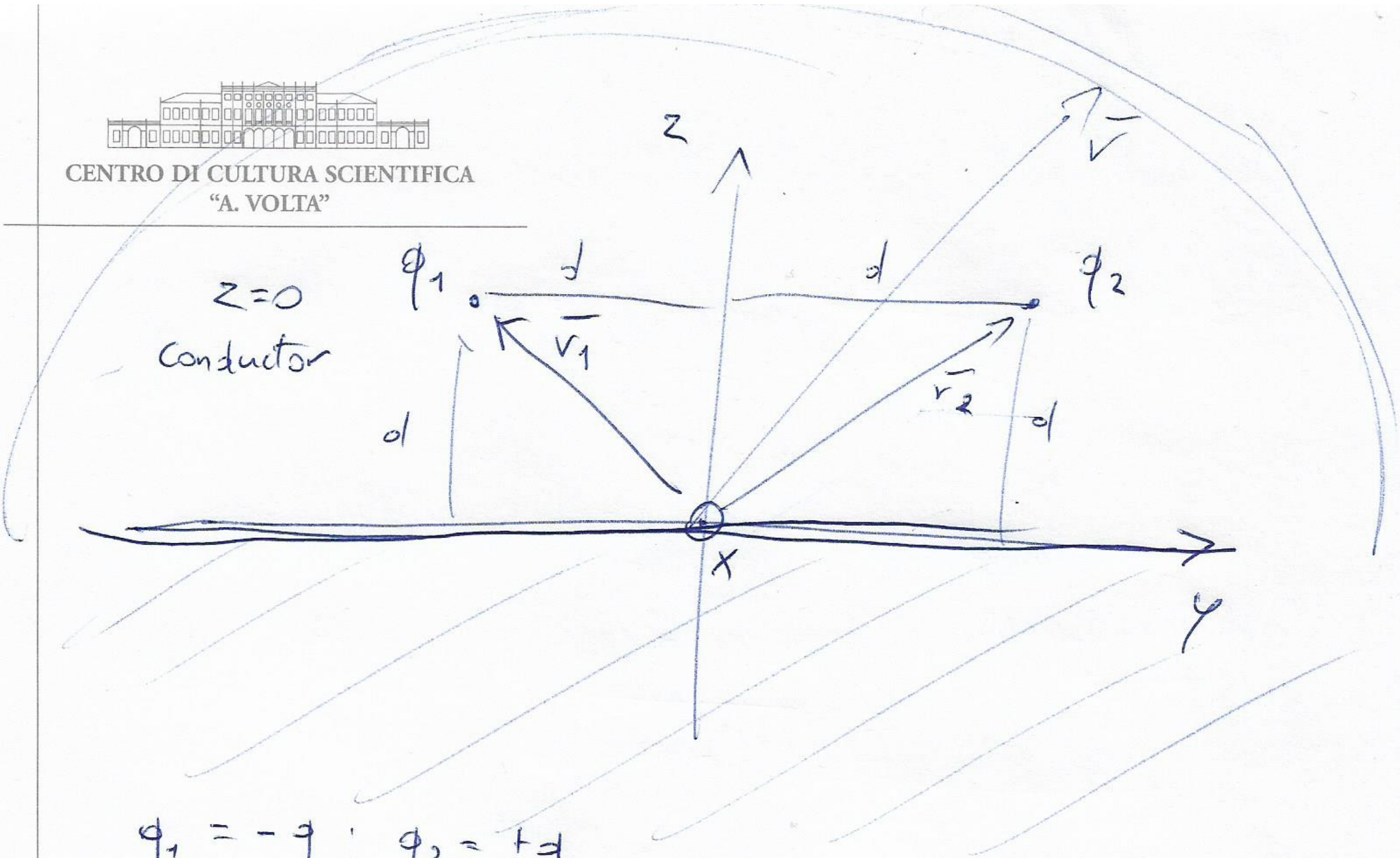

$q_1 = -q \; ; \; q_2 = +q$

a) Poisson ; dominio : semispazio $z \geq 0$

$$\nabla^2 \phi = -\frac{\rho(\bar{r})}{\varepsilon_0} \qquad \rho(\bar{r}) = q\,\delta(\bar{r}-\bar{r}_2) - q\,\delta(\bar{r}-\bar{r}_1)$$

$\bar{r}_1 = (0, -d, d) \; ; \; \bar{r}_2\,(0, d, d)$

$\boxed{B.C.1}$ $\phi(x, y, z=0) = 0$

$\boxed{B.C.2}$ $\lim_{|\bar{r}| \to +\infty} \phi(\bar{r}) = 0$ in domain $z \geq 0$

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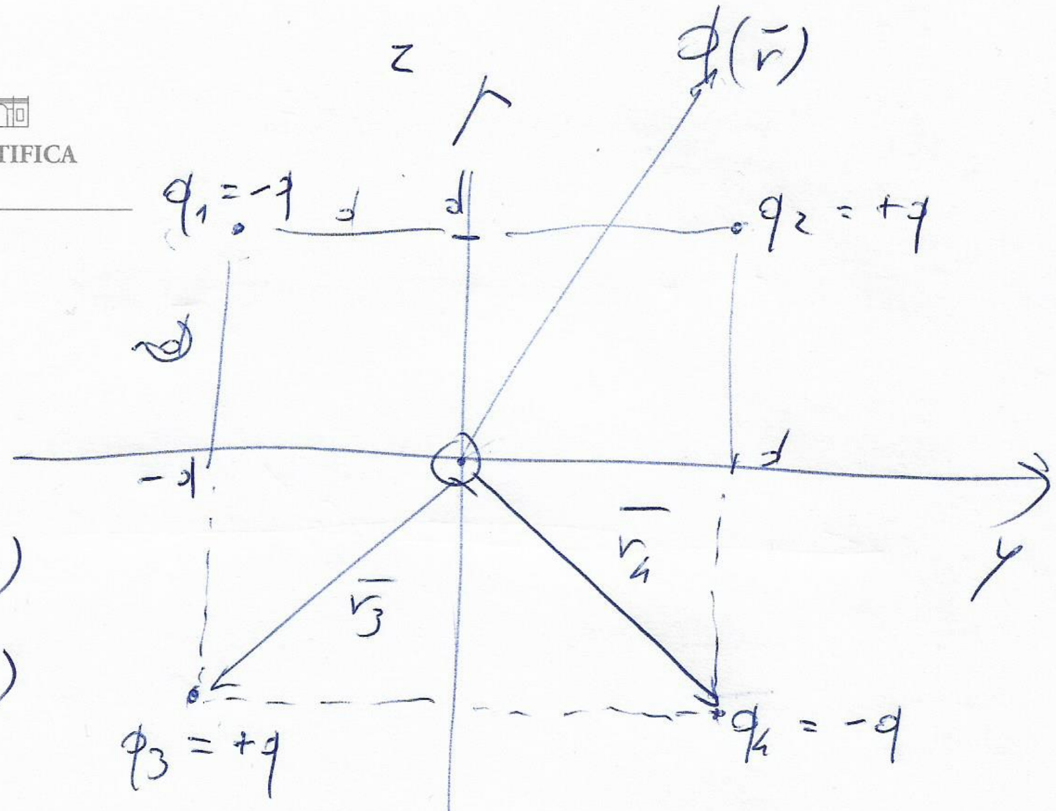
b) Images



$\bar r_3 = (0, -d, -d)$

$\bar r_4 = (0, +d, -d)$

$$\phi(\bar r) = \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{|\bar r - \bar r_1|} + \frac{1}{|\bar r - \bar r_2|} + \frac{1}{|\bar r - \bar r_3|} - \frac{1}{|\bar r - \bar r_4|}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{x^2+(y+d)^2+(z-d)^2}} + \frac{1}{\sqrt{x^2+(y-d)^2+(z-d)^2}} +\right.$$

$$\left. + \frac{1}{\sqrt{x^2+(y+d)^2+(z+d)^2}} - \frac{1}{\sqrt{x^2+(y-d)^2+(z+d)^2}}\right)$$

$\phi(x, y, 0) = 0$

$\phi(\bar r \to +\infty,\ z \geq 0) = 0$

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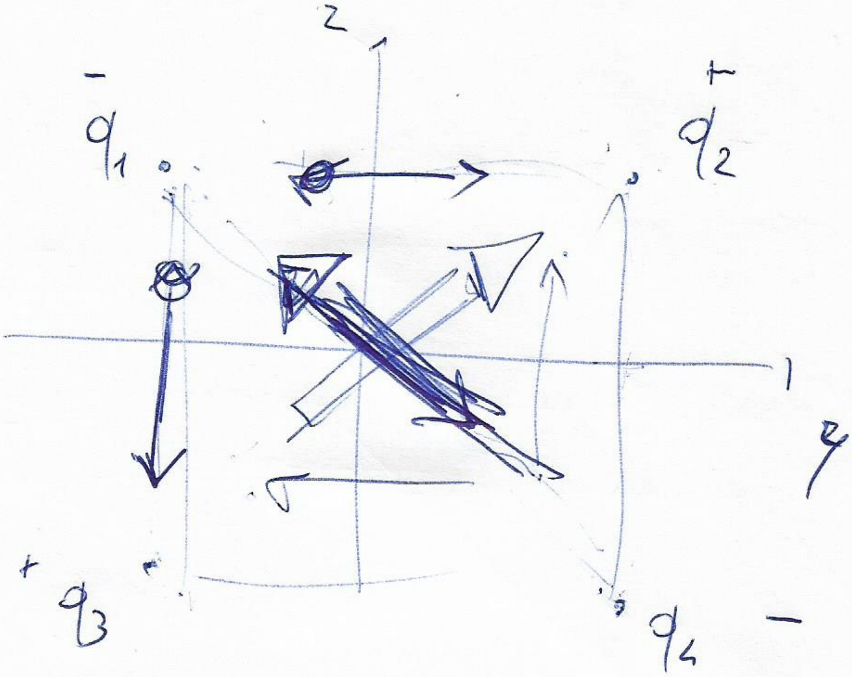
c)



$$\overline{F_1} = \underbrace{\frac{q^2}{4\pi\varepsilon_0}\frac{1}{(2d)^2}}(-\hat{e}_z) + \underbrace{\frac{q^2}{4\pi\varepsilon_0}\frac{1}{(2d)^2}}(\hat{e}_y) +$$

$$+ \frac{q^2}{4\pi\varepsilon_0}\frac{1}{(2\sqrt{2}\,d)^2}\left(\frac{\hat{e}_z - \hat{e}_y}{\sqrt{2}}\right) =$$

$$= \frac{q^2}{16\pi\varepsilon_0 d^2}\left(\sqrt{2} - \frac{1}{2}\right)\frac{(\hat{e}_y - \hat{e}_z)}{\sqrt{2}}$$

$\overline{F_2}$ $\qquad \hat{e}_y \to -\hat{e}_y$ reflection

$$\overline{F_2} = -\frac{q^2}{16\pi\varepsilon_0 d^2}\left(1 - \frac{1}{2\sqrt{2}}\right)(\hat{e}_y + \hat{e}_z)$$

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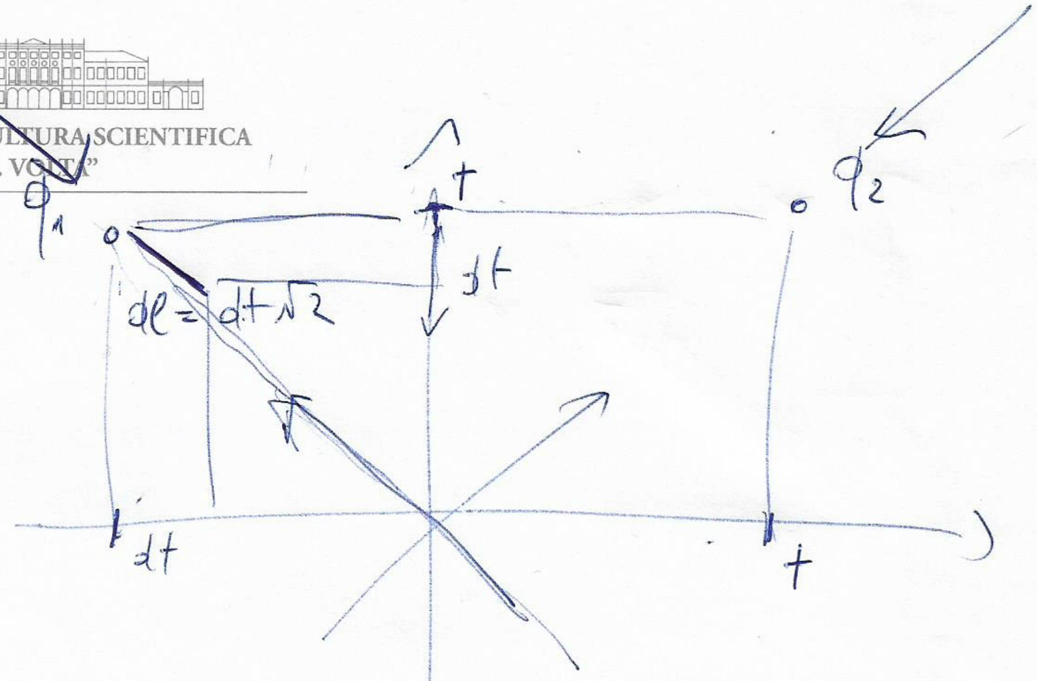
d)



$+\infty \Rightarrow t \to d$

$$dL = \overline{F}_1 \cdot d\overline{l}_1$$

$$\overline{F}_1 = \frac{q^2}{16\pi\varepsilon_0 d^2}(2\sqrt{2}-1)\underbrace{\left(\frac{\hat{e}_y - \hat{e}_z}{\sqrt{2}}\right)}_{\text{Versore}}$$

$$d\overline{l}_1 = dl_1\left(\frac{\hat{e}_y - \hat{e}_z}{\sqrt{2}}\right) \qquad dl = \sqrt{2}\,dt$$

$$\overline{F}_1 \cdot d\overline{l}_1 = \frac{q^2}{16\pi\varepsilon_0 t^2}(2\sqrt{2}-1)\left(\frac{\hat{e}_y - \hat{e}_z}{\sqrt{2}}\right) \cdot \sqrt{2}\,dt\left(\frac{\hat{e}_y - \hat{e}_z}{\sqrt{2}}\right) =$$

$$= \frac{q^2}{16\pi\varepsilon_0}\,\frac{2\sqrt{2}-1}{2}\,\sqrt{2}\,dt$$

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$$\mathcal{L}_1 = \int_{+\infty}^{d} \vec{F}_1 \cdot d\vec{l}_1 = \frac{q^2}{32\pi\varepsilon_0}(4-\sqrt{2})\int_{+\infty}^{d}\frac{dr}{r^2} = \frac{q^2(4-\sqrt{2})}{32\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{+\infty}^{d}$$

$$\Rightarrow \boxed{\mathcal{L}_1 = \frac{q^2}{32\pi\varepsilon_0}\,\frac{4-\sqrt{2}}{d}}$$

$$\mathcal{L}_2 = \mathcal{L}_1$$

$$U_{final} = -(\mathcal{L}_1+\mathcal{L}_2) = -\frac{q^2}{4\pi\varepsilon_0}\,\frac{4-\sqrt{2}}{2}\,\frac{1}{2d}$$

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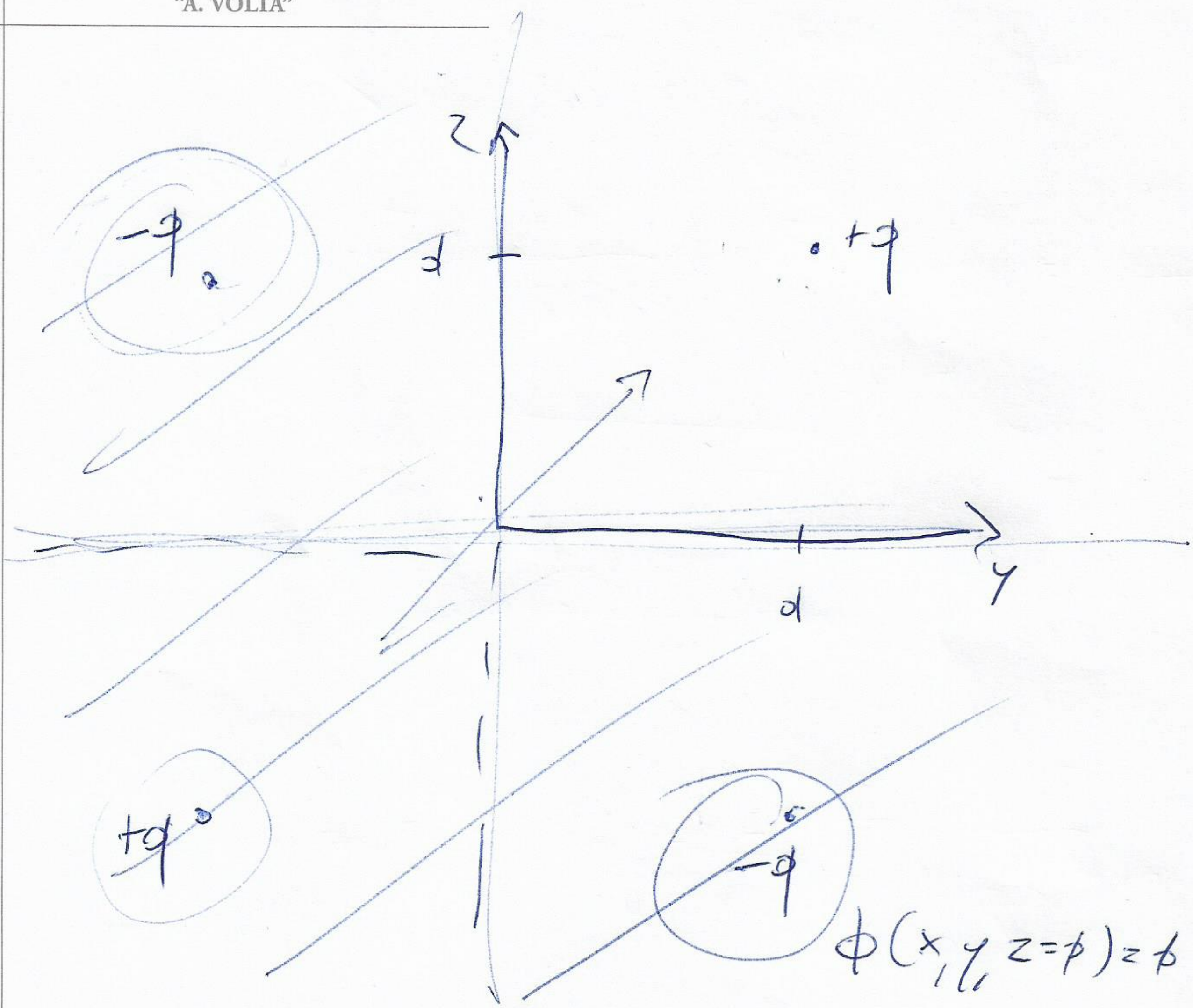

$$\phi(x, y, z=0) = 0$$

$$\phi(x, y=0, z) = 0$$

$\bar{F}_2$ as before

$\mathcal{L}_2$ = as before

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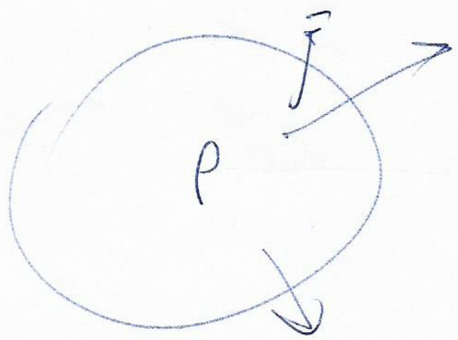
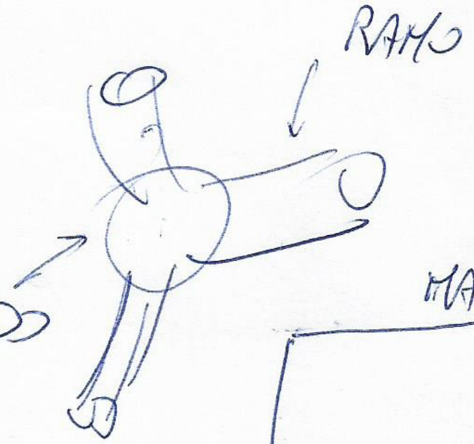
I [A] $\bar{J} = nq\bar{v}_d$ $[A/m^2]$

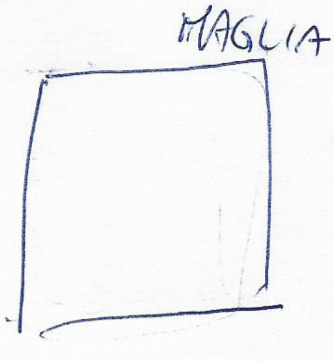
$$\frac{\partial \rho}{\partial t} + \bar{\nabla}\cdot\bar{J} = \emptyset$$ eq. continuita'



I L. KIRCHHOFF



$$\sum_i I_i = \emptyset$$ (NODI)

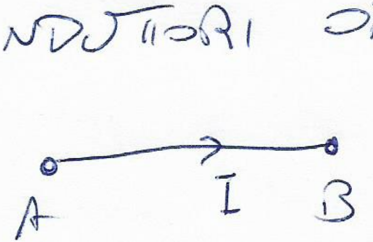


II L. KIRCHHOFF (MAGLIE)

$\bar{E}_s$ conservativo

$$\sum_i V_i = \emptyset$$

CONDUTTORI OHMICI



$$\Delta V = V_A - V_B = RI$$

$[R] = [\Omega]$ Ohm

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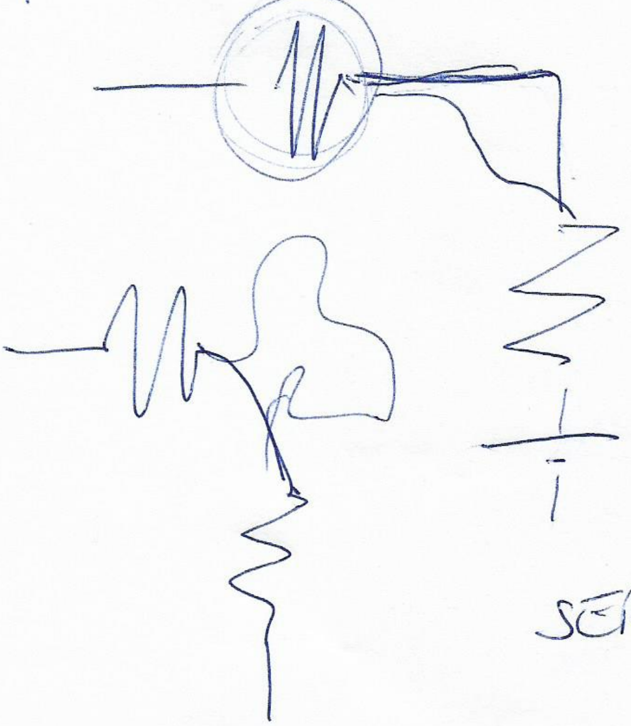
$\frac{1}{R} = G$ conduttanza $[\Omega^{-1}] = [S]$

$\boxed{\Delta V = RI}$ I L. OHM

$R = \rho \frac{l}{S} = \frac{1}{\sigma} \frac{l}{S}$ II L. OHM

resistività conducibilità

parametri concentrati



R_1 R_2 ...

SERIE

$R_{eq} = \sum_i R_i$

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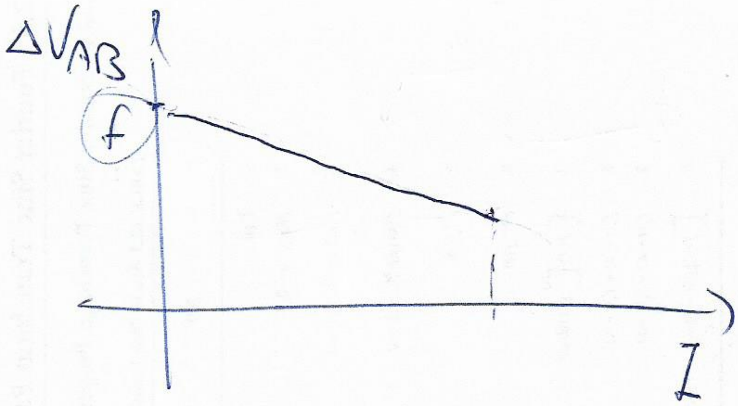
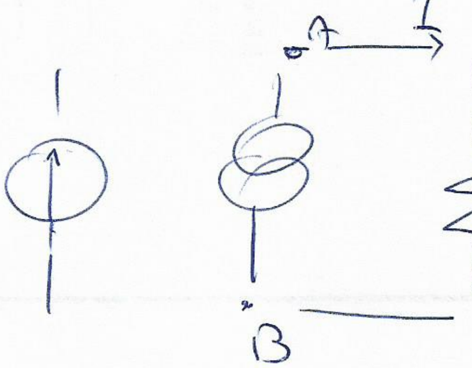


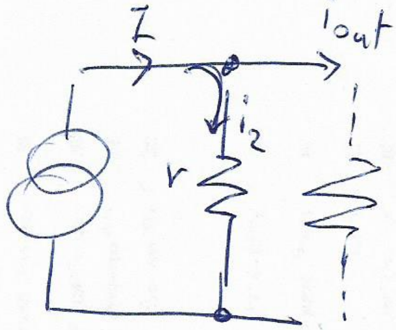
G di corrente



$\Delta V_{AB} = f = RI$

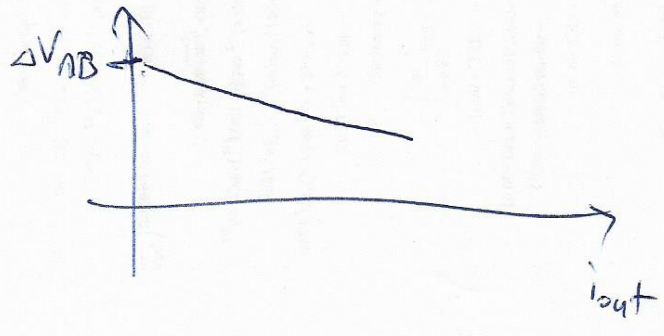
ideale

Reale

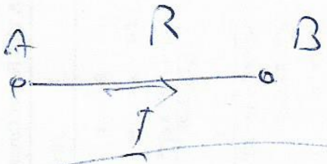


$i_{out} = I - i_2 = I - \frac{\Delta V_{AB}}{r}$

$\Delta V_{AB} = r\left[I - i_{out}\right]$



Lavoro - energia - potenza



$$d\mathcal{L} = dq\,(V_A - V_B) = \Delta V\, I\, dt$$

$$P = W = \frac{d\mathcal{L}}{dt} = I\Delta V = RI^2 = \frac{\Delta V^2}{R}$$

pot. ohmica dissipata sul conduttore
(effetto Joule) → stufa

$$d\mathcal{L} = n\,d\tau\, q\, \bar{E}\cdot \bar{dl} = n\,d\tau\, q\, \bar{E}\cdot \bar{v}_d\, dt$$

$d\tau$

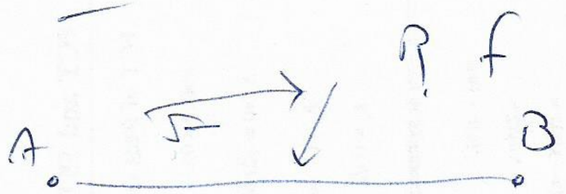
$$\bar{dl} = \bar{v}_d\, dt$$

$$\frac{1}{d\tau}\frac{d\mathcal{L}}{dt} = \frac{W}{d\tau} = w = \overbrace{qn\bar{v}_d}^{\bar{J}}\cdot \bar{E} = \bar{E}\cdot \bar{J}$$

densità di potenza

$$\boxed{w = \bar{E}\cdot \bar{J}}$$

II L. KIRCHHOFF GENERALIZZATA



$$\int_A^B \bar{E}_s \cdot d\bar{l} + \underbrace{\int_A^B \bar{E}_e \cdot d\bar{l}}_{\downarrow} = \int_A^B \rho \bar{J} \cdot d\bar{l}$$

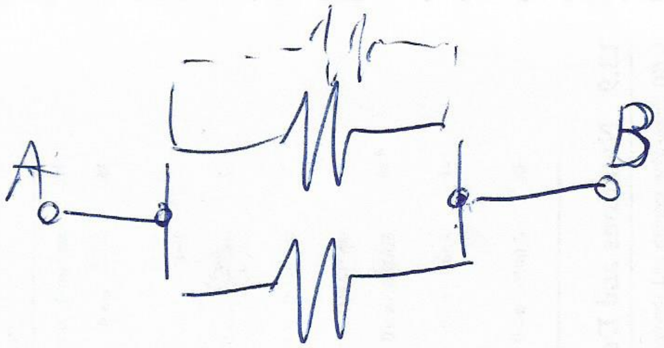
J unif.

$$V_A - V_B + \sum_{\substack{i \\ A \to B}} f_i = \int_A^B \rho \bar{J} \cdot d\bar{l} = I \underbrace{\int_A^B \rho \frac{dl}{S}}_{R_{AB}} = I R_{AB}$$

$$\Delta V = RI$$

Maglia $A = B$

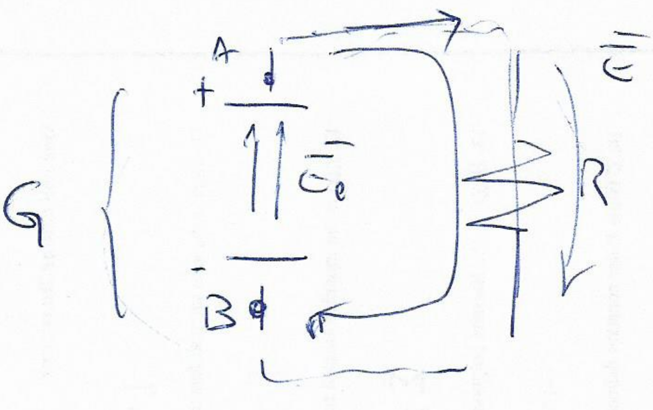
$$\boxed{\sum_i f_i = \sum_j R_j I_j}$$ II K. GENER.



Parallelo $\quad \frac{1}{R_{eq\,//}} = \sum_i 1/R_i$

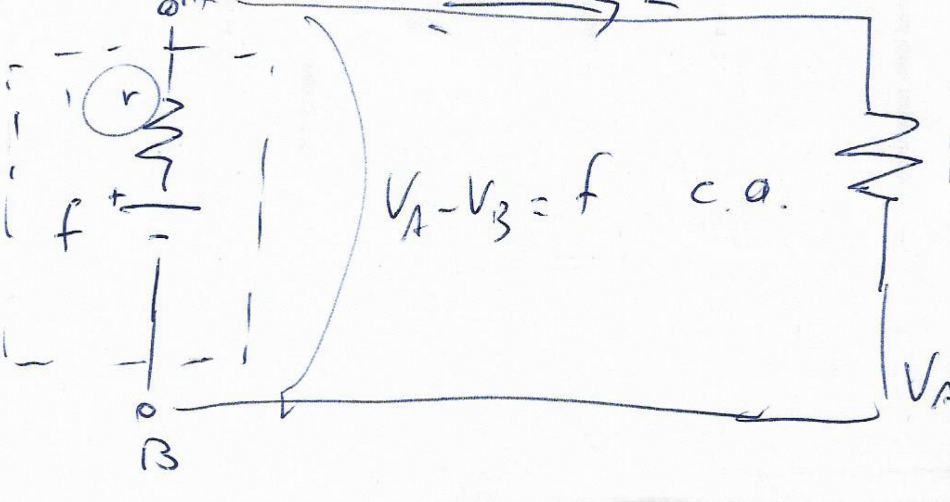
Generatori

$$f_e = \int_B^A \bar{E}_e \cdot d\bar{l} = V_A - V_B \text{ a circuito aperto}$$



ideale

$$V_A - V_B = f = RI$$



$V_A - V_B = f$ c.a.

$V_A - V_B$

$$f = rI + RI$$

$$V_A - V_B = f - rI$$