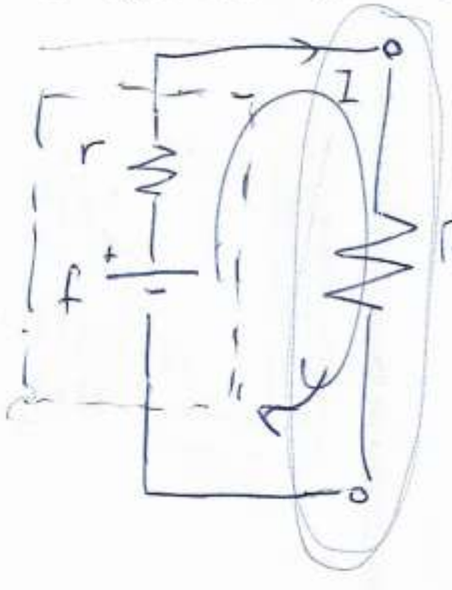
ESERCITAZIONE 12/03/2020

CIRCUITI IN REGIME STAZIONARIO



carico (load)

$$\sum_i f_i = \sum_j R_j I_j$$

$$f = rI + RI = (r+R)I$$

$$I = f/(r+R)$$

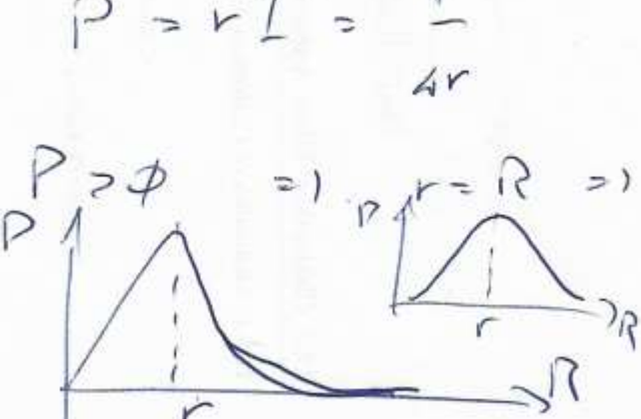
$$P = RI^2 = R\left(\frac{f}{r+R}\right)^2 > \phi$$

$$\frac{dP}{dR} = f^2 \cdot \frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} = \dots = \frac{r^2-R^2}{(r+R)^4} =$$

$$= \frac{r-R}{(r+R)^3} = \phi \qquad R = r$$

$$P = rI^2 = \frac{f^2}{4r}$$

$\lim_{\substack{R\to\phi \\ R\to+\infty}} P = \phi$, $P > \phi \Rightarrow r = R \Rightarrow \max P$

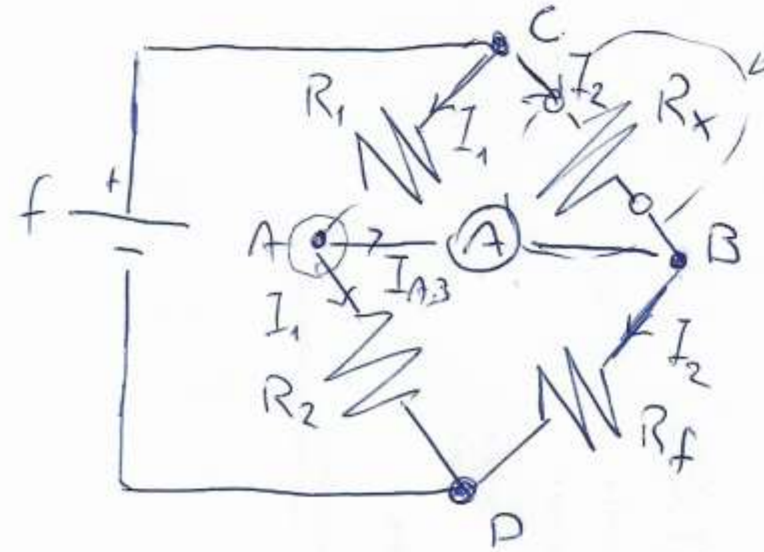


$$\eta = \frac{P_R}{P_T} = \frac{P_R}{P_R + P_r} = \frac{RI^2}{(r+R)I^2} = \frac{R}{r+R}$$

$\cancel{\eta = 1}$ $\quad R \to +\infty \iff P_R = \emptyset$

$P_{R\,max} \iff R = r \iff \eta = 50\%$

Ponte di Wheatstone R_x incognita



R_1, R_2 variabili

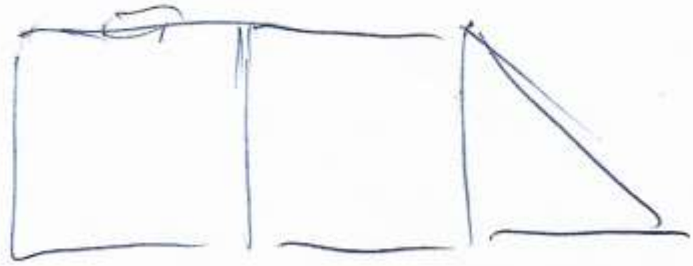
R_f fissa

$I_{AB} = \phi \qquad \Delta V_{AB} = V_A - V_B = \phi$

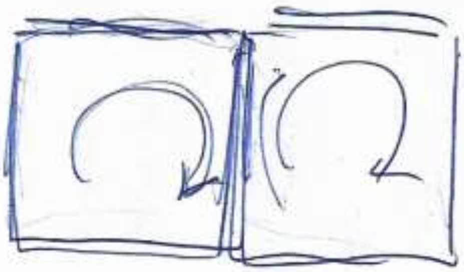
$V_C - V_A = V_C - V_B \longrightarrow R_1 I_1 = R_x I_2$

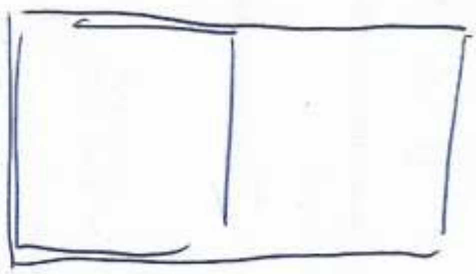
$V_A - V_D = V_B - V_D \qquad R_2 I_1 = R_f I_2$

$$\frac{R_1}{R_2} = \frac{R_x}{R_f} \Rightarrow \boxed{R_x = \frac{R_1}{R_2} R_f}$$



(1) # maglie indipendenti = # rami − # nodi + 1

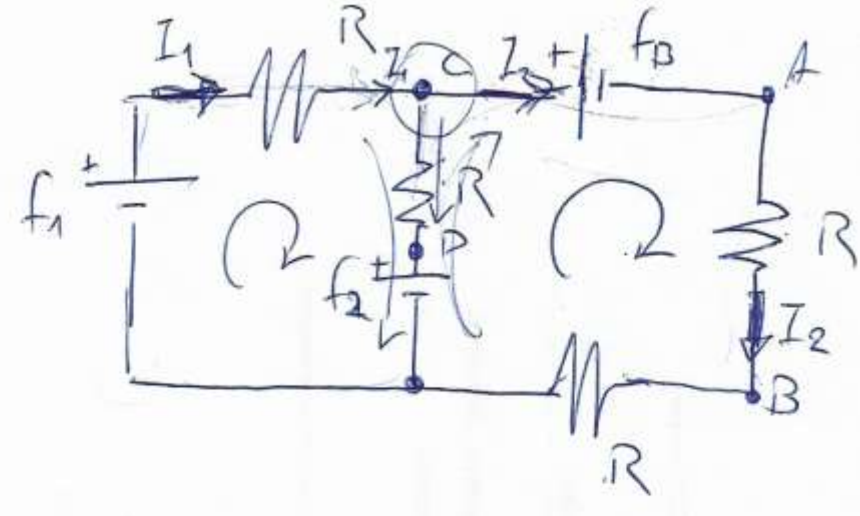




(2) Assegnare versi alle correnti di maglia

(3) Kirchhoff eq. alle maglie (+ eq. ai nodi)

(4) Risolvere → correnti...



$\Delta V_{AB} = ?$
$\Delta V_{CD} = ?$

$f_1 = 12\,V$
$f_2 = 3\,V$
$f_3 = 5\,V$
$R = 1\,k\Omega$

$$\begin{cases} f_1 - f_2 = RI_1 + R(I_1 - I_2) \\ f_2 - f_3 = R(I_2 - I_1) + 2RI_2 \end{cases}$$

$$\begin{cases} f_1 - f_2 = 2RI_1 - RI_2 & [1] \\ f_2 - f_3 = -RI_1 + 3RI_2 & [2] \end{cases}$$

$[1] + 2[2]$ $\quad f_1 + f_2 - 2f_3 = 5RI_2 \Rightarrow I_2 = \dfrac{f_1 + f_2 - 2f_3}{5R} =$

$= 2.2 \cdot 10^{-3} A$

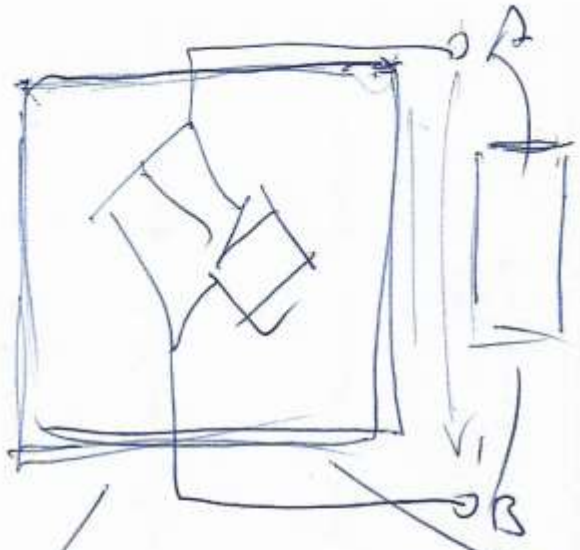
$3 \cdot [1] + [2]$

$$3f_1 - 2f_2 - f_3 = 5RI_1 \Rightarrow I_1 = \frac{3f_1 - 2f_2 - f_3}{5R} = 2.6 \cdot 10^{-3} A$$

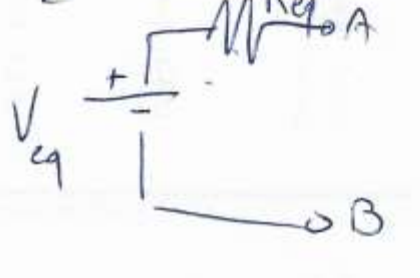
$\Delta V_{AB} = RI_2 = 2.2\,V$

$\Delta V_{CD} = R(I_1 - I_2) = 0.4\,V$

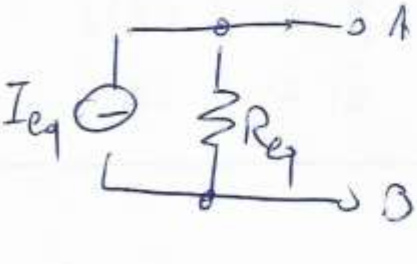
Thévenin e Norton



Th.



Norton



- V_{AB} c.a. $= V_{eq}$
- I_{AB} c.c. $A - B$

$\Updownarrow$

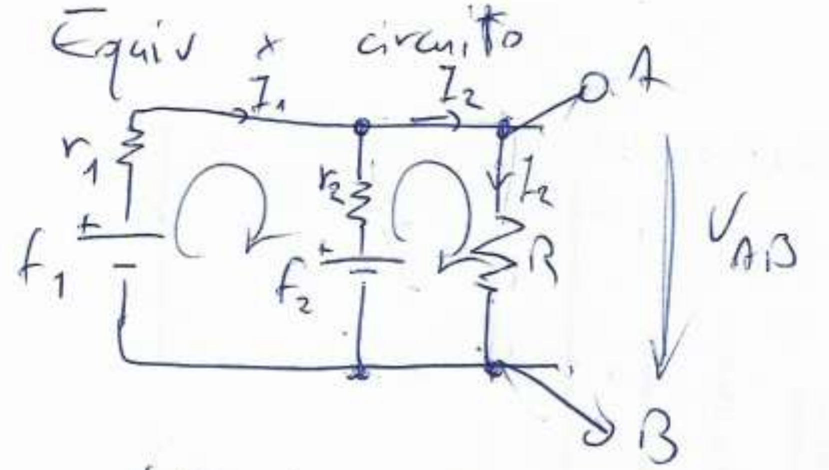
R_{eq} a generatori disattivati

- $V_{gen} \to$ c.c.
  $I_{gen} \to$ c.a.

$\longrightarrow \quad I_{eq} = \dfrac{V_{eq}}{R_{eq}}$

$R_{eq\,Norton} = R_{eq\,Th}$

Equiv x circuito



$f_1 = 9V$

$f_2 = 12V$

$r_1 = 5\Omega$

$r_2 = 3\Omega$

$R = 150\Omega$

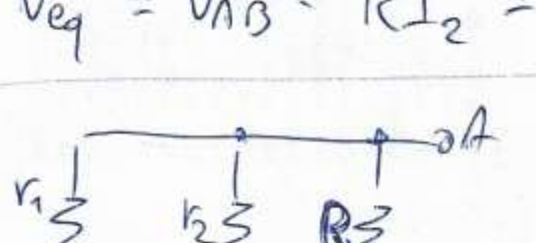
$$\begin{cases} f_1 - f_2 = r_1 I_1 + r_2 (I_1 - I_2) \\ f_2 = r_2 (I_2 - I_1) + R I_2 \end{cases}$$

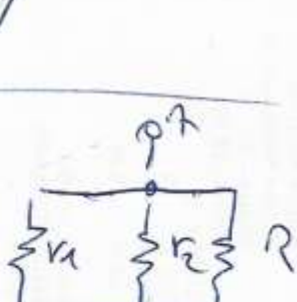
$$f_1 = r_1 I_1 + R I_2 \Rightarrow I_1 = \frac{f_1 - R I_2}{r_1}$$

$$f_2 = -r_2 \frac{f_1 - R I_2}{r_1} + (r_2 + R) I_2$$

$$\boxed{I_2 = \frac{r_2 f_1 + r_1 f_2}{r_1 r_2 + r_1 R + r_2 R}} = 71.6\,mA$$

$$V_{eq} = V_{AB} = R I_2 = 10.74\,V$$

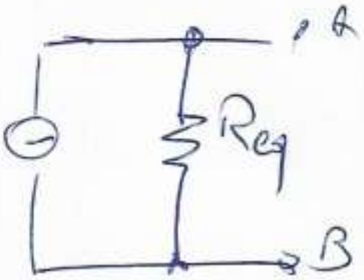




$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R} \Rightarrow$$

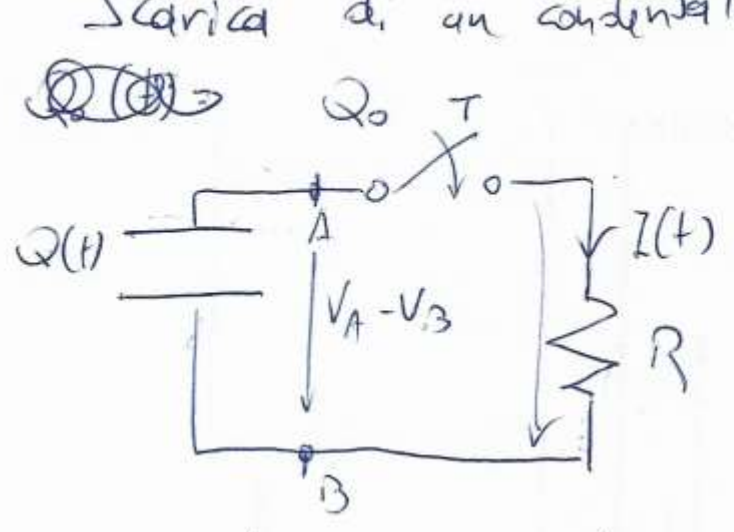
$$R_{eq} = \frac{r_1 r_2 R}{r_1 r_2 + r_1 R + r_2 R} = 1.85\ \Omega$$

$I_{eq} = \frac{V_{eq}}{R_{eq}}$



$$= \frac{r_2 f_1 + r_1 f_2}{r_1 r_2} = \frac{f_1}{r_1} + \frac{f_2}{r_2}$$

Scarica di un condensatore



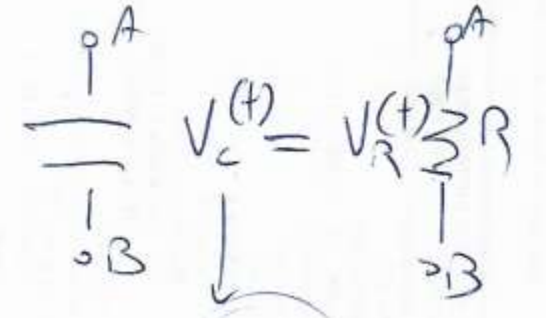
T chiuso $t = \phi$

$$\begin{cases} Q(t=0) = Q_0 \\ V_0 = Q_0/C \\ I_0 = V_0/R \end{cases}$$

$t \to +\infty$

$V_\infty = \dfrac{Q_\infty}{C} = \phi$

$I_\infty = \phi$



$$\frac{Q(t)}{C} = I(t)\,R \qquad I(t) = \frac{dq}{dt}$$

$$\cancel{Q = q} \qquad Q = -q$$

$$I(t) = -\frac{dQ(t)}{dt}$$

$$\frac{Q(t)}{C} = -\frac{dQ(t)}{dt}\,R$$

$$\frac{1}{RC}\int dt = -\int \frac{dQ}{Q}$$

$$\frac{1}{RC}\,t = -\log Q(t) + \log A = -\log\left(\frac{Q(t)}{A}\right)$$

$$Q(t) = A e^{-t/RC}$$

$$Q(t=\phi) = \boxed{Q_0 = A} \Rightarrow Q(t) = Q_0 e^{-t/RC}$$

$$\tau = RC \longrightarrow 100\ pF/m \qquad 6+2-12$$

$$1\ M\Omega \qquad 10^6 \cdot 10^2 \cdot 10^{-12} = 10^{-4}\ s$$

$$c \approx 10^8\ m/s \qquad 10\ ns$$

$$I(t) = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

$$U_j = \int_\phi^{+\infty} R I^2(t)\,dt = \frac{Q_0^2}{RC} \int_\phi^{+\infty} e^{-2t/RC}\,dt = -\frac{Q_0^2}{RC} \frac{RC}{2} \left[e^{-2t/RC}\right]_\phi^{+\infty} =$$

$$\phi - 1$$

$$= \frac{1}{2} \frac{Q_0^2}{C} = U_{0c}$$

$$U_j(\phi - t^*) = \frac{1}{2C} \left[Q_0^2 - Q^2(t^*)\right]$$

$$U_c(t^*) = \frac{Q^2(t^*)}{2C}$$

Carica

$t = \phi$ chiude circuito

$$f = V_c(t) + RI(t) = \frac{Q(t)}{C} + R\frac{dQ(t)}{dt}$$

$$I = \frac{dq}{dt} = \frac{dQ}{dt}$$

$$fC - Q(t) = RC\frac{dQ}{dt} \rightsquigarrow \boxed{\frac{dt}{RC} = \frac{dQ}{fC - Q(t)}}$$

$t = \phi$	$t = \infty$
$Q_0 = \phi$	$Q_\infty = fC$
$V_0 = \phi$	$V_\infty = f$
$I_0 = f/R$	$I_\infty = \phi$

$$y \doteq fC - Q$$

$$dy = -dQ$$

$$\frac{dt}{RC} = -\frac{dy}{y} \Rightarrow y(t) = Ae^{-t/RC}$$

$$\tau = RC$$

$$fC - Q(t) = Ae^{-t/RC}$$

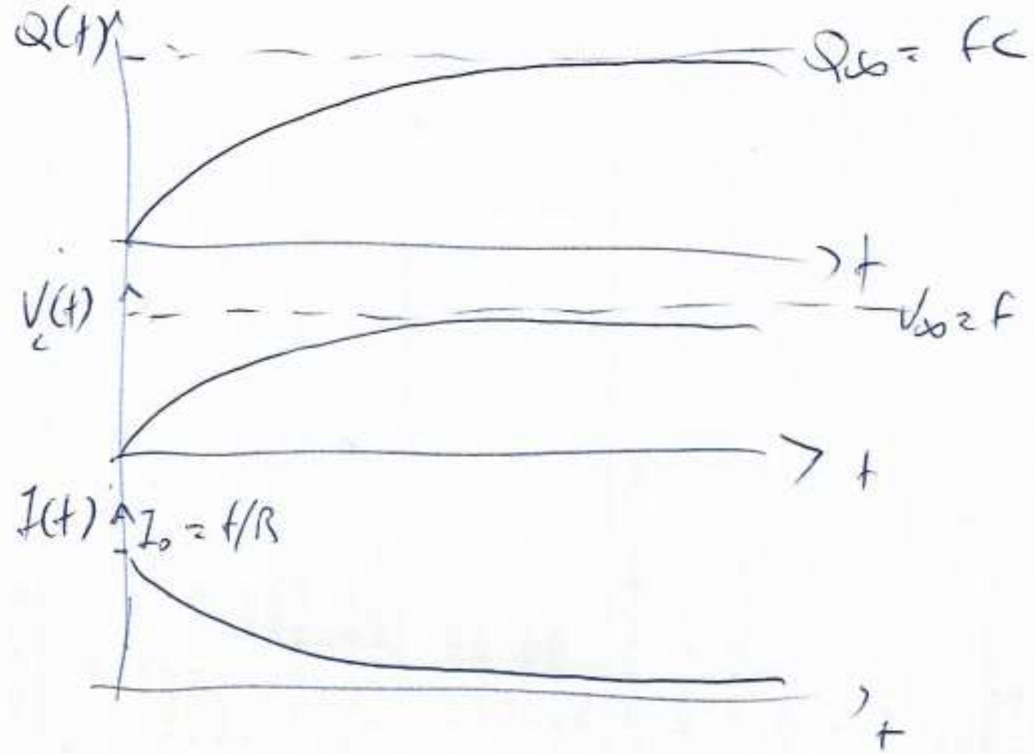
$$Q(t) = fC - Ae^{-t/RC}$$

c.i. $Q_0 = \phi = fC - A \Rightarrow A = fC$

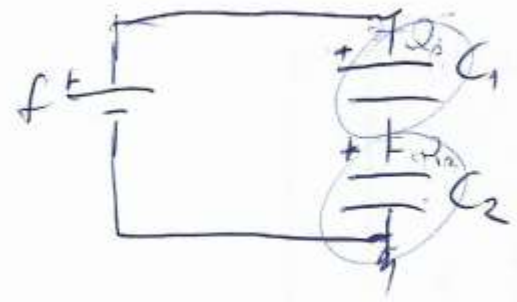
$$Q(t) = Q_\infty(1 - e^{-t/RC}) = fC(1 - e^{-t/RC})$$

$$V_c(t) = \frac{Q(t)}{C} = f(1 - e^{-t/RC})$$

$$I(t) = \frac{dQ}{dt} = fC\frac{1}{RC}e^{-t/RC} = \frac{f}{R}e^{-t/RC} = I_0 e^{-t/RC}$$



Q(t)
Q∞ = εC
t
V_c(t)
V∞ = ε
t
I(t)
I_0 = ε/R
t



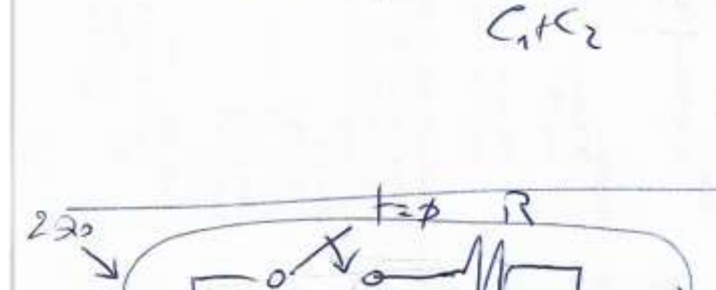
$$Q_0 = V_1 C_1 = V_2 C_2$$

$$f = V_1 + V_2 = Q_0 \left(\frac{1}{C_1} + \frac{1}{C_2} \right) =$$

$$= Q_0 \cdot \frac{1}{C_s} = Q_0 \frac{C_1 + C_2}{C_1 C_2}$$

$$Q_0 = f \frac{C_1 C_2}{C_1 + C_2} = f C_s$$

$$U_0 = \frac{1}{2} \frac{Q_0^2}{C_s}$$



$$V_{10} = V_1(t=0) = \frac{Q_0}{C_1}$$

$$V_{20} = V_2(t=0) = \frac{Q_0}{C_2}$$

$$V_1(t) - V_2(t) = R\, i(t)$$

$$\frac{Q_1(t)}{C_1} - \frac{Q_2(t)}{C_2} = R\, i(t) \leftarrow \frac{d}{dt}$$

$$i(t) = -\frac{dQ_1(t)}{dt} = \frac{dQ_2(t)}{dt}$$

$$\frac{1}{C_1} \frac{dQ_1}{dt} - \frac{1}{C_2} \frac{dQ_2}{dt} = R \frac{di}{dt}$$

$$-\frac{i(t)}{C_1} - \frac{i(t)}{C_2} = R\frac{di(t)}{dt}$$

$$-\frac{1}{RC_s}\, i(t) = \frac{di(t)}{dt} \rightarrow i(t) = A e^{-t/RC_s}$$

$$R\, i(0) = \left(\frac{1}{C_1} - \frac{1}{C_2}\right) Q_0 = V_{10} - V_{20} = RA$$

$$A = \frac{V_{10} - V_{20}}{R} \quad (= I_0)$$

$$i(t) = \frac{V_{10} - V_{20}}{R}\, e^{-t/RC_s}$$

$$\frac{dQ_2(t)}{dt} = i(t) \qquad Q_2(t) = \int_0^t i(t')\,dt' = -\frac{V_{10}-V_{20}}{R} RC_s\, e^{-t/RC_s} + K$$

$$Q_2(t=0) = -C_s(V_{10} - V_{20}) + K = Q_0$$

$$\Rightarrow K = Q_0 + C_s(V_{10} - V_{20})$$

$$\Rightarrow Q_2(t) = Q_0 + C_s(V_{10} - V_{20})\left(1 - e^{-t/RC_s}\right)$$

$$Q_{2\infty} = Q_0 + C_s(\underset{\uparrow\, Q_0/C_1}{V_{10}} - \underset{\nwarrow\, Q_0/C_2}{V_{20}}) = 2Q_0 \frac{C_2}{C_1 + C_2}$$

$$Q_1(t) + Q_2(t) = Q_{10} + Q_{20} = \underline{\underline{2Q_0}}$$

$$Q_1(t) = 2Q_0 - Q_2(t)$$

$$Q_1(t) = Q_0 - C_1(V_{10} - V_{20})\left(1 - e^{-t/RC_1}\right)$$

$$Q_{1\infty} = \dots = 2Q_0 \frac{C_1}{C_1 + C_2}$$

$$V_1(t) = \frac{Q_1(t)}{C_1} \qquad V_{1\infty} = \frac{Q_{1\infty}}{C_1} = \frac{2Q_0}{C_1 + C_2}$$

$$V_2(t) = \frac{Q_2(t)}{C_2} \qquad V_{2\infty} = \frac{Q_{2\infty}}{C_2} = \frac{2Q_0}{C_1 + C_2}$$

$$U_f = \frac{1}{2}\frac{(2Q_0)^2}{C_1} = \frac{1}{2}\frac{4Q_0^2}{C_1 + C_2} = \frac{2Q_0^2}{C_1 + C_2} < U_0$$