ESERCITAZIONE 16/11/2020

CAMPO $\bar B$ — FORZA DI LORENTZ

$$\bar F_L = q\,\bar v \times \bar B$$

$$\bar B' = \bar B + \delta \bar B \quad / \quad \delta \bar B \parallel \bar v$$

$$\bar F_L(\bar B) = \bar F_L(\bar B')$$

$\bar F_{L1}$ $\bar F_{L2}$

$q\ \bar v_1$ $\quad q\ \bar v_2 \quad \bar v_1 \perp \bar v_2$

$$\begin{cases} \bar F_1 = q\,\bar v_1 \times \bar B & \leftarrow \times\, \bar v_1 \ / q v_1^2 \\ \bar F_2 = q\,\bar v_2 \times \bar B & \leftarrow \times\, \bar v_2 \ / q v_2^2 \end{cases}$$

$$(\bar a \times \bar b) \times \bar c = (\bar a \cdot \bar c)\,\bar b - (\bar b \cdot \bar c)\,\bar a$$

$$(\bar v_1 \times \bar B) \times \bar v_1 = (\bar v_1 \cdot \bar v_1)\,\bar B - (\bar B \cdot \bar v_1)\,\bar v_1 = v_1^2\,\bar B - (\bar B \cdot \bar v_1)\,\bar v_1$$

$$\begin{cases} \dfrac{\bar F_1 \times \bar v_1}{q v_1^2} = \bar B - \dfrac{\bar B \cdot \bar v_1}{v_1^2}\,\bar v_1 \\[2ex] \dfrac{\bar F_2 \times \bar v_2}{q v_2^2} = \bar B - \dfrac{\bar B \cdot \bar v_2}{v_2^2}\,\bar v_2 & \leftarrow \cdot\, \bar v_1 \end{cases}$$

$\bar B \cdot \bar v_1$ $\qquad \bar v_1 \cdot \bar v_2 = \emptyset$

$\Rightarrow$

$$\bar{B} = \frac{\bar{F}_1 \times \bar{v}_1}{q v_2^2} \cdot \bar{v}_1 + \frac{\bar{B} \cdot \bar{v}_2}{v_2^2} \underbrace{\bar{v}_2 \cdot \bar{v}_1}_{\to \emptyset}$$

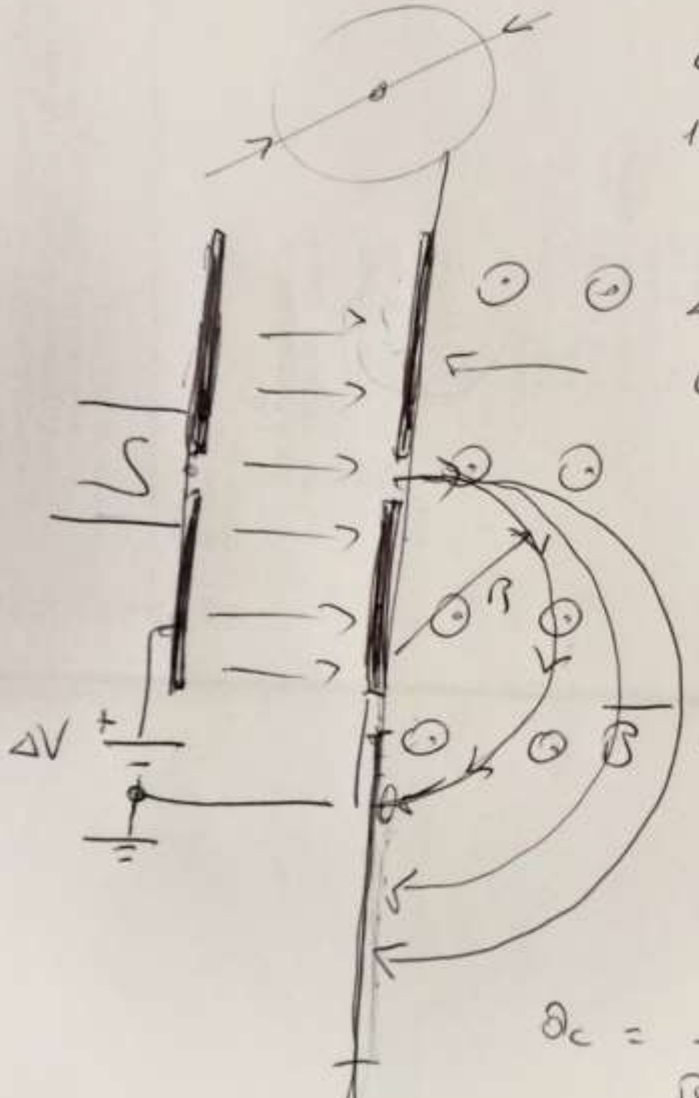
$$\bar{B} = \frac{\bar{F}_1 \times \bar{v}_1}{q v_1^2} + \left(\frac{(\bar{F} \times \bar{v}_2) \cdot \bar{v}_1}{q v_2^2} \right) \frac{\bar{v}_1}{v_1^2}$$

$$\boxed{\bar{B} = \frac{1}{q v_1^2} \left[\bar{F}_1 \times \bar{v}_1 + \frac{(\bar{F}_2 \times \bar{v}_2) \cdot \bar{v}_1}{v_2^2} \bar{v}_1 \right]}$$

Spettrometro di Dempster (1918)

Fascio collimato e monoenergetico

$\Longrightarrow$



cocktail beam

$^{12}C^+,\ ^{13}C^+,\ ^{14}C^+$

$\Delta V = 1\,kV$

$E_k = q\Delta V = 1\,keV$

$\bar{B} = B_z \hat{e}_z$

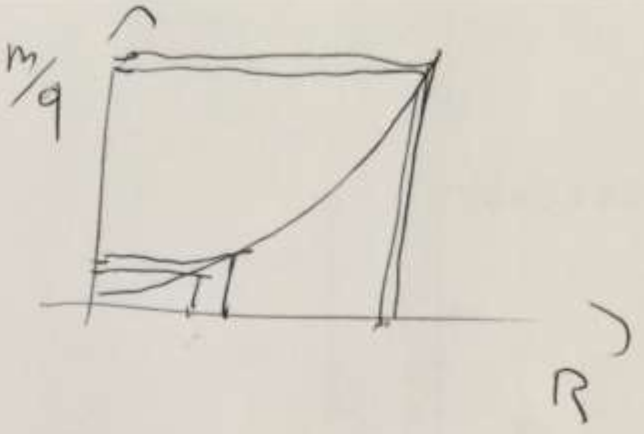
$\bar{F} = q\bar{v} \times \bar{B}$

$$a_c = \frac{v^2}{R} = \frac{F_L}{m} = \frac{qvB}{m}$$

$$\Rightarrow R = \frac{mv}{qB} = \frac{v}{\omega_c} = \frac{m}{qB}\left(\frac{2E_k}{m}\right)^{\frac{1}{2}} = \left(\frac{m}{q}\right)^{\frac{1}{2}}\left(\frac{2\Delta V}{B^2}\right)^{\frac{1}{2}}$$

$\omega_c \doteq \frac{qB}{m}$ (angular) cyclotron frequency

$\nu_c = \omega_c / 2\pi$



$\Delta R \rightsquigarrow \Delta m$

$\bar{m}$ = massa nucleone = $1.67 \cdot 10^{-27}$ kg

A = # di massa (nucleoni)

$$R = \left(\frac{m}{q}\right)^{\frac{1}{2}} \left(\frac{2\Delta V}{B^2}\right)^{\frac{1}{2}} = \sqrt{A}\ 4.57 \cdot 10^{-2}\ m$$

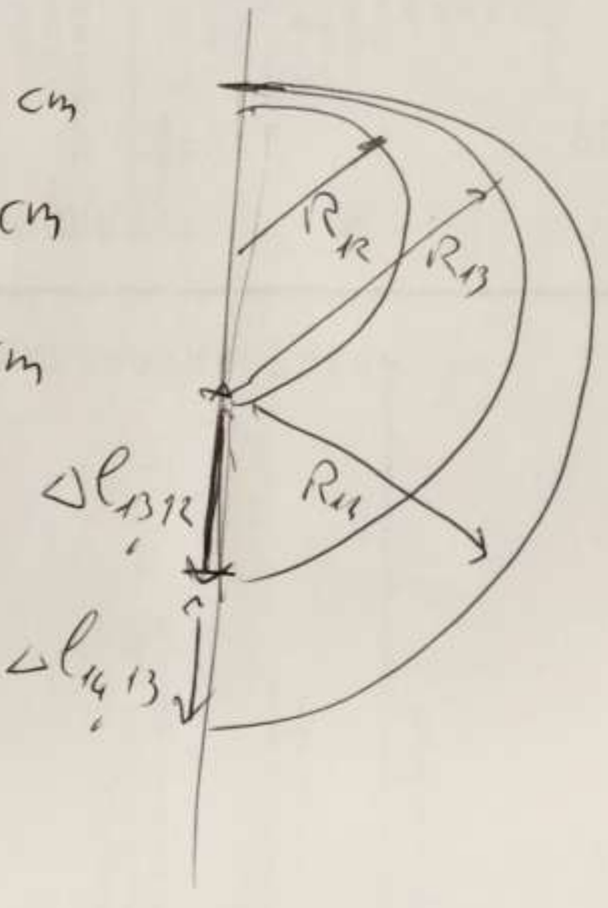
$R(^{12}C^+) = 15.83$ cm

$R(^{13}C^+) = 16.48$ cm

$R(^{14}C^+) = 17.10$ cm

$\Delta \ell_{13,12} = 1.3$ cm

$\Delta \ell_{14,13} = 1.24$ cm

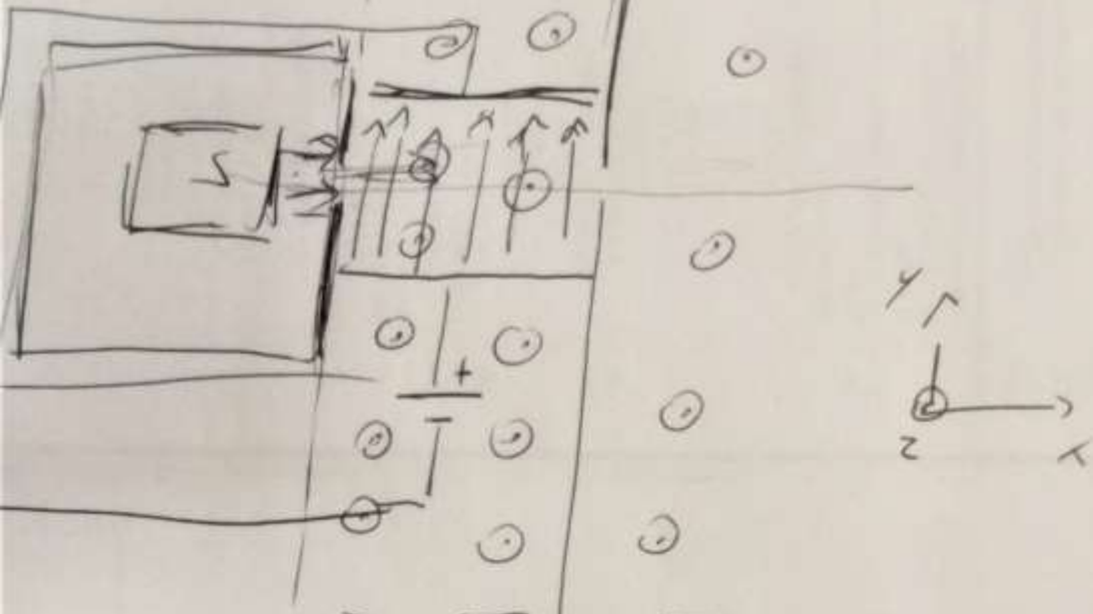


Spettrometro di Bainbridge (1933)

(+) selettore di velocità

Dempster monoenergetico $= \overline{E}_k$

Bainbridge! $= |\bar{v}|$



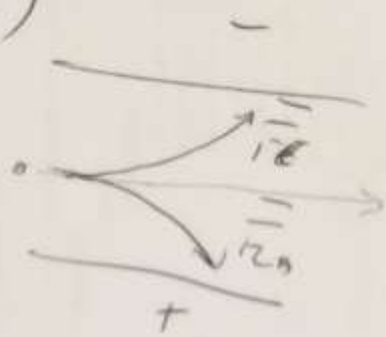
$\overline{E} + \overline{B}$

$\overline{B} = B_z \hat{e}_z$

$\overline{E} = E_y \hat{e}_y$

$\overline{F}_L = q(\overline{E} + \bar{v} \times \overline{B})$

$\bar{v} = v \hat{e}_x$



$F_y = 0 \qquad q E_y - q v_x B_z = 0 \Rightarrow E_y = v_x B_z$

$^{12}C^+, \; ^{13}C^+, \; ^{14}C^+$

$v = 10^5\, m/s \qquad E_y = ? \qquad B_z = 0.1\, T$

$E_y = v_x B_z = 10^4\, V/m$

$\bar{F}_L = q\bar{v} \times \bar{B}$

$a_c = \frac{v^2}{R} = \frac{F_L}{m} = \frac{q v B_z}{m}$

$R = \frac{mv}{qB} = \left(\frac{m}{q}\right)\left(\frac{v}{B}\right)$

$R = A \frac{\bar{m} v}{qB} = A \cdot 1.04 \cdot 10^{-2}\, m$

$R_{12} = 12.48$ cm

$R_{13} = 13.52$ cm

$R_{14} = 14.56$ cm

$\Delta l_{13,12} = \Delta l_{14,13} = 2.08$ cm

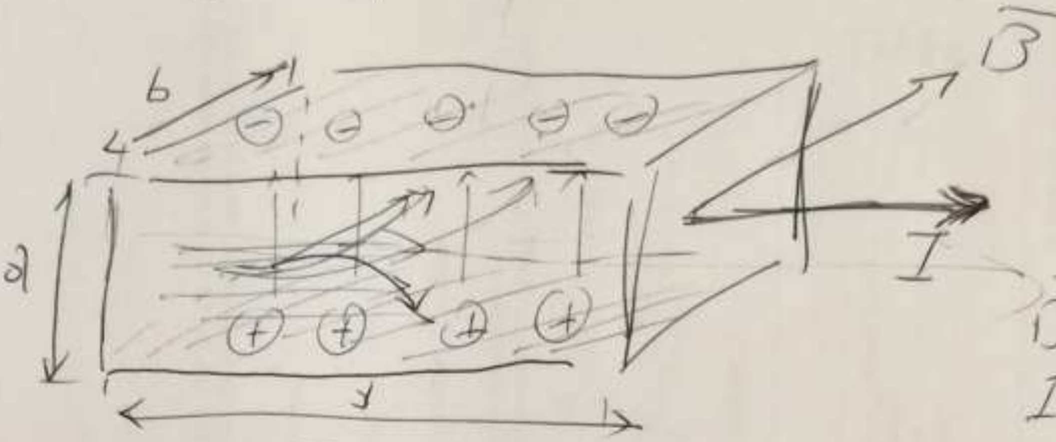
E_K → 624 eV (12)

E_K → 676 eV (13)

E_K → 728 eV (14)

$\frac{1}{2} m v^2$

Effetto Hall



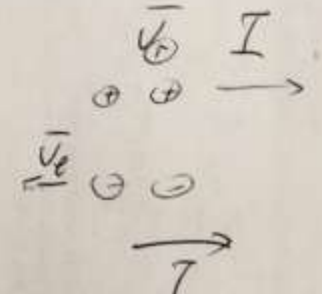
$a = 5\ mm$

$b = 2\ mm$

$d = 30\ mm$

$\bar{B} = 2\ T$

$I = 3\ A$



$$\bar{F}_L = q\bar{v} \times \bar{B}$$

$$\bar{v} \to \qquad q\bar{v} \to$$

$$(-e)(-\bar{v}) \to \qquad e\vec{v}_e \to$$

$\bar{F}_L \uparrow$

$$\bar{F}_L = q\bar{E} + q\bar{v}_d \times \bar{B} \qquad \bar{F}_e = q\bar{E} = -eE\hat{e}_z \downarrow$$

$$\bar{F}_m = q\bar{v}_d \times \bar{B} = e v_d B \hat{e}_z \uparrow$$

$$\bar{F}_e + \bar{F}_m = \emptyset \qquad E = v_d B$$

$$V_H = Ea = v_d B a$$ potenziale di Hall

$$I = Jab = qnv_d ab$$

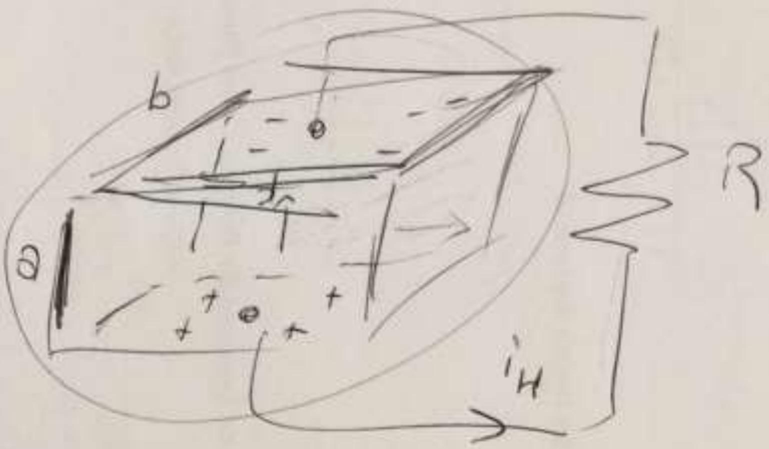
$$V_H = \frac{BI}{qnb} = R_H \frac{BI}{b}$$

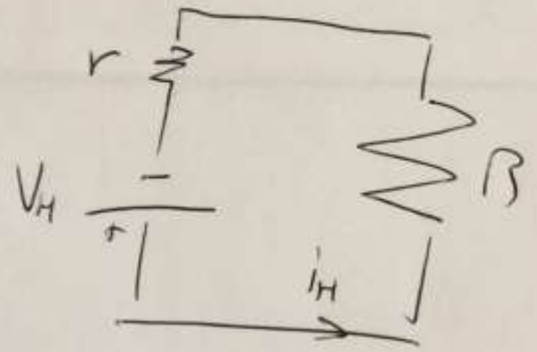
$$R_H \doteq \frac{1}{qn}$$ costante di Hall $[m^3/C]$

$n = 8.5 \cdot 10^{28}\ m^{-3}$ (e^-)

$V_H = R_H\, BI/b = -2.21 \cdot 10^{-7}\ V$

Hall probe





$V_H = (r+R)\, i_H \Rightarrow i_H = \frac{V_H}{r+R}$

$r = \rho \frac{l}{S} = \rho \frac{a}{bd} = 1.39 \cdot 10^{-6}\ \Omega$

$\rho_{Cu} = 1.67 \cdot 10^{-8}\ \Omega \cdot m$

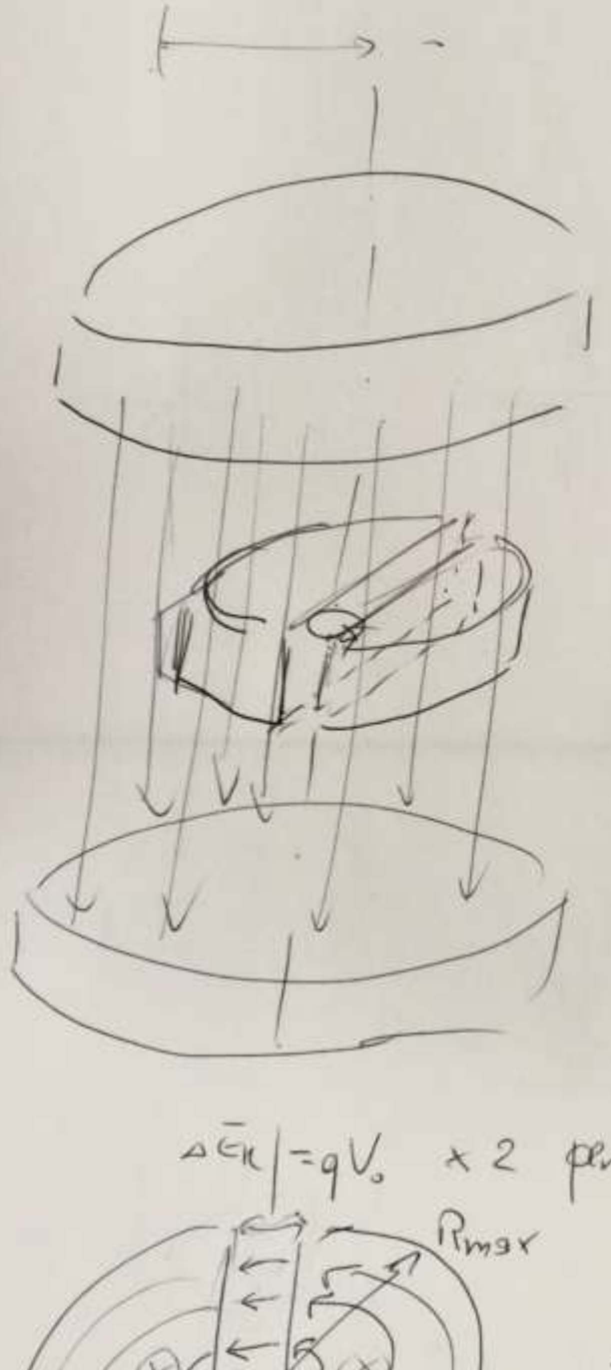
$R = 10^{-4}\ \Omega$

$\Rightarrow i_H = 2.18 \cdot 10^{-3}\ A$

Ciclotrone

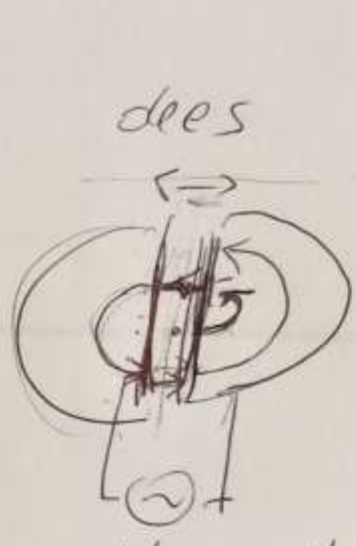
Acceleratore circolare

ADROTERAPIA
(CNAO)



$\bar{B}$

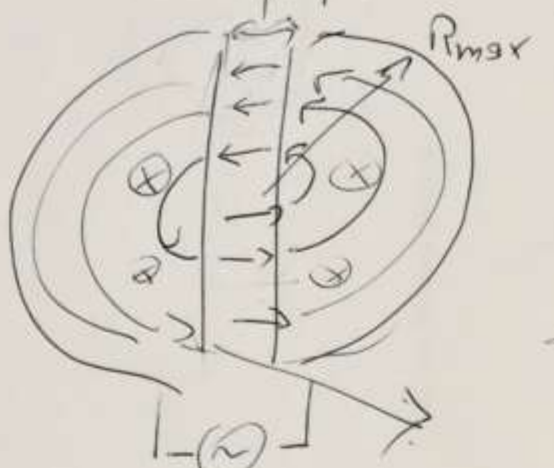
dees



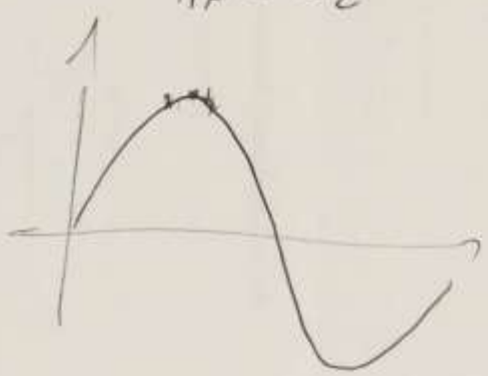
$V_{RF}(t) = V_0\, f(\omega_{RF} t + \varphi)$

$\cancel{\sin(\omega_{RF} t)}$

$|\Delta E_K| = qV_0 \times 2$ per revolution $\quad \sin(\omega_{RF} t + \varphi)$



$\omega_{RF} = \omega_c$



$$R_{max} = \frac{m v_{max}}{qB}$$

$$v_{max} = \frac{qBR_{max}}{m}$$

$$E_{kmax} = \frac{1}{2} m v_{max}^2 = \frac{q^2 B^2 R_{max}^2}{2m} = \boxed{2nqV_0}$$

$$n = \frac{qB^2R_{max}^2}{4mV_0} \qquad n T_c = n \frac{2\pi}{\omega_c} = \frac{\pi q B R_{max}^2}{2V_0}$$

$^{12}C^+$; $R_{max} = 1m$; $B = 1.5\,T$ $V_0 = 10\,kV$

$E_{kmax} = 8.98\,MeV$; $n = 449$ giri

$\nu_{RF} = \nu_c = 1.91\,MHz$; $T_c = 5.25 \cdot 10^{-7}\,s$

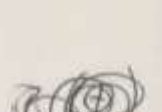
$t_{tot} = nT_c = 2.36 \cdot 10^{-4}\,s$

$v \lll c$

$$\omega_c = \frac{qB}{m} \rightsquigarrow m = \gamma m_0 = \frac{1}{\sqrt{1-\beta^2}} m_0$$

$\beta \doteq v/c$

$\omega_c = qB/\gamma m_0$

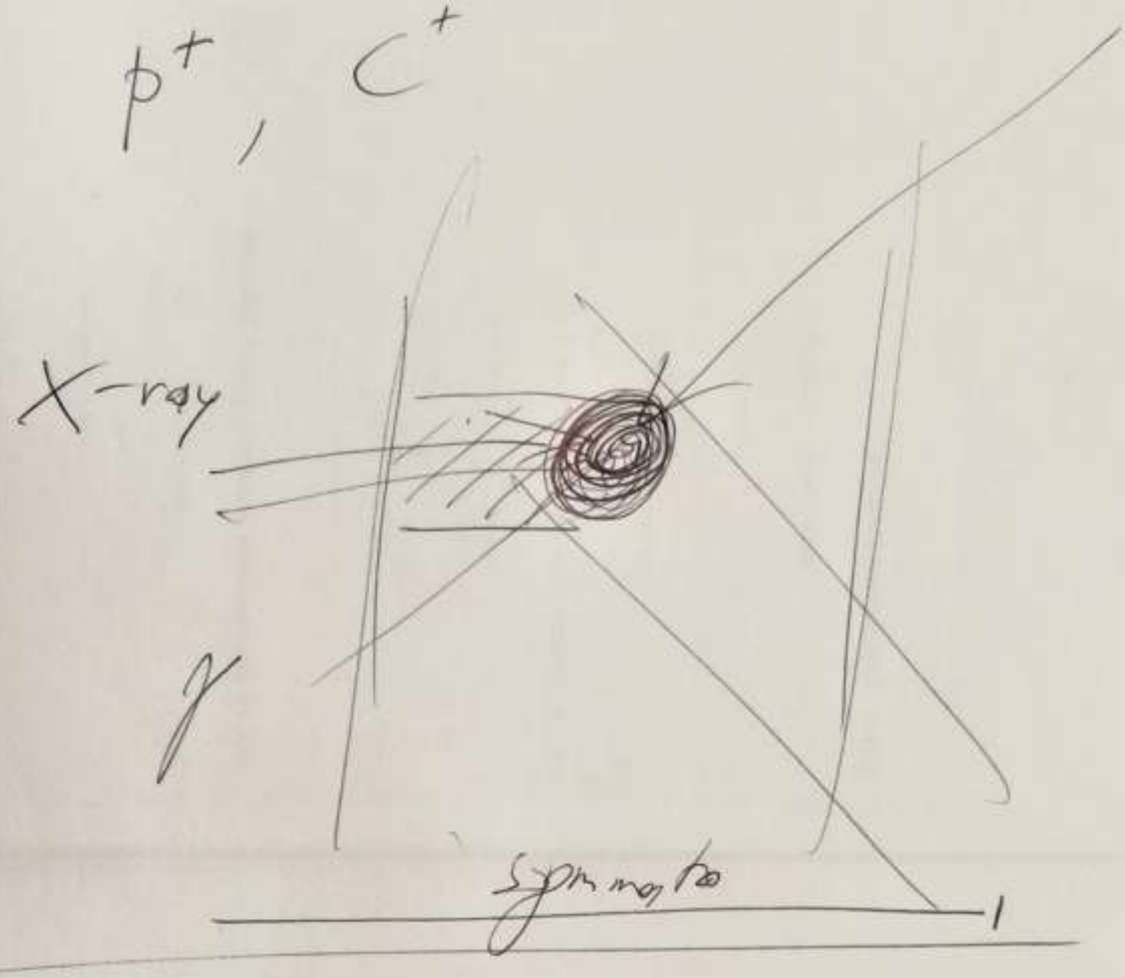


$\nearrow V_{RF}(t)\ (\omega_{RF})$ SINCROCICLOTRONE

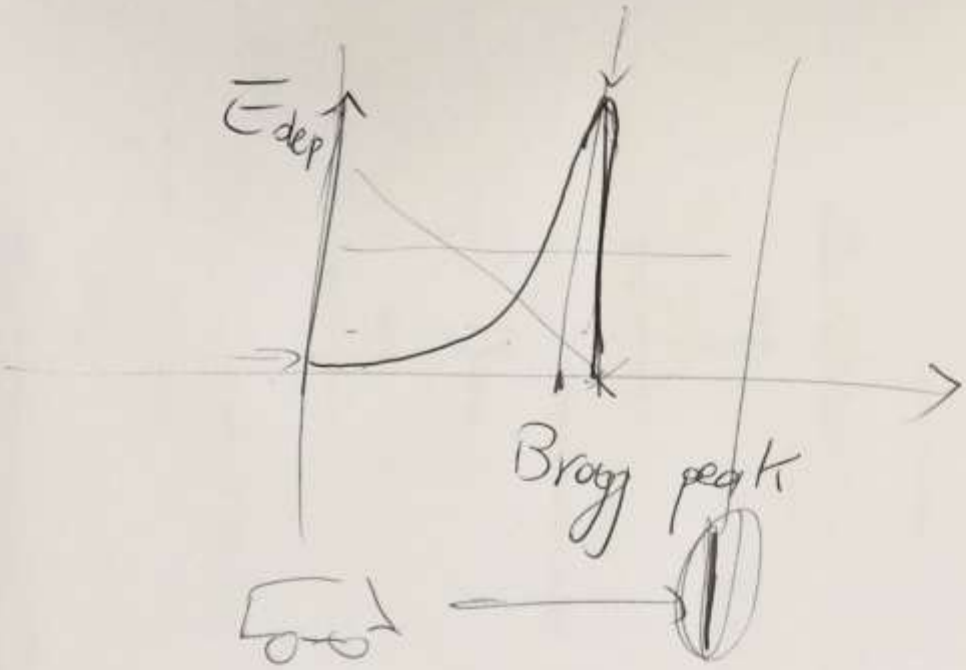
$\searrow B(r)$ CICLOTRONE ISOCRONO

ADROTERAPIA

p^+, C^+



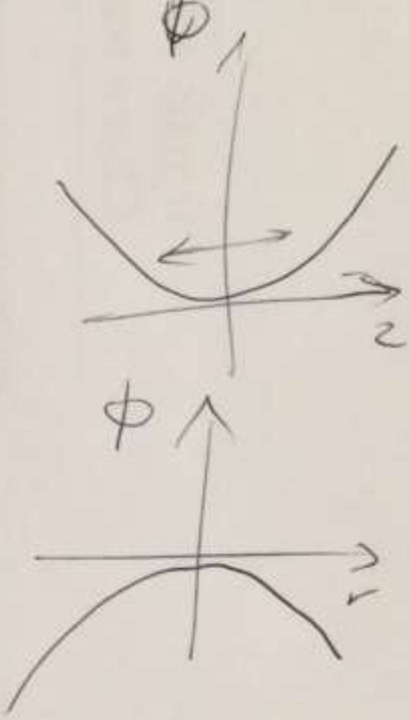
ion — matter



$\bar{E} \times \bar{B}$

Trappola di Penning

$$\nabla^2 \phi = \phi$$



$$\begin{cases} \phi_0(r, z) = \dfrac{V_0}{2d_0^2}\left(z^2 - \dfrac{1}{2} r^2\right) \\ \bar{B}_0 = B_0 \hat{e}_z \end{cases}$$

$$\phi_0(z, r) = \frac{V_0}{2d_0^2}\left(z^2 - \frac{1}{2}x^2 - \frac{1}{2}y^2\right)$$

$$\begin{cases} E_{0x,y} = -\dfrac{\partial \phi_0}{\partial x(y)} = \dfrac{V_0}{2d_0^2} x\,(y) \\ \bar{E}_{0z} = -\dfrac{\partial \phi_0}{\partial z} = -\dfrac{V_0}{d_0^2} z \end{cases}$$

$$\bar{v} = v_x \hat{e}_x + v_y \hat{e}_y + v_z \hat{e}_z = \dot{x}\hat{e}_x + \dot{y}\hat{e}_y + \dot{z}\hat{e}_z$$

$$\bar{F}_L = q(\bar{E}_0 + \bar{v} \times \bar{B}_0)$$

$$\begin{cases} m\ddot{x} = \frac{qV_0}{2d_0^2}x + qB_0\dot{y} \\ m\ddot{y} = \frac{qV_0}{2d_0^2}y - qB_0\dot{x} \\ m\ddot{z} = -\frac{qV_0}{d_0^2}z \end{cases}$$

$$\ddot{z} = -\underbrace{\frac{qV_0}{md_0^2}}_{\omega_z^2} z$$

$$\rightarrow \begin{cases} (1)\ \ddot{x} = \frac{\omega_z^2}{2}x + \omega_c\dot{y} \\ (2)\ \ddot{y} = \frac{\omega_z^2}{2}y - \omega_c\dot{x} \\ (3)\ \ddot{z} = -\omega_z^2 z \end{cases}$$

$$\omega_c = \frac{qB_0}{m}$$

$$z \sim \sin(\omega_z t)$$

$u \doteq x + iy$ $\quad (1) + i(2)$

$$\underbrace{\ddot{x} + i\ddot{y}}_{\ddot{u}} = \frac{\omega_z^2}{2}\underbrace{(x+iy)}_{u} + \omega_c\underbrace{(\dot{y} - i\dot{x})}_{-i(\dot{x}+i\dot{y}) = -i\dot{u}}$$

$$\boxed{\ddot{u} + i\omega_c\dot{u} - \frac{\omega_z^2}{2}u = 0}$$

(x, y) plane: oscillatory

ipotesi $\quad u \sim \exp(-i\omega t)$

$$(-i\omega)^2 u - i\omega\cdot\omega_c u - \frac{\omega_z^2}{2}u = 0$$

$$\omega^2 - \omega_c\omega + \frac{\omega_z^2}{2} = 0$$

$$\boxed{\omega_\pm = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}}$$

ω_+ = modified cyclotron freq.

ω_- = magnetron freq.

$\exp(-i\omega t)$

$\uparrow$ ω Re

$\Delta \geq \emptyset \rightarrow$ $\omega_c^2 \geq 2\omega_z^2$

confinement condition

$\omega_c \gg \omega_z \Rightarrow \omega_+ \simeq \omega_c$

$$\boxed{\omega_c, \omega_+ \gg \omega_z \gg \omega_-}$$

$\omega_+ + \omega_- = \omega_c$ $\leftarrow$ exact

$\omega_+ \omega_- = \omega_z^2/2$

$\omega_- = \omega_z^2/2\omega_+ \simeq \omega_z^2/2\omega_c = \dfrac{V_0}{eB_0 d_0^2}$ indep. on (m/q)

$\omega_c = \dfrac{qB}{m}$ $\quad \omega_z = \sqrt{\dfrac{qV_0}{m d_0^2}}$

$$\boxed{\omega_c^2 = \omega_z^2 + \omega_+^2 + \omega_-^2}$$ | Brown-Gabrielse invariance theorem

(eigenfrequencies)

trap as a mass spectrometer

$$u(t) = A_+ e^{-i\omega_+ t} + A_- e^{-i\omega_- t}$$

$$A_\pm = R_\pm e^{-i\alpha_{0\pm} t}$$

moti circolari ampiezza $R_\pm$, $\nu_\pm = \omega_\pm / 2\pi$

$\alpha_{0\pm}$

$u = x + iy$

$$e^{ik} = \cos k + i \sin k$$

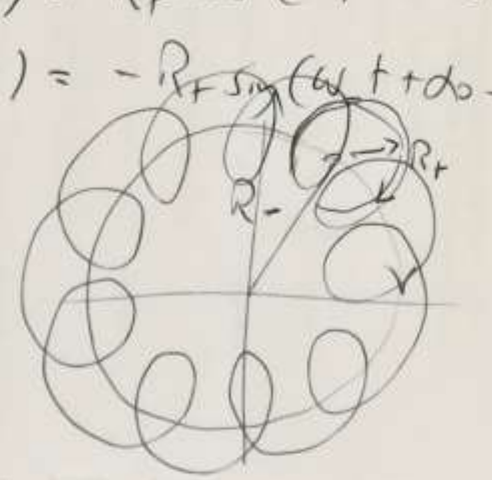
$$u(t) = R_+ e^{-i(\omega_+ t + \alpha_{0+})} + R_- e^{-i(\omega_- t + \alpha_{0-})} =$$

$$= R_+ \cos(\omega_+ t + \alpha_{0+}) - i R_+ \sin(\omega_+ t + \alpha_{0+}) +$$

$$+ R_- \cos(\omega_- t + \alpha_{0-}) - i R_- \sin(\omega_- t + \alpha_{0-})$$

$$x(t) = R_+ \cos(\omega_+ t + \alpha_{0+}) + R_- \cos(\omega_- t + \alpha_{0-})$$

$$y(t) = -R_+ \sin(\omega_+ t + \alpha_{0-}) - R_- \sin(\omega_- t + \alpha_{0-})$$



cicloide (trocoide)