Trappola di Penning

24/III/2020

(Penning)

I FORMULA DI LAPLACE E CALCOLO DI $\bar{B}_0$

$V_0 = 12\ V \quad B = 3\ T \quad d_0 = 10\ mm$

$^{12}C^+$

$\nu_c = 3.835\ MHz$

$\nu_+ = 3.832\ MHz$

$\nu_z = 156.252\ kHz$

$\nu_- = 3.186\ kHz$

$\omega_c = \frac{qB}{\boxed{m}}$ L

$V_0 \rightarrow -V_0$

$e^- \quad \nu_c = 83.931\ GHz = \nu_+$

$\nu_z = 83.110\ MHz$

$\nu_- = 3.183\ kHz$

$$\omega_- = \omega_z^2 / 2\omega_+ \simeq \frac{\omega_z^2}{2\omega_c} = \frac{V_0}{eB_0 d_0^2}$$

$\bar{r}$ vettore posizione nel piano trasversale

$$m\ddot{\bar{r}} = q\bar{E} + q\dot{\bar{r}} \times \bar{B}$$

$\times \bar{B} \qquad \times \bar{B}$

$$m\ddot{\bar{r}} \times \bar{B} = q\bar{E} \times \bar{B} + q\underbrace{\dot{\bar{r}} \times \bar{B} \times \bar{B}}_{-qB^2\dot{\bar{r}}}$$

$$\dot{\bar{r}} = \bar{v}_\perp = \frac{\bar{E} \times \bar{B}}{B^2} - \frac{m}{qB^2}\ddot{\bar{r}} \times \bar{B}$$

$$\ddot{\bar r}$$

$$\bar v_\perp = \frac{\bar E \times \bar B}{B^2} - \frac{\bar a \times \hat e_z}{\omega_c}$$

$$\frac{m \bar B}{q B^2} = \frac{1}{\omega_c}\hat e_z$$

Corrente (integrale, estensiva)

I

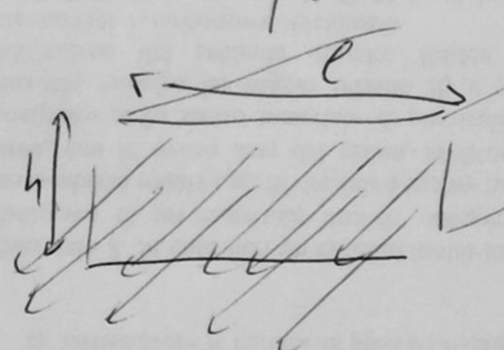
densità intensive

$\bar{J}$ densità di corrente volumica

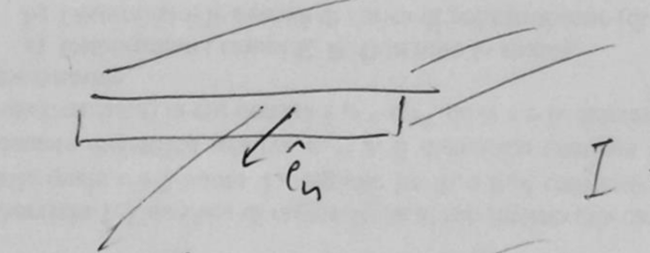
$[\bar{J}] = [A/m^2]$ $\frac{\text{corr.}}{\text{sup.}}$

$$I = \int_S \bar{J} \cdot d\bar{S} = \int_S \bar{J} \cdot \hat{e}_n \, dS$$

Densità di corrente superficiale

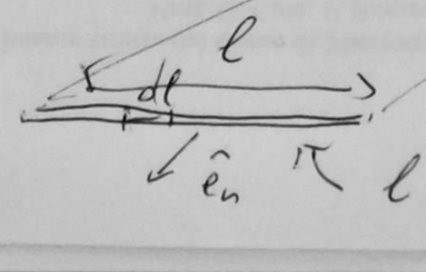


$$I = JA = Jhl$$



$$I = \int \bar{J} \cdot \hat{e}_n \, dS$$

I

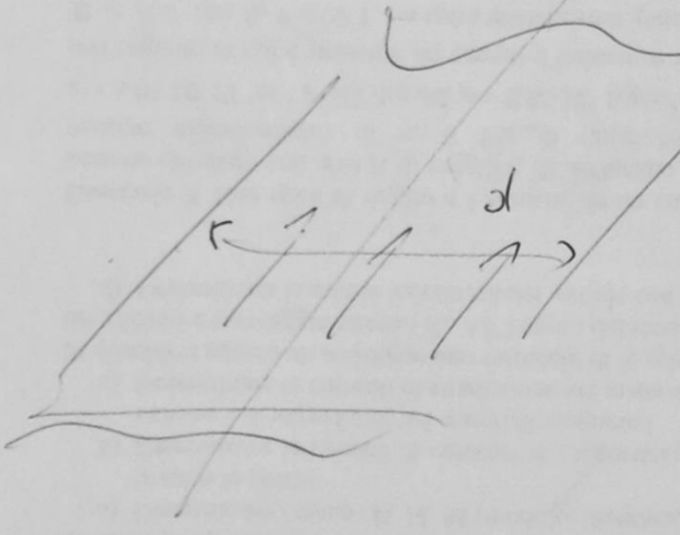


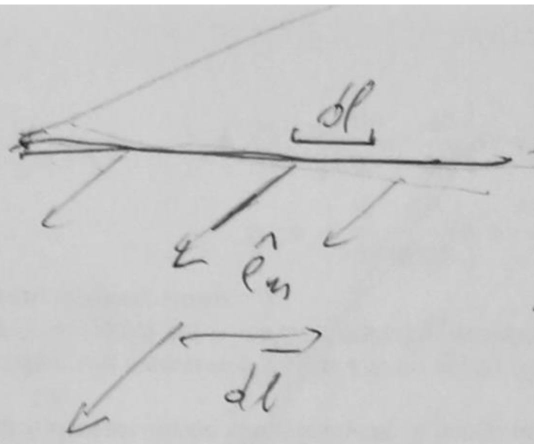
$\bar{B} \leftarrow$ sorgenti (correnti)

$$\boxed{d\bar{B}_0(\bar{r}) = \frac{\mu_0}{4\pi} I \frac{d\bar{l} \times \Delta\bar{r}}{|\Delta\bar{r}|^3}}$$

$I\,d\bar{l}(\bar{r}')$ $\qquad \Delta\bar{r} = \bar{r} - \bar{r}'$

Lamina sottile

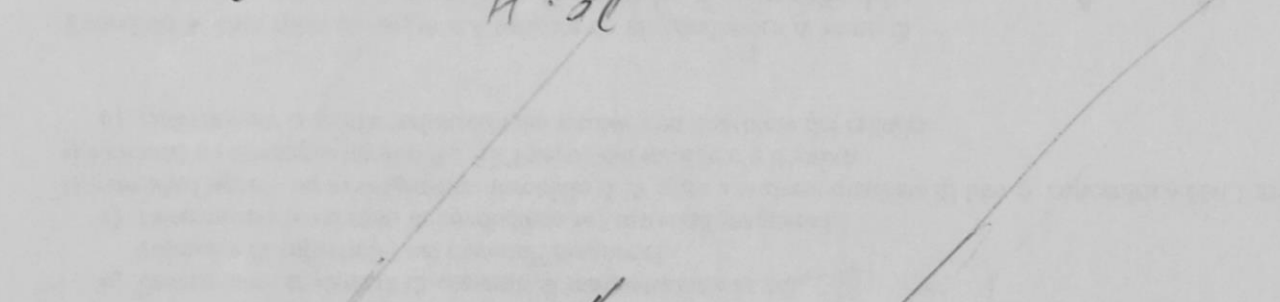




$$I = \int_l \bar{J}_s \cdot \hat{e}_n \, dl = \int_l \bar{H} \cdot \hat{e}_n \, dl$$

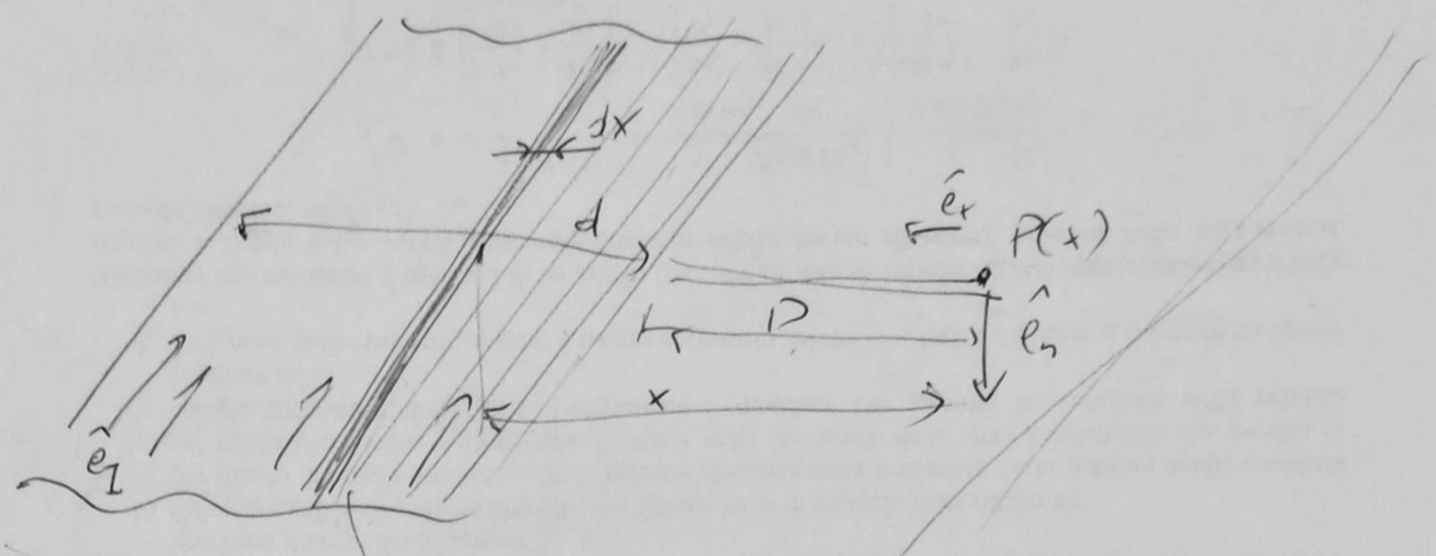
$$\cancel{I = \int \bar{H} \cdot d\bar{l}}$$

Hl $\qquad$ $\bar{H} \cdot d\bar{l}$



Densità di corrente
da integrare
lungo la varietà NORMALE
alla sua dir. di propagazione

$$\begin{cases} \text{sup.} & \bar{J} \\ \text{linea} & \bar{H} \end{cases}$$



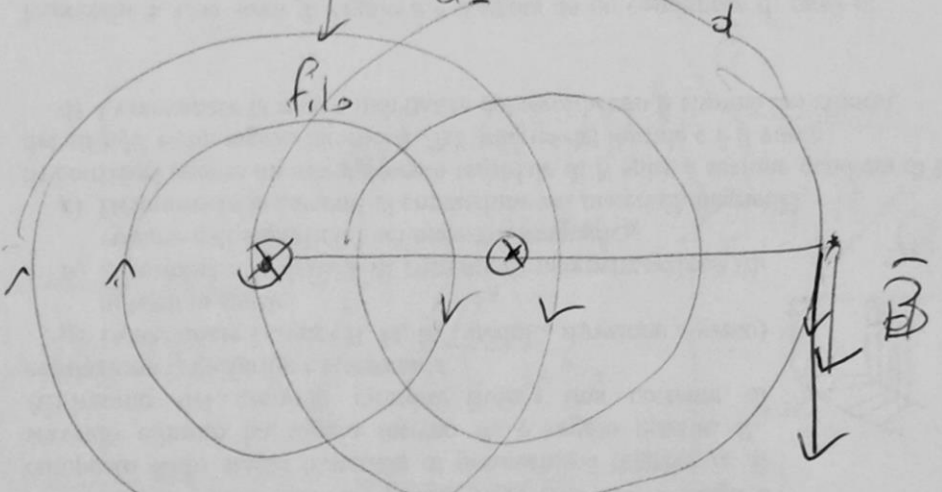
$I = 1A$

$d = 20$ mm

$D = 100$ mm

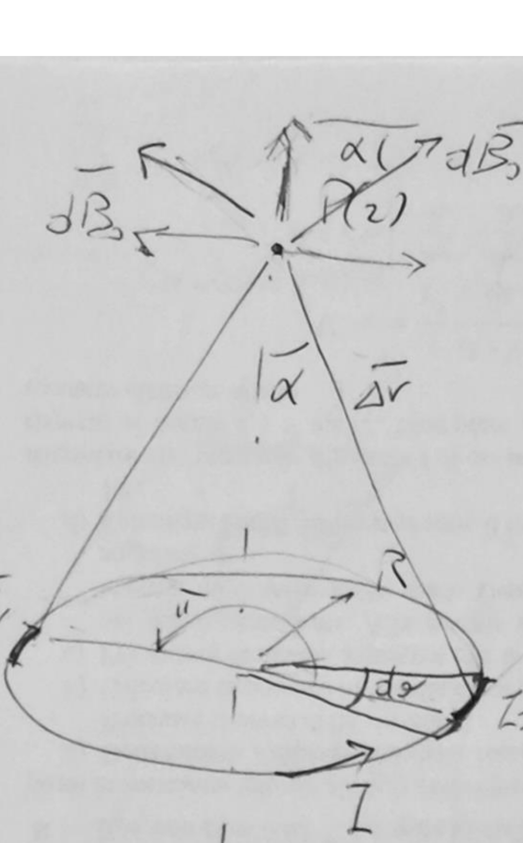
$$\bar{K} = \frac{I}{d}\hat{e}_I$$

$$dI = K\,dx = \frac{I}{d}\,dx$$



$$dB_{on} = \frac{\mu_0}{2\pi}\frac{dI}{x} = \frac{\mu_0 I}{2\pi d}\frac{dx}{x}$$

$$B_{on} = \int dB_{on} = \frac{\mu_0 I}{2\pi d}\int_D^{D+d}\frac{dx}{x} = \frac{\mu_0 I}{2\pi d}\log\left(\frac{D+d}{D}\right) = 1.82\cdot 10^{-6}\,T$$



$$\bar{dB_0} = dB_{0z} = dB_0 \sin\alpha$$

$$d\bar{B}_0 = \frac{\mu_0}{4\pi} I \frac{d\bar{l} \times \Delta\bar{r}}{\Delta r^3}$$

$$dl = R\,d\theta$$

$$B_{0z}(z) = \int dB_{0z} = \frac{\mu_0 I}{4\pi} \oint_C \frac{dl}{\Delta r^2} \sin\alpha = \frac{\mu_0 I R \sin\alpha}{2\Delta r^2}$$

$$\Delta r = (R^2+z^2)^{\frac{1}{2}}$$

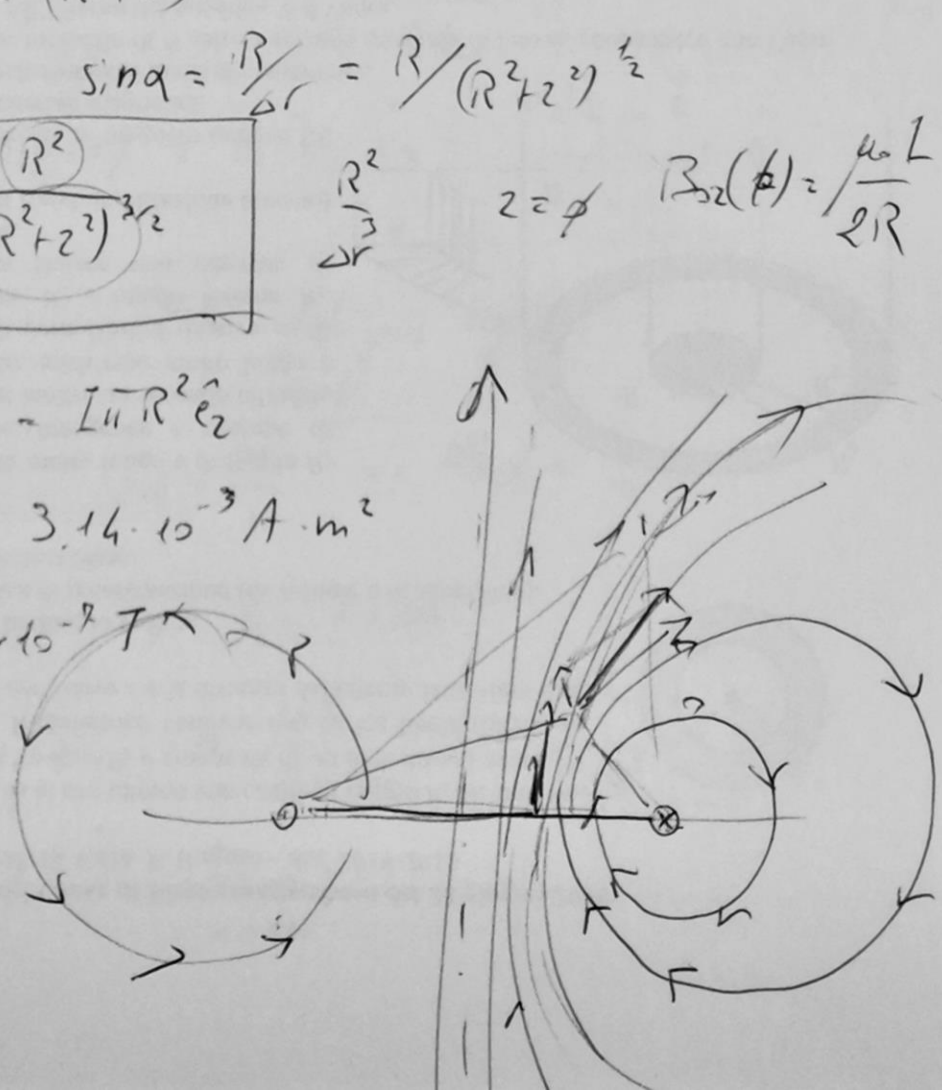
$$\sin\alpha = R/\Delta r = R/(R^2+z^2)^{\frac{1}{2}}$$

$$\boxed{B_{0z}(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}} \qquad \frac{R^2}{\Delta r^3} \qquad z = 0 \qquad B_{0z}(0) = \frac{\mu_0 I}{2R}$$

$$\bar{m} = I S \hat{e}_z = I\pi R^2 \hat{e}_z$$

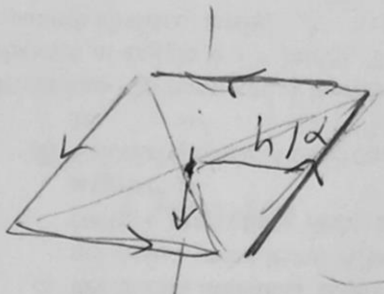
$$m = 3.14 \cdot 10^{-3}\ A \cdot m^2$$

$$B_0(0) = 6.28 \cdot 10^{-7}\ T$$



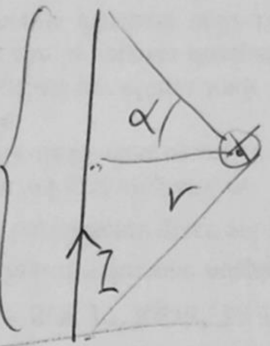
$\bar{B}_0$ centro spira

Quadrata L, I



Filo rettilineo finito

$L = 2\ell$



$$B_0(r) = \frac{\mu_0 I}{2\pi r}\sin\alpha = \frac{\mu_0 I}{2\pi r}\frac{\ell}{\sqrt{r^2+\ell^2}}$$

$$\boxed{r = h = L/2 = \ell \,, \quad \alpha = \frac{\pi}{4}}$$

$$B_{0L}(\phi) = \frac{\mu_0 I}{2\pi h}\sin\alpha = \frac{\sqrt{2}}{2}\frac{\mu_0 I}{\pi L}$$

$$\bar{B}_0(z=0, x=0, y=0) = 4 B_{0L}\,\hat{e}_z = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}\,\hat{e}_z$$

Generalizziamo n lati poligono regolare

$$2\alpha = \frac{2\pi}{n} \Rightarrow \alpha = \frac{\pi}{n}$$

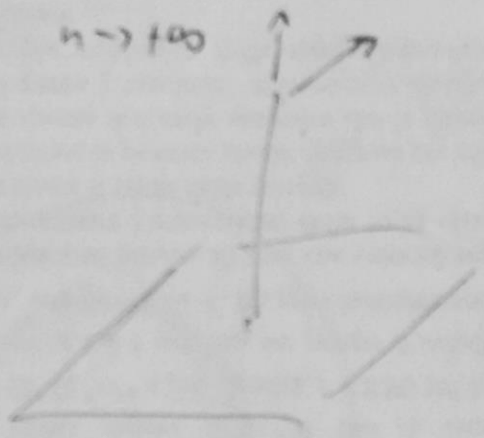
$$r = h = R\cos\alpha$$

$$B_0(\phi) = n B_{0L} = n\frac{\mu_0 I}{2\pi h}\sin\alpha = \frac{\mu_0 I}{2\pi R}\, n\, \mathrm{tg}\left(\frac{\pi}{n}\right)$$

$n \to \infty$ $\quad y \doteq \pi/n \quad \lim_{n\to+\infty} y = 0$

$$\lim_{y\to 0} B_{0pol} = \lim_{y\to 0} \frac{\mu_0 I}{2R}\,\frac{\operatorname{tg} y}{y} = \frac{\mu_0 I}{2R}\,\frac{y+\frac{y^3}{3}+\dots}{y} = \qquad \searrow 1$$

$$\lim_{n\to +\infty} B_{0pol} = \frac{\mu_0 I}{2R}$$



Disco di Rowland

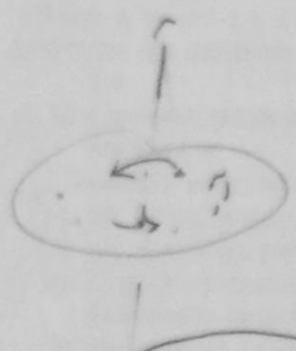
Disco R dieletrico σ uniforme

Rotazione ω

$\sigma = 10^{-6}\ C/m^2$

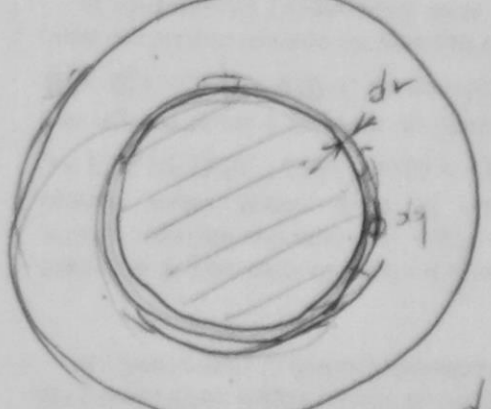
$\omega = 2\pi \cdot 10^3$ rad/s

$R = 10$ cm



B lungo asse di rotazione

$\bar{m}$



$$dq \rightarrow dI = \frac{dq}{T} = dq\,\frac{\omega}{2\pi}$$

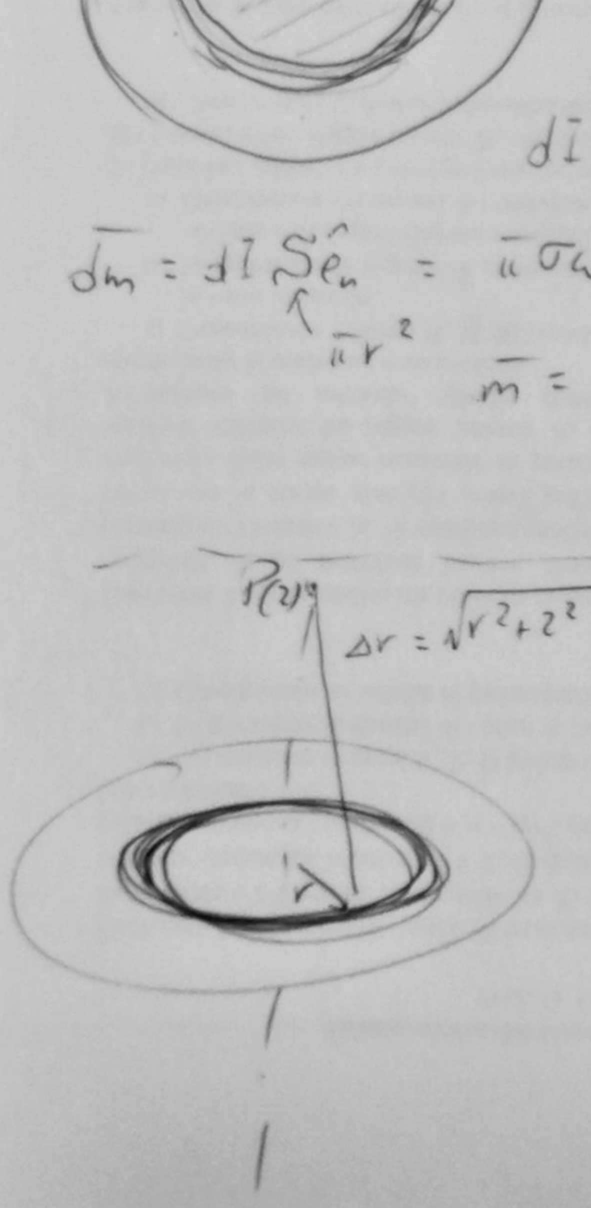
$$dq = \sigma\, dS = \sigma\, 2\pi r\, dr$$

$$dI = \frac{dq}{T} = \sigma \omega r\, dr \qquad Q = \pi R^2 \sigma$$

$$d\bar{m} = dI\, \underbrace{S}_{\pi r^2}\, \hat{e}_n = \pi \sigma \omega r^3 dr\, \hat{e}_n$$

$$\bar{m} = \int_0^R \pi \sigma \omega r^3 dr\, \hat{e}_n = \sigma \omega \pi \frac{R^4}{4} \hat{e}_n = \frac{Q \omega R^2}{4} \hat{e}_n$$

$$m = 4.93 \cdot 10^{-7}\ A \cdot m^2$$



$$dB_z(z) = \int \frac{\mu_0\, dI}{2} \frac{r^2}{(r^2+z^2)^{3/2}} = \int \frac{\mu_0 \sigma \omega}{2} \frac{r^3\, dr}{(r^2+z^2)^{3/2}}$$

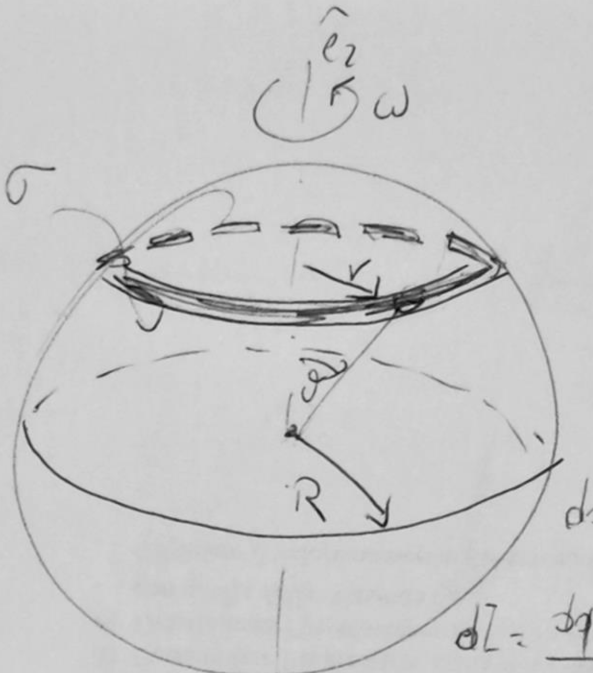
$$B_{oz}(z) = \frac{\mu_0 \sigma \omega}{2} \underbrace{\int_{\phi}^{R} \frac{r^3}{(r^2+z^2)^{3/2}}\, dr}$$

$$\int_{\phi}^{R} \frac{r^3}{(r^2+z^2)^{3/2}}\, dr = \int \underbrace{\frac{r}{(r^2+z^2)^{3/2}}}_{f'(r)} \cdot \underbrace{r^2}_{g(r)}\, dr = \underbrace{-\frac{1}{(r^2+z^2)^{1/2}} \cdot r^2 \Big|_{\phi}^{R}}_{f \cdot g} + \int_{\phi}^{R} \frac{1}{(r^2+z^2)^{1/2}}\, 2r\, dr =$$

$$= -\frac{r^2}{(r^2+z^2)^{1/2}} + 2(r^2+z^2)^{1/2} = \boxed{\frac{r^2+2z^2}{(r^2+z^2)^{1/2}}}$$

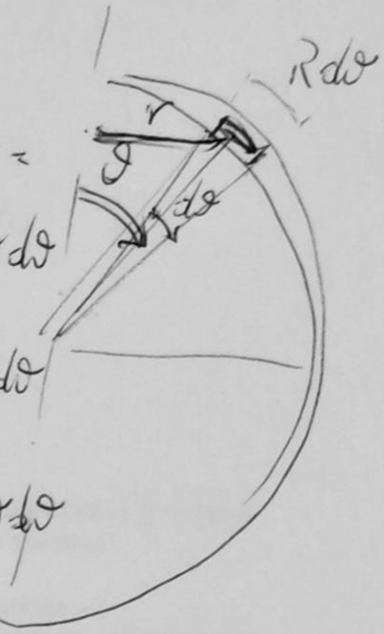
$$B_{oz}(z) = \frac{\mu_0 \sigma \omega}{2} \left[\frac{r^2+2z^2}{(r^2+z^2)^{1/2}} \right]_{\phi}^{R} = \frac{\mu_0 \sigma \omega}{2} \left[\frac{R^2+2z^2}{(R^2+z^2)^{1/2}} - 2z \right]$$

$$B_{oz}(\phi) = 4\pi^2 \cdot 10^{-11}\, T \simeq 4 \cdot 10^{-10}\, T$$



$r = R\sin\vartheta$

$dS = 2\pi r\, R\, d\vartheta = 2\pi R^2 \sin\vartheta\, d\vartheta$



$dq = \sigma\, dS = 2\pi\sigma R^2 \sin\vartheta\, d\vartheta$

$dI = \frac{dq}{T} = dq\,\frac{\omega}{2\pi} = \sigma\omega R^2 \sin\vartheta\, d\vartheta$

$d\bar{m} = dI\, S\, \hat{e}_z = \pi\omega\sigma R^4 \sin^3\vartheta\, d\vartheta\, \hat{e}_z$

$S = \pi r^2$

$$\bar{m} = \int_0^{\pi} \pi\omega\sigma R^4 \sin^3\vartheta\, d\vartheta\, \hat{e}_z = \pi\omega\sigma R^4 \int_0^{\pi} \left(\sin\vartheta - \sin\vartheta\cos^2\vartheta\right) d\vartheta\, \hat{e}_z =$$

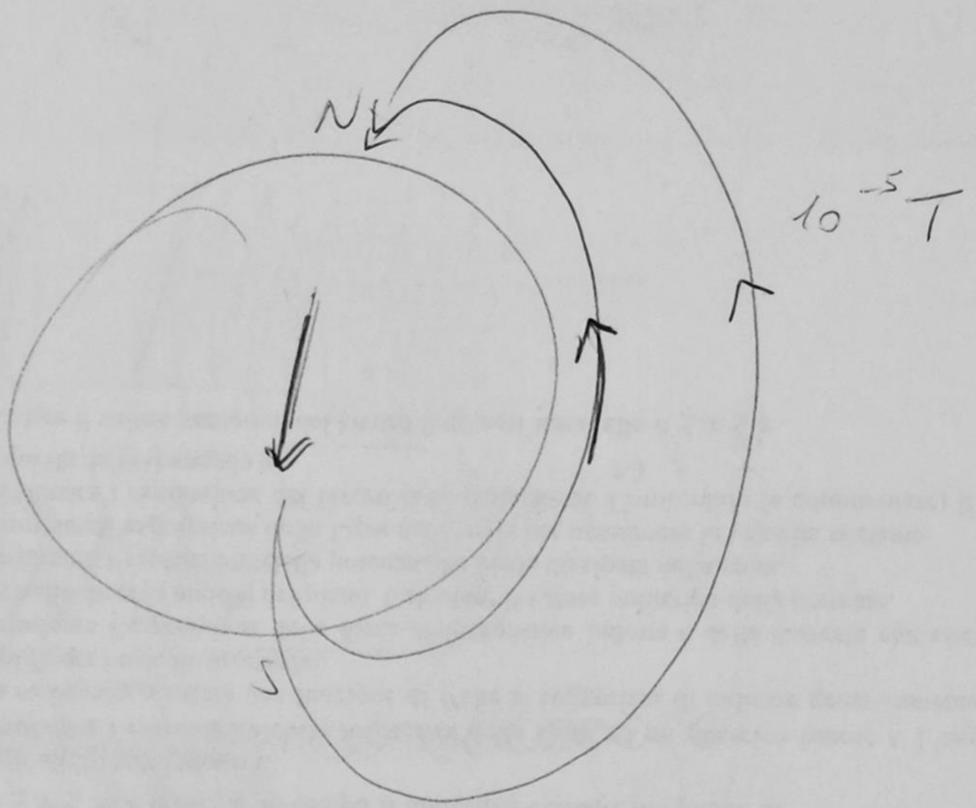
$\sin\vartheta \cdot \sin^2\vartheta = \sin\vartheta\,(1-\cos^2\vartheta)$

$$= \pi\omega\sigma R^4 \underbrace{\left[-\cos\vartheta + \frac{1}{3}\cos^3\vartheta\right]_0^{\pi}}_{4/3} \hat{e}_z = \frac{4\pi}{3}\omega\sigma R^4 \hat{e}_z = \frac{Q\omega R^2}{3}\hat{e}_z$$

$Q \doteq \sigma\, \underline{4\pi R^2}$

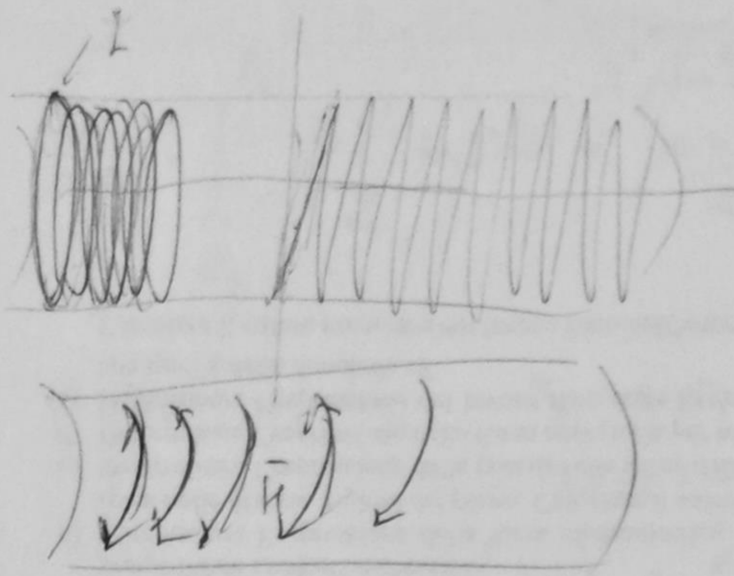
$$dB_{0z}(0) = \frac{\mu_0\, dI}{2}\,\frac{r^2}{R^3} = \frac{\mu_0}{2}\,\omega\sigma R \sin^3\vartheta\, d\vartheta$$

$$B_{0z}(0) = \int_0^{\pi} \mu_0 \frac{\omega\sigma R}{2} \sin^3\vartheta\, d\vartheta = \frac{2}{3}\mu_0\omega\sigma R = \frac{\mu_0}{2\pi}\,\frac{m}{R^3}$$

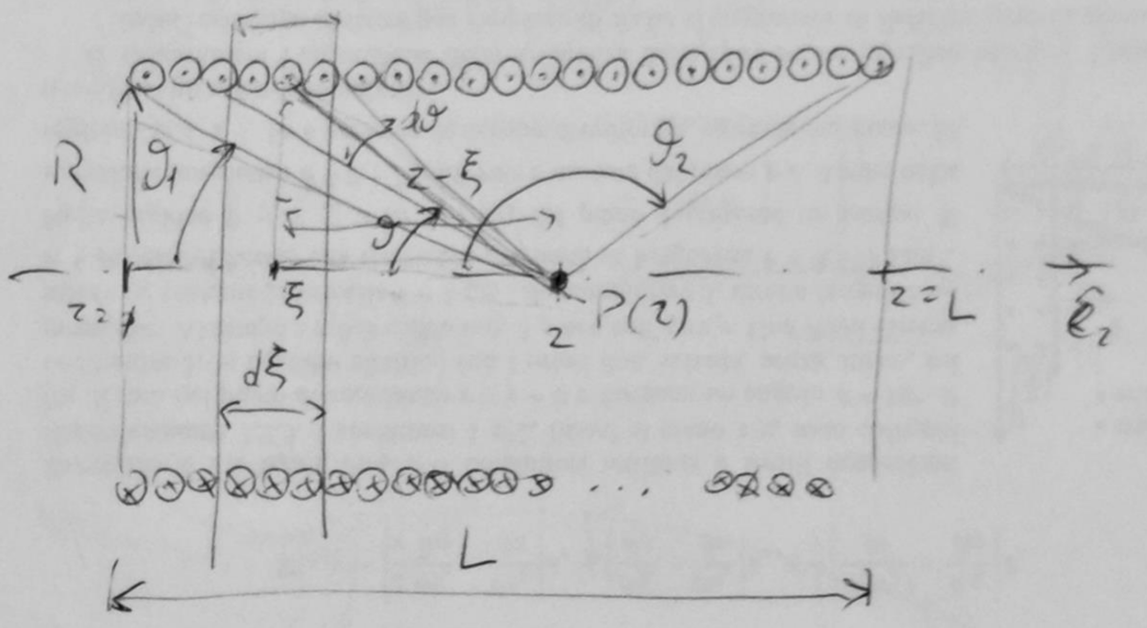

N
S
10^{-5} T

Solenoide rettilineo sottile

L , N , I $\quad B_{oz}$(asse)



$n = \frac{N}{L}$



$\frac{N}{L} d\xi = n d\xi$ spire $\qquad dI = n d\xi \cdot I \qquad \frac{dI}{d\xi} = K$

$$dB_{oz}(z) = \frac{\mu_0 n d\xi I}{2} \frac{R^2}{\left[R^2+(z-\xi)^2\right]^{3/2}}$$

$R = (z-\xi)\, tg\vartheta$; $\quad z-\xi = R/tg\vartheta$

$$-\frac{d\xi}{d\vartheta} = -\frac{R}{tg^2\vartheta}\frac{1}{\cos^2\vartheta} \Rightarrow d\xi = \frac{R\, d\vartheta}{\sin^2\vartheta}$$

$$dB_{0z}(z) = \frac{\mu_0 n I R^2}{2}\, \frac{1}{\underbrace{\left[R^2 + R^2/\tan^2\vartheta\right]}_{R^2/\sin^2\vartheta}{}^{3/2}}\, \frac{R\,d\vartheta}{\sin^2\vartheta} = \frac{\mu_0 n I}{2}\sin\vartheta\,d\vartheta$$

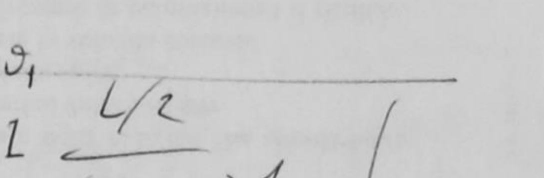
$$B_z(z) = \int_{\vartheta_1}^{\vartheta_2} \frac{\mu_0 n I}{2}\sin\vartheta\,d\vartheta = \frac{\mu_0 n I}{2}\left(\cos\vartheta_1 - \cos\vartheta_2\right)$$

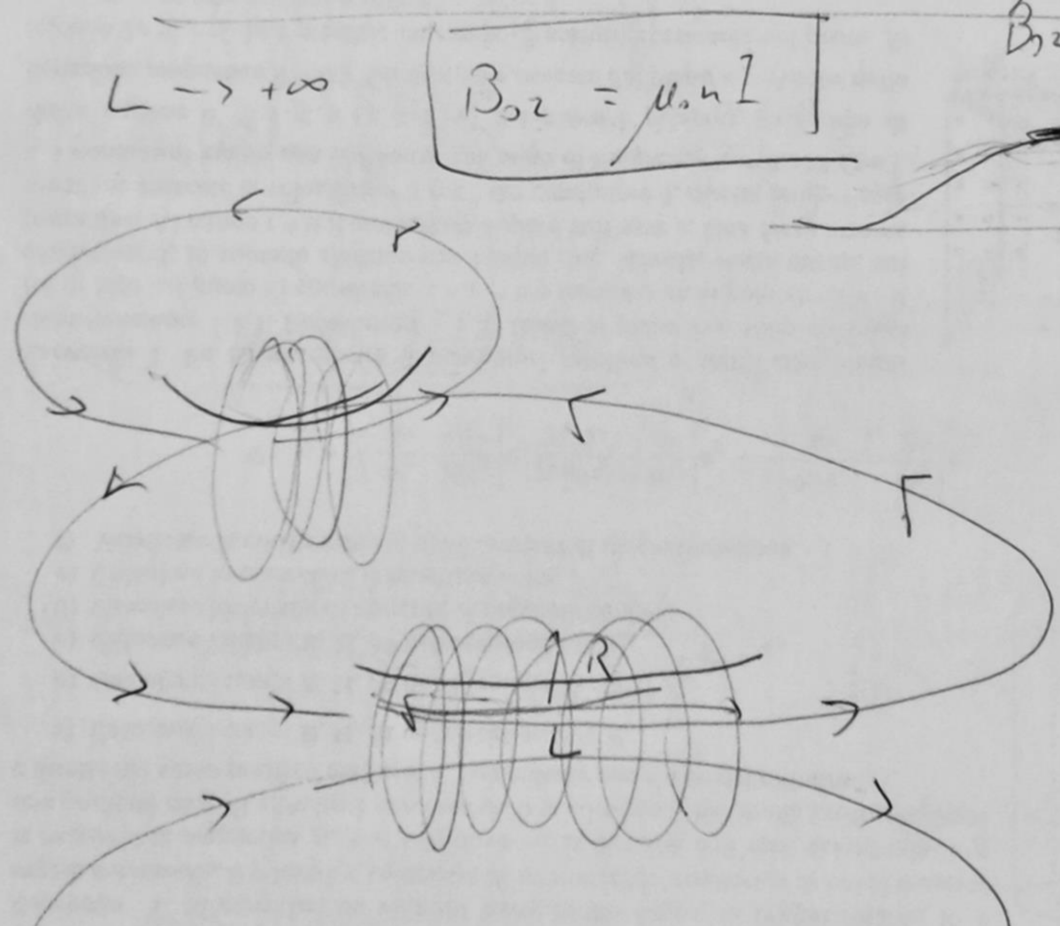
Centro : $z = L/2$; $\vartheta_2 = \pi - \vartheta_1$

$\cos\vartheta_2 = -\cos\vartheta_1$

$$\boxed{B_{0z}(L/2) = \mu_0 n I \cos\vartheta_1 = \mu_0 n I \frac{L/2}{\left(R^2 + L^2/4\right)^{1/2}}}$$

$L \to +\infty$ $\boxed{B_{0z} = \mu_0 n I}$

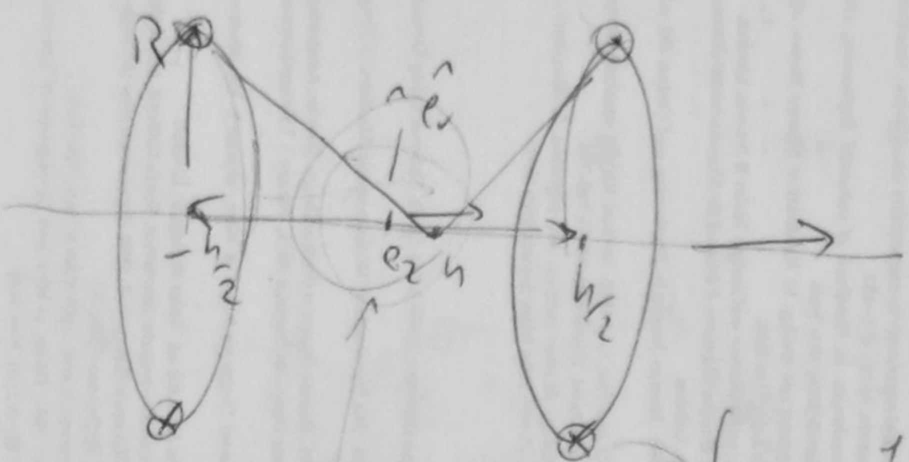




$\frac{L}{R} \uparrow$

$L \to \infty$

Bobine di Helmholtz (H. coils)



$$B_{0z}(z) = \frac{\mu_0 I R^2}{2}\left\{ \frac{1}{\left[R^2+\left(z+\frac{h}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2+\left(z-\frac{h}{2}\right)^2\right]^{3/2}} \right\}$$

$$B_0(z=0) = \mu_0 I R^2 \Big/ \left(R^2+h^2/4\right)^{3/2}$$

$$B_{0z}(z) = B_{0z}(0) + \cancel{\left.\frac{\partial B_{0z}}{\partial z}\right|_{z=0}} \cdot z + \frac{1}{2}\cancel{\left.\frac{\partial^2 B_{0z}}{\partial z^2}\right|_{z=0}} z^2 + \frac{1}{3!}\cancel{\left.\frac{\partial^3 B_{0z}}{\partial z^3}\right|_{z=0}} z^3 + \frac{1}{4!}\left.\frac{\partial^4 B_{0z}}{\partial z^4}\right|_{z=0} z^4 + \dots$$

$$\frac{\partial B_{0z}}{\partial z} = -\frac{3\mu_0 I R^2}{2}\left\{ \frac{z+h/2}{\left[R^2+(z+h/2)^2\right]^{5/2}} + \frac{z-h/2}{\left[R^2+(z-h/2)^2\right]^{5/2}} \right\}\Bigg|_{z=0} = 0$$

$$\frac{\partial^2 B_{0z}}{\partial z^2} = -\frac{3\mu_0 I R^2}{2}\left\{ \frac{R^2-4(z+h/2)^2}{\left[R^2+(z+h/2)^2\right]^{7/2}} + \frac{R^2-4(z-h/2)^2}{\left[R^2+(z-h/2)^2\right]^{7/2}} \right\}$$

$$\left.\frac{\partial^2 B_{0z}}{\partial z^2}\right|_{z=0} = 0 \quad \longrightarrow \quad R^2 - 4\frac{h^2}{4} + R^2 - 4\frac{h^2}{4} = 0$$

$$R^2 = h^2$$

$$\boxed{R = h}$$

$$\frac{\partial^4 B_{0z}}{\partial z^4} = \frac{3\mu_0 I R^2}{2}\left\{ \frac{\left[15R^2 - 60\left(z+\frac{h}{2}\right)^2\right]\left[R^2+\left(z+\frac{h}{2}\right)^2\right] - 3\left[15R^2-20\left(z+\frac{h}{2}\right)\right]\left(z+\frac{h}{2}\right)^2}{\left[R^2+\left(z+\frac{h}{2}\right)^2\right]^{11/2}} + \ldots \text{idem on } \left(z-\frac{h}{2}\right) \right\}$$

In $z=0$, con $h=R$

$$B_{0z}(z) = B_{0z}(0) + \frac{1}{24}\left.\frac{\partial^4 B}{\partial z^4}\right|_{z=0} z^4 = \; ?$$

$$= \mu_0 \frac{8}{5\sqrt{5}} \frac{I}{R}\left[1 - \frac{144}{125}\frac{z^4}{R^4}\right]$$

$$B_{0z}\left(z = \pm\frac{h}{2}\right) = B_{0z}(0)\left[1 - 0.072\right]$$

Winding coil