ESERCITAZIONE 30/11/2020
LEGGE DI AMPÈRE

Ampère

$\bar{B} \leftarrow \bar{j}$

=

Gauss

$\bar{E} \leftarrow q, \rho$

$\int_{S=\partial\tau} \bar{E}\cdot d\bar{S} = \frac{1}{\varepsilon_0}\int_\tau \rho \, d\tau$

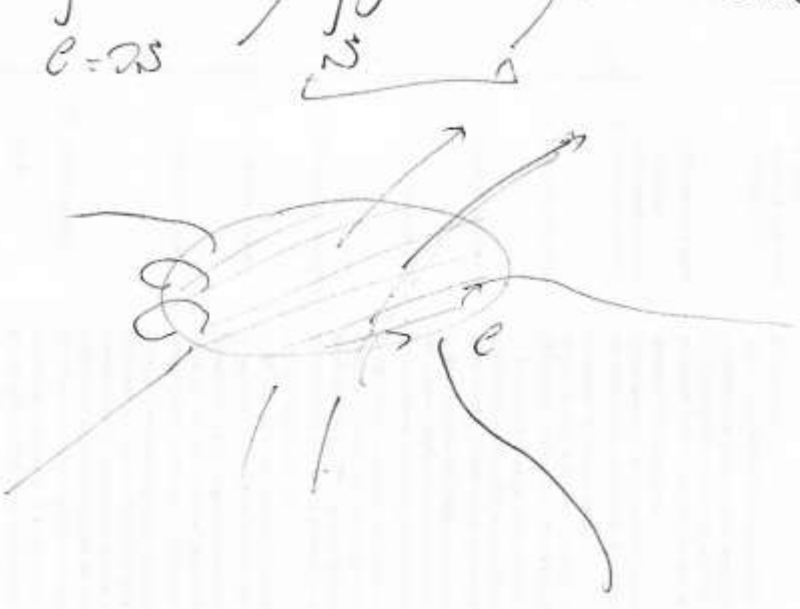
+

SIMMETRIA INVARIANZA DEL PROBLEMA

Magnetostatica ($\bar{j}$ stazionaria)

$\bar{\nabla}\times\bar{B} = \mu_0 \bar{j}$

$$\oint_{\ell=\partial S} \bar{B}\cdot d\bar{\ell} = \mu_0 \int_S \bar{j}\cdot d\bar{S} = \mu_0 \sum I_{conc}$$



Filo rettilineo indefinito (I staz.)

↳ raggio finito R $\qquad \bar{B}_0 \ \forall r \begin{smallmatrix} < R \\ > R \end{smallmatrix}$

invarianza $\hat{e}_\vartheta$, $\hat{e}_z$

$\bar{B}_0(r, \cancel{\vartheta}, \cancel{z})$

$\bar{B}_0 = \left(B_{0r}(r), \underline{B_{0\vartheta}(r)}, \cancel{B_{0z}(r)} \right)$

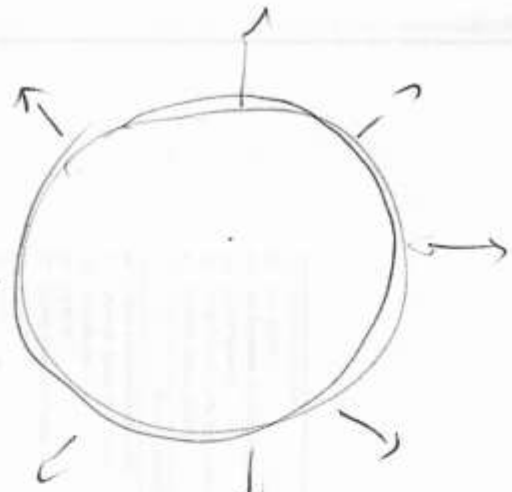
$d\bar{B}_0 \propto \underline{I d\bar{l}} \times \Delta\bar{r} \Rightarrow dB_{0z} = \emptyset \ \forall \ I d\bar{l}$

$\bar{\nabla} \cdot \bar{B}_0 = \emptyset$

$\int \bar{B}_0 \cdot d\bar{S} = \emptyset$

$B_{0r}(r)$

$\left(B_{0\vartheta}(r) \right) \leftarrow$

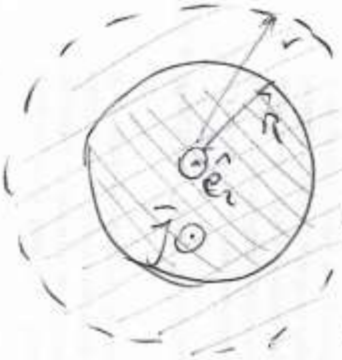


$B_{0r}(r) = \emptyset$

$\bar{B}_0(r) = B_{0\vartheta}(r)\, \hat{e}_\vartheta$

$\oint_{\ell = \partial S} \bar{B}_0 \cdot d\bar{l} = \mu \int_S \bar{J} \cdot d\bar{S}$

$\boxed{r > R}$

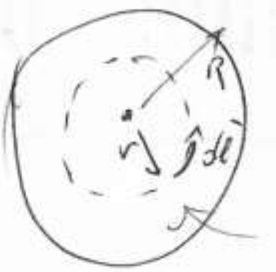


$$\oint \bar{B}_3^{(r)} \cdot d\bar{l} = \int_0^{2\pi} B_{3\vartheta}\, r\, d\vartheta = 2\pi r B_3(r) = \mu_0 \int \bar{J} \cdot d\bar{S} = \mu_0 I$$

$d\bar{l} = dl\, \hat{e}_\vartheta = r\, d\vartheta\, \hat{e}_\vartheta$

$$\boxed{\bar{B}_3(r) = \frac{\mu_0 I}{2\pi r} \hat{e}_\vartheta}$$

$\boxed{r < R}$



$$\oint_\ell \bar{B}_3 \cdot d\bar{l} = 2\pi r B_3(r) = \mu_0 \int \bar{J} \cdot d\bar{S} = \mu_0 I \frac{r^2}{R^2}$$

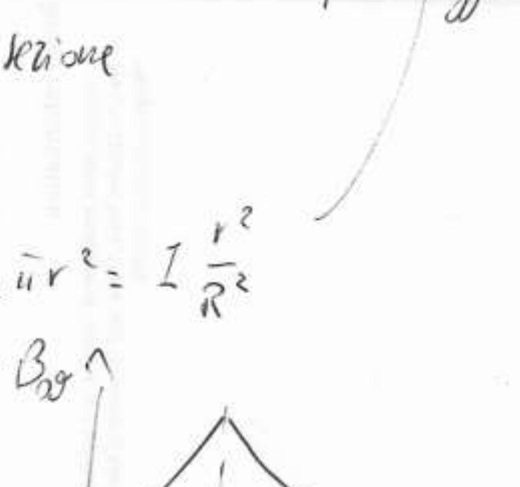
sup. di raggio r

Hyp.: $\bar{J}$ uniforme sulla sezione

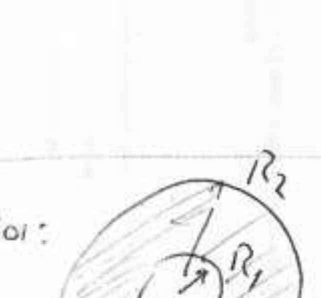
$$J_z = \frac{I}{\pi R^2}$$

$$I(0 \to r) = J_z \pi r^2 = I \frac{r^2}{R^2}$$

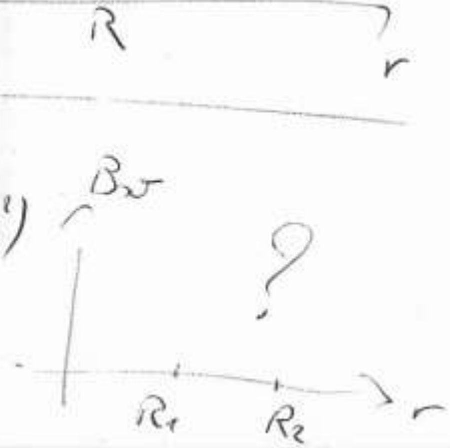
$$\Rightarrow \boxed{\bar{B}_3(r) = \frac{\mu_0 I}{2\pi R^2} r\, \hat{e}_\vartheta}$$



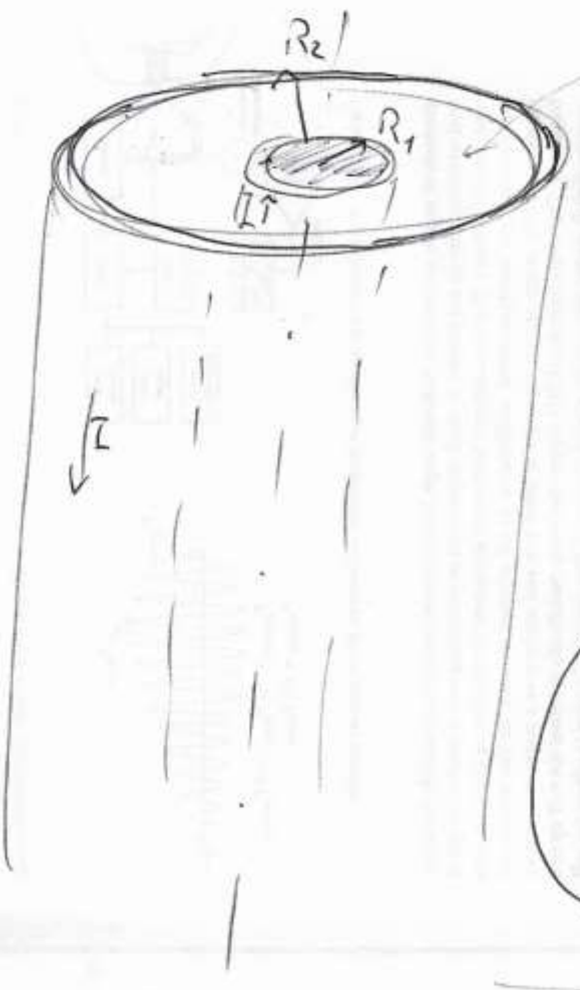
Es. per casa: filo lungo cavo



$$J_z = \frac{I}{\pi (R_2^2 - R_1^2)}$$



Cavo coassiale (indefinit. lungo, simm. cilindrica)



isolante magneticamente trasparente (μ_0 come il vuoto)

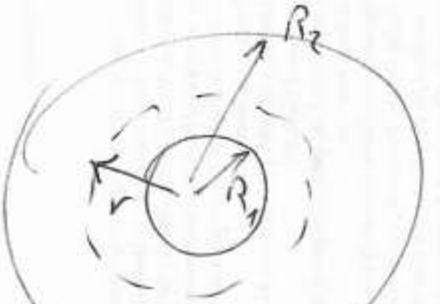
$$\vec{B}_0 = B_{0\theta}(r)\hat{e}_\theta$$

$r < R_1$

$$2\pi r B_{0\theta}(r) = \mu_0 I \frac{r^2}{R_1^2}$$

$$B_{0\theta}(r) = \frac{\mu_0 I}{2\pi R_1^2} r$$

$R_1 < r < R_2$



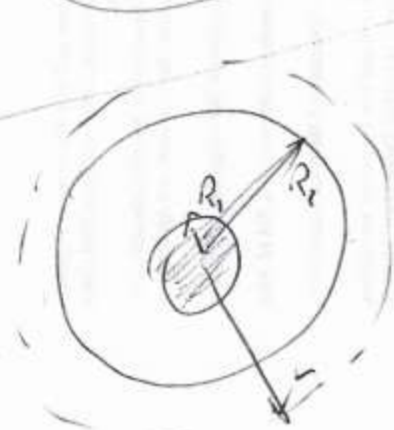
$$2\pi B_{0\theta}(r) = \mu_0 I$$

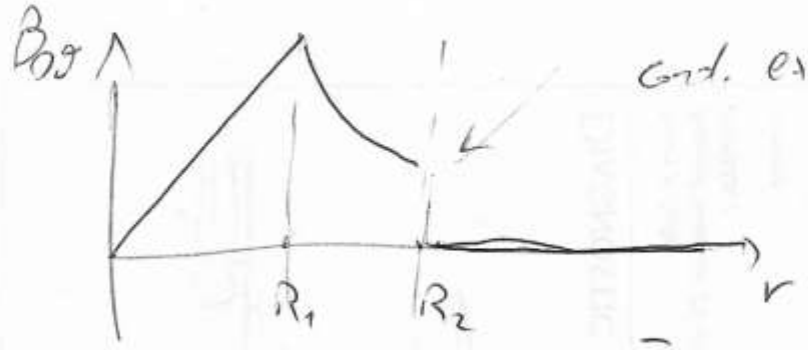
$$B_{0\theta}(r) = \frac{\mu_0 I}{2\pi r}$$

$r > R_2$

$$2\pi r B_{0\theta}(r) = \mu_0 (I - I) = \phi$$

$$\vec{B}_0 = \phi$$





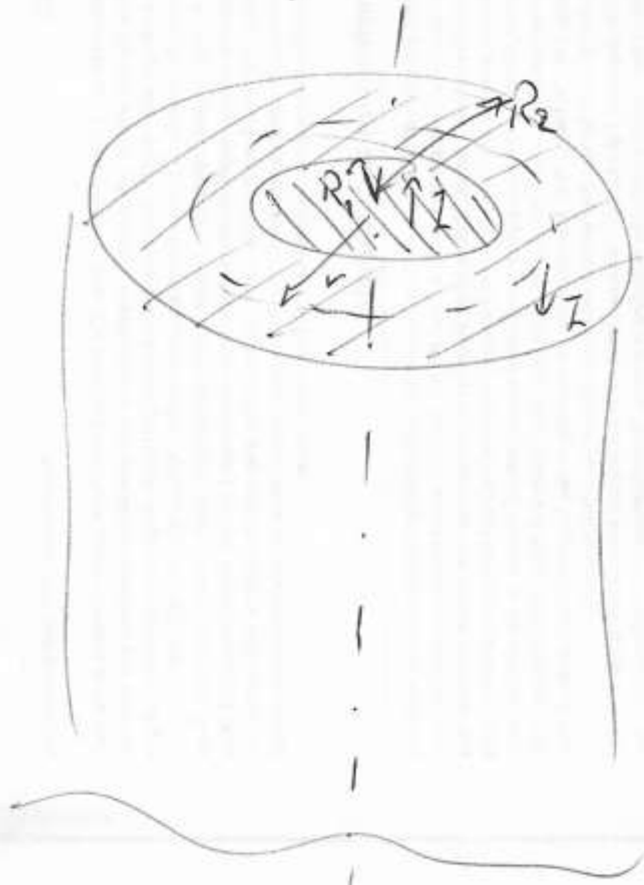
cond. est $\rightsquigarrow \bar{K}$

discontinuità di $\bar{E} \Leftarrow \sigma$

$\bar{B} \Leftarrow \bar{K}$

$\Delta B = \mu_0 K$ discontinuità di $\bar{B}$.

Cavo coassiale a conduttori spessi
(guaina comunque sottile)



$$\bar{B}_o(\bar{r}) = B_{o\vartheta}(r)\hat{e}_\vartheta$$

$\boxed{r < R_1}$

$$2\pi r B_{o\vartheta} = \mu_0 I \frac{r^2}{R_1^2} \Rightarrow B_{o\vartheta}(r) = \frac{\mu_0 I}{2\pi R_1^2} r$$

$R_1 < r < R_2$

$$I_{in} = I$$

$$I_{out}(R_1 \to r) = -\frac{\cancel{\pi}(r^2 - R_1^2)}{\cancel{\pi}(R_2^2 - R_1^2)} I$$

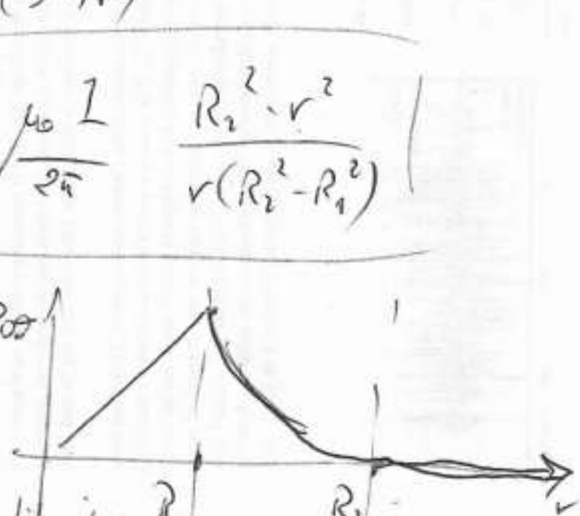
$$I(0 \to r) = I_{in} + I_{out}(R_1 \to r) = I - \frac{r^2 - R_1^2}{R_2^2 - R_1^2} I$$

$$\Rightarrow I(0 \to r) = \frac{R_2^2 - r^2}{R_2^2 - R_1^2} I$$

Ampère: $2\pi r B_{o\vartheta}(r) = \mu_0 I(0 \to r)$

$$\Rightarrow \boxed{B_{o\vartheta}(r) = \frac{\mu_0 I}{2\pi} \frac{R_2^2 - r^2}{r(R_2^2 - R_1^2)}}$$

$r > R_2$ $\quad I_{conc} = 0$ $\quad \bar{B}_o = 0$



$\exists \bar{j}, \nexists \bar{\kappa} \Rightarrow \nexists$ discontinuità di $\bar{B}_o$

Solenoide rettilineo ∞mente esteso R, I, n

Invarianza in θ, z

$$\bar{B}_0 = \bar{B}_0(r)$$

$$B_{0r}(r),\ B_{0\theta}(r),\ B_{0z}(r)$$

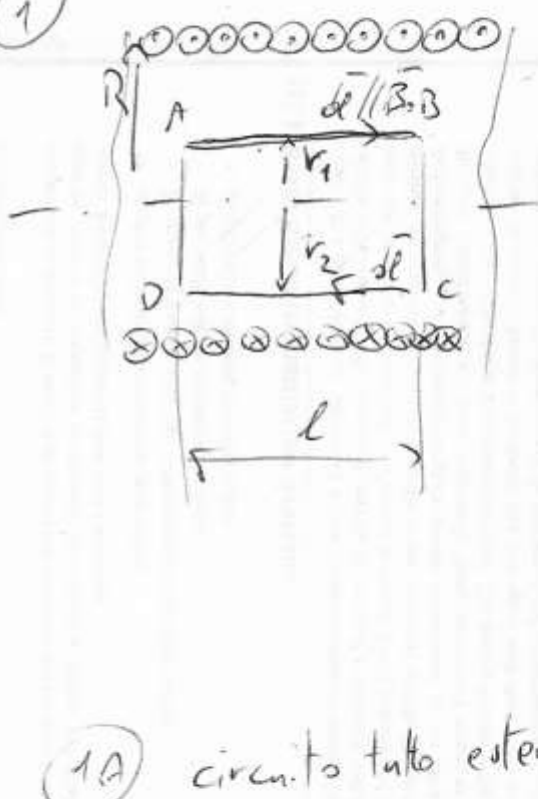
Sorgenti $\sim \hat{e}_\theta \Rightarrow \bar{B} \perp \hat{e}_\theta \quad B_{0\theta} = \emptyset$

$$B_{0r}(r),\ B_{0z}(r)$$

solenoidalità di $\bar{B}$ + invarianza $\theta, z \Rightarrow B_{0r}(r) = \emptyset$

$$\bar{B}_0 = B_{0z}(r)\hat{e}_z$$

(1)



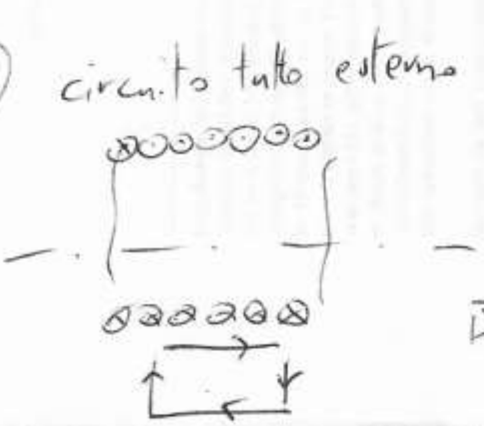
$$\oint \bar{B}_0 \cdot d\bar{l} = \underbrace{\int_A^B \bar{B}_0 \cdot d\bar{l}}_{B_{0z}(r_1)l} + \underbrace{\int_B^C \bar{B}_0 \cdot d\bar{l}}_{\emptyset} + \underbrace{\int_C^D \bar{B}_0 \cdot d\bar{l}}_{-B_{0z}(r_2)l} + \underbrace{\int_D^A \bar{B}_0 \cdot d\bar{l}}_{\emptyset} =$$

$$\mu_0 \iint \bar{J} \cdot d\bar{S} = \emptyset$$

$$B_{0z}(r_1) = B_{0z}(r_2) \quad \forall r_1, r_2$$

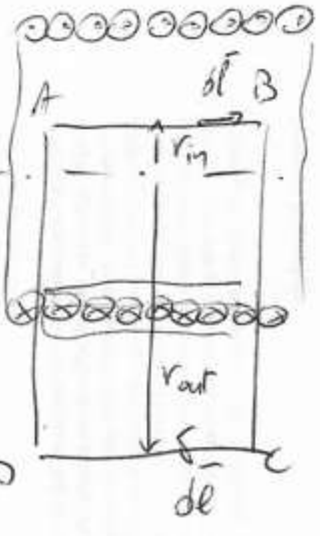
$$B_{0z} = \text{uniforme } r < R$$

(1A) circuito tutto esterno



B_{0z} uniforme $\forall r > R$

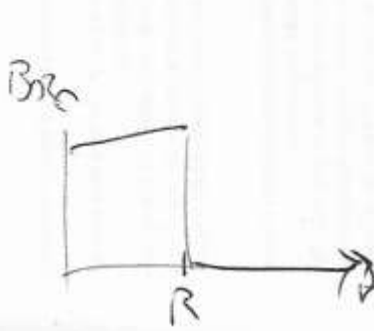
2



$$\underbrace{\oint \bar{B}_0 \cdot d\bar{l}}_{\downarrow} = \int_A^B \bar{B}_0 \cdot d\bar{l} + \underbrace{\int_B^C}_{\bar{B}_0 \perp d\bar{l} \; = \phi} + \underbrace{\int_C^D}_{\downarrow} + \underbrace{\int_D^A}_{\bar{B}_0 \perp d\bar{l} \; = \phi} =$$

$B_{0z}(r_{in})\,l$

$-B_{0z}(r_{out})\,l = \phi$



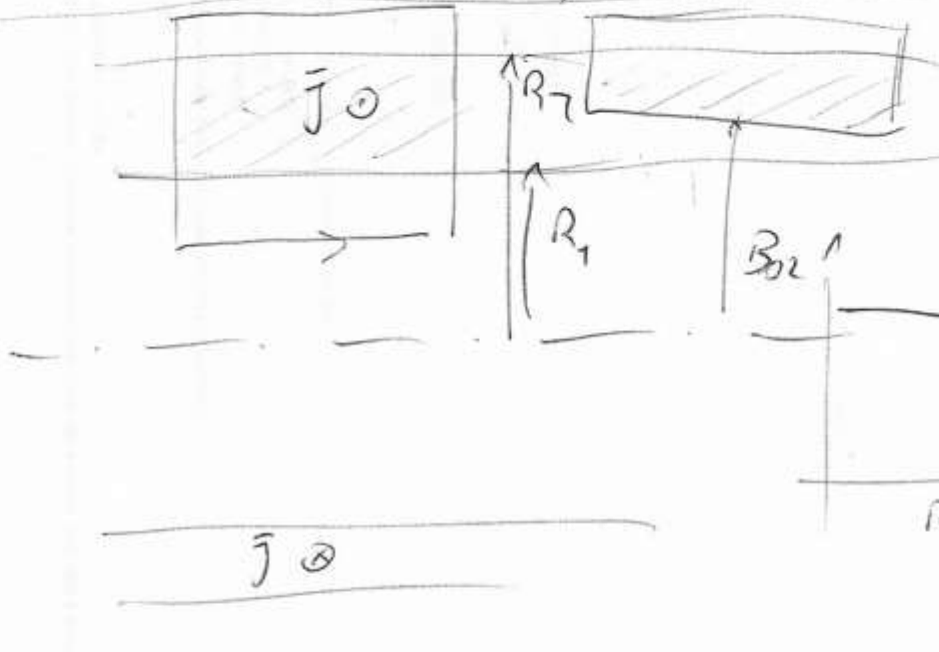
$\mu_0 I = \mu_0 k l$

$\#$ spire $= nl$

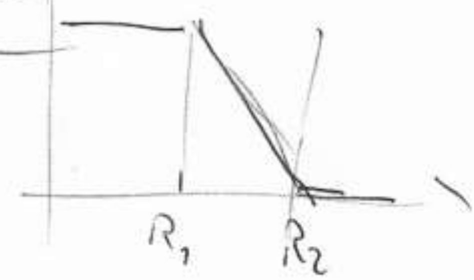
corr. concatenata nlI

$B_{0z}(r_{in})\,l = \mu_0 n l I$

$\boxed{\bar{B}_{0z} = \mu_0 n I \hat{e}_z}$ dentro il solenoide

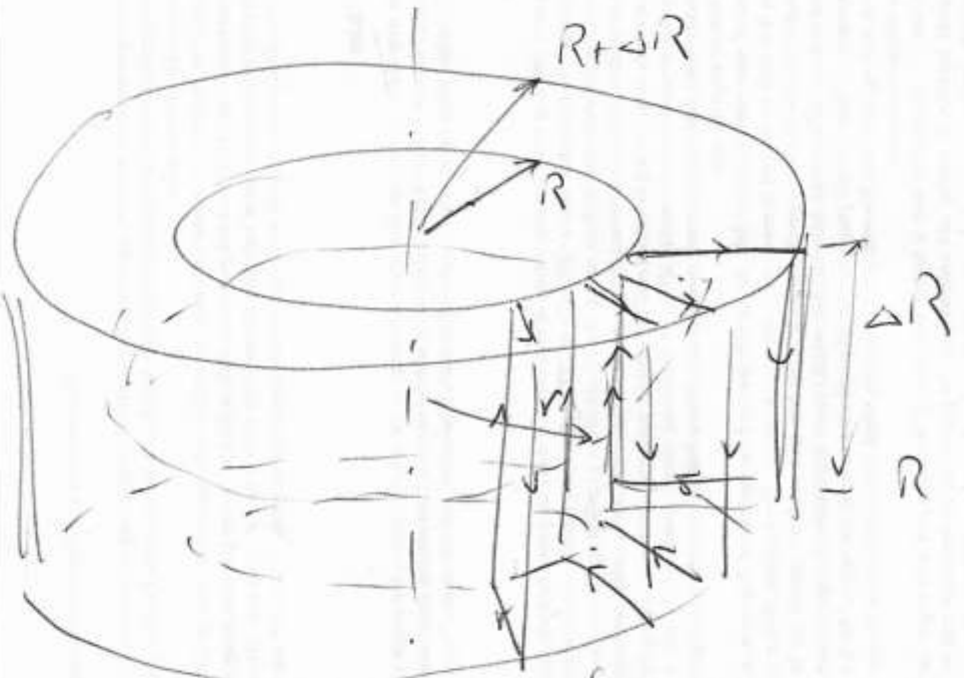


Solenoide spesso



Toroide = solenoide chiuso a ciambella

Toroide a sezione quadrata N spire quadrate



$$\vec{B}_0 = B_{0\theta}(r)\,\hat{e}_\theta$$

$$R < r < R+\Delta R$$

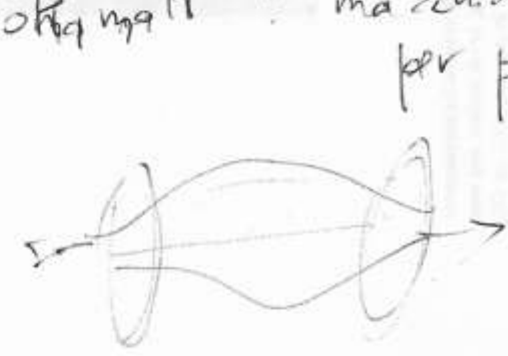
$$\oint \vec{B}_0 \cdot d\vec{l} = 2\pi r\, B_{0\theta}(r) = \mu_0 N I$$

$$B_{0\theta}(r) = \frac{\mu_0 N I}{2\pi r} \quad \text{solo interno}$$

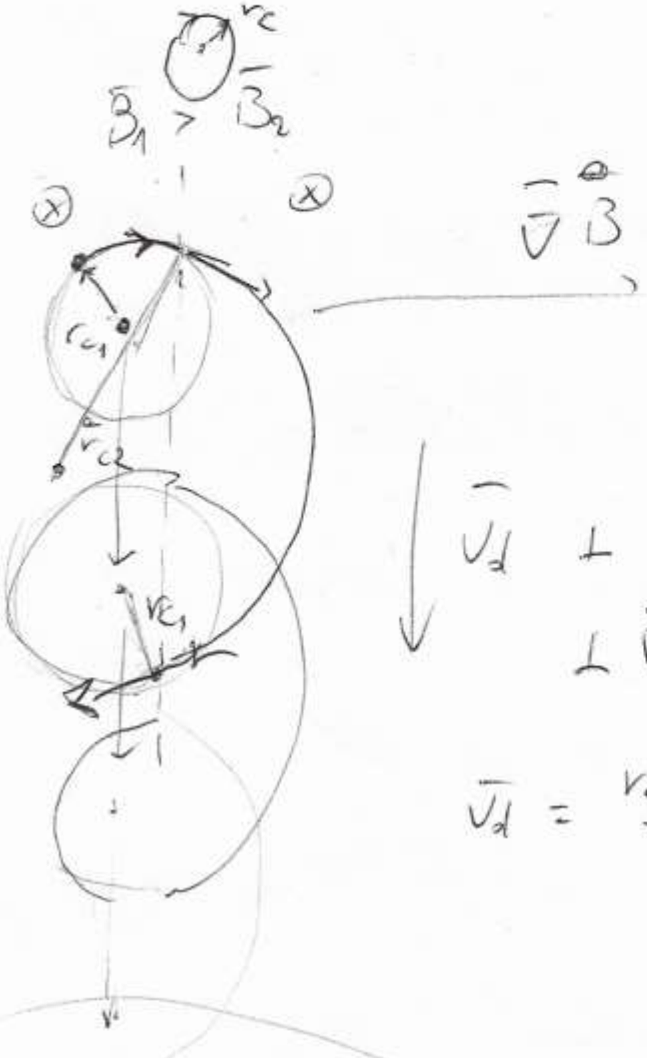
$$r > R+\Delta R \qquad I_{conc} = \emptyset$$

$$\frac{\Delta B_0}{B_0} = \frac{B_0(R) - B_0(R+\Delta R)}{B_0(R)} = R\left[\frac{1}{R} - \frac{1}{R+\Delta R}\right] = \frac{\Delta R}{R+\Delta R} \simeq \frac{\Delta R}{R}$$

Tokamak : macchina a confinamento magnetico per plasma da fusione



charge + $\bar{B}$ → cyclotron motion



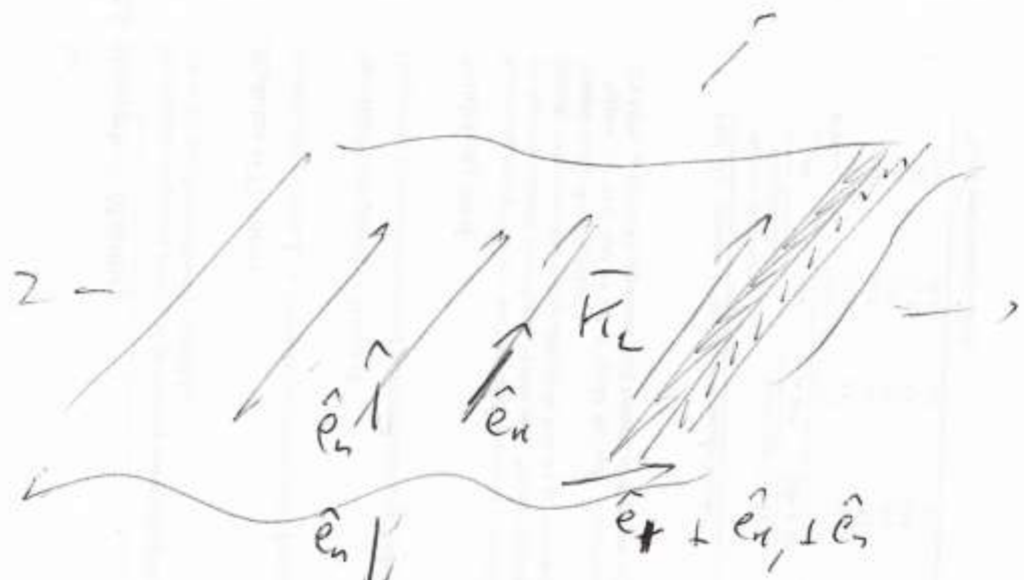
$\bar{v}_d \perp \bar{B}$ d = drift

$\perp \bar{\nabla} B$

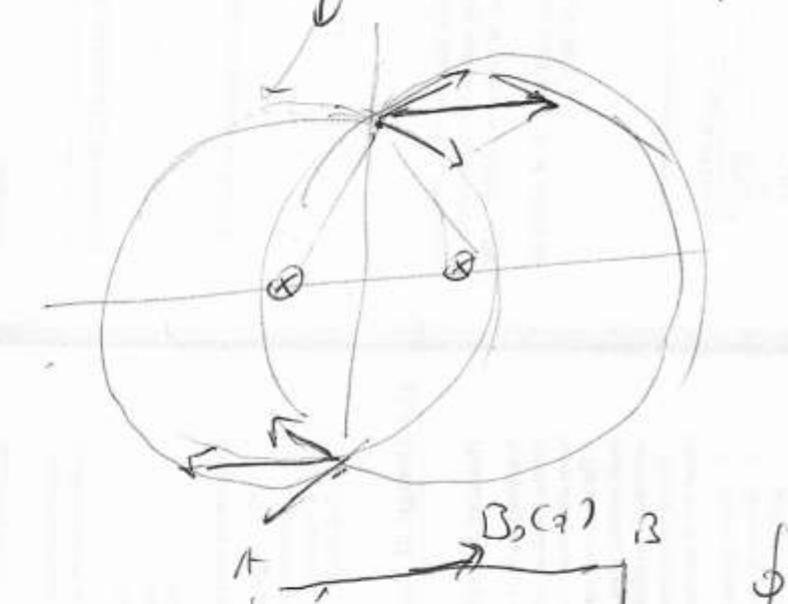
$$\bar{v}_d = \frac{r_c v_\perp}{2} \frac{\bar{B} \times \bar{\nabla} B}{B^2}$$

POLITECNICO DI MILANO
150°

Corrente piana indefinita (lamina ∞)



$\bar{B} = B\,\hat{e}_t$



$\oint \bar{B}_0 \cdot d\bar{l} = \int_A^B \bar{B}_0 \cdot d\bar{l} + \int_B^C + \int_C^D + \int_D^A$

$\int_A^B \to B_0(+)\,l$

$\int_B^C = \phi$

$\int_C^D = B_0(+)\,l$

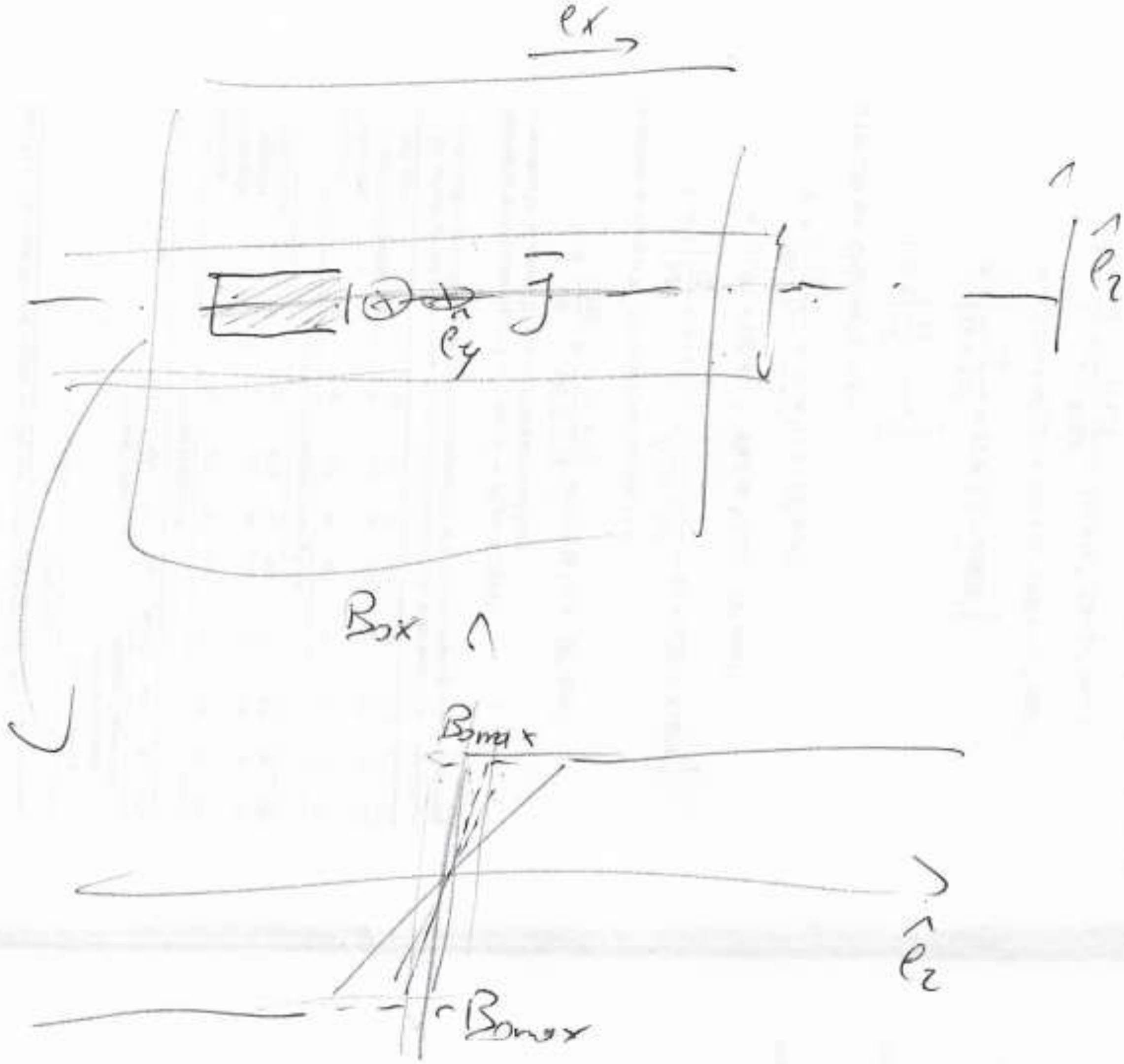
$\int_D^A = \phi$

$\oint \bar{B}_0 \cdot d\bar{l} = 2 B_0(+)\,l =$

$= \mu_0 K_L\, l$

$\Rightarrow B_0 = \frac{1}{2}\mu_0 K_L$ uniforme

$\bar{B}_0 = \frac{1}{2}\mu_0 \bar{K}_L \times \hat{e}_n$



e_x
$\vec{J}$
$\hat{e}_y$
$\hat{e}_z$
B_{0x}
B_{0max}
$-B_{0max}$
$\hat{e}_z$