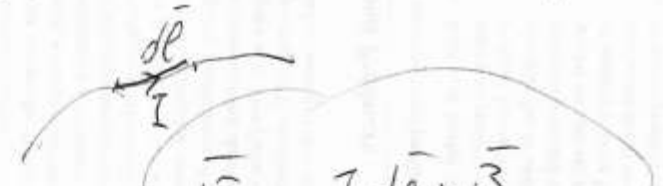
ESERC. 31/111/2020

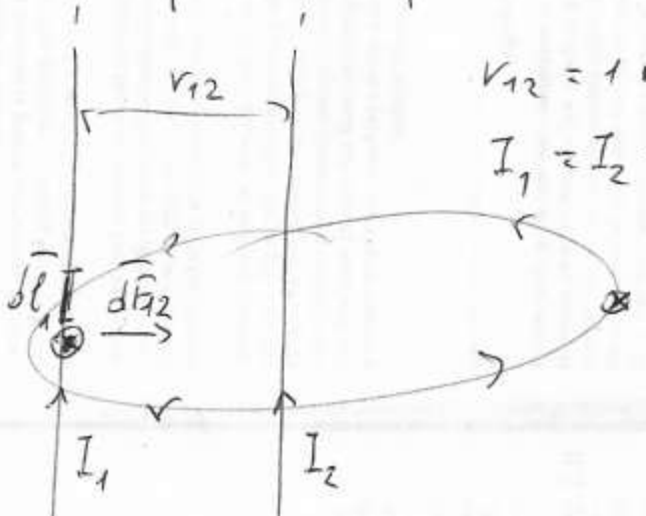
FORZA MAGNETICA

$I\,d\bar{l}$



$\bar{B}_0$ $\qquad d\bar{F}_M = I\,d\bar{l} \times \bar{B}_0$

Ampère ; operativamente definito tramite $\bar{F}_M$



$r_{12} = 1$ m

$I_1 = I_2 = 1$ A

$$\frac{\bar{F}_M}{L} = \frac{\mu_0}{2\pi} = 2 \cdot 10^{-7}\ \text{N/m}$$

$$dF_{12} = I_1 dl_1\, B_2(r_{12}) = I_1 dl_1 \frac{\mu_0 I_2}{2\pi r_{12}}$$

$$\frac{dF_{12}}{dl_1} = \frac{\mu_0 I_1 I_2}{2\pi r_{12}} = \frac{\mu_0}{2\pi}$$

1 m $\qquad I_1, I_2 = 1$ A

FORZA SU DIPOLI MAGNETICI

$\bar{m}$ $\quad\longleftrightarrow\quad$ $\bar{p}$

$$\begin{cases} U_M = -\bar{m}\cdot\bar{B} \\ \bar{F}_M = -\bar{\nabla} U = \bar{\nabla}(\bar{m}\cdot\bar{B}) \\ \bar{M}_M = \bar{m}\times\bar{B} \end{cases}$$

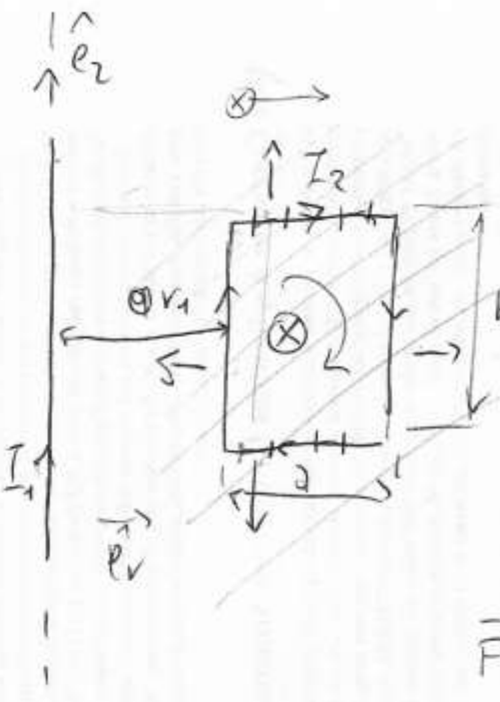
$$U_E = -\bar{p}\cdot\bar{E}$$

$$\bar{F}_E = -\bar{\nabla} U_E = \bar{\nabla}(\bar{p}\cdot\bar{E}) = (\bar{p}\cdot\bar{\nabla})\bar{E}$$

$$\bar{M}_E = \bar{p}\times\bar{E}$$

$$\boxed{\vec F_m = \vec\nabla(\vec m\cdot\vec B)} = \dots = m\times\overset{\neq \emptyset}{(\vec\nabla\times\vec B)} + (\vec m\cdot\vec\nabla)\vec B$$

$$\vec F_E = \vec\nabla(\vec p\cdot\vec E) = \quad = \underbrace{\cancel{\vec\nabla\times\vec E}}_{\emptyset} + (\vec p\cdot\vec\nabla)\vec E$$



$a = 5\,cm$

$b = 20\,cm$

$r_1 = 5\,cm$

$I_1 = 10\,A$

$I_2 = 1\,A$

$$\bar{F} = \oint_\gamma I_2 d\bar{l}_2 \times \bar{B}_1$$

$$B_{1\theta}(r) = \frac{\mu_0 I_1}{2\pi r}$$

$F_z = \phi$ complessivamente

$$\bar{F}_{sx} = \int I_2 d\bar{l} \times \bar{B}_1(r_1) = -I_2 b B_1(r_1)\hat{e}_r$$

$$\bar{F}_{dx} = \int I_2 d\bar{l} \times B_1(r_1+a) = I_2 b B_1(r_1+a)\hat{e}_r$$

$$B_1(r_1) = \frac{\mu_0 I_1}{2\pi r_1};$$

$$B_1(r_1+a) = \frac{\mu_0 I_1}{2\pi(r_1+a)}$$

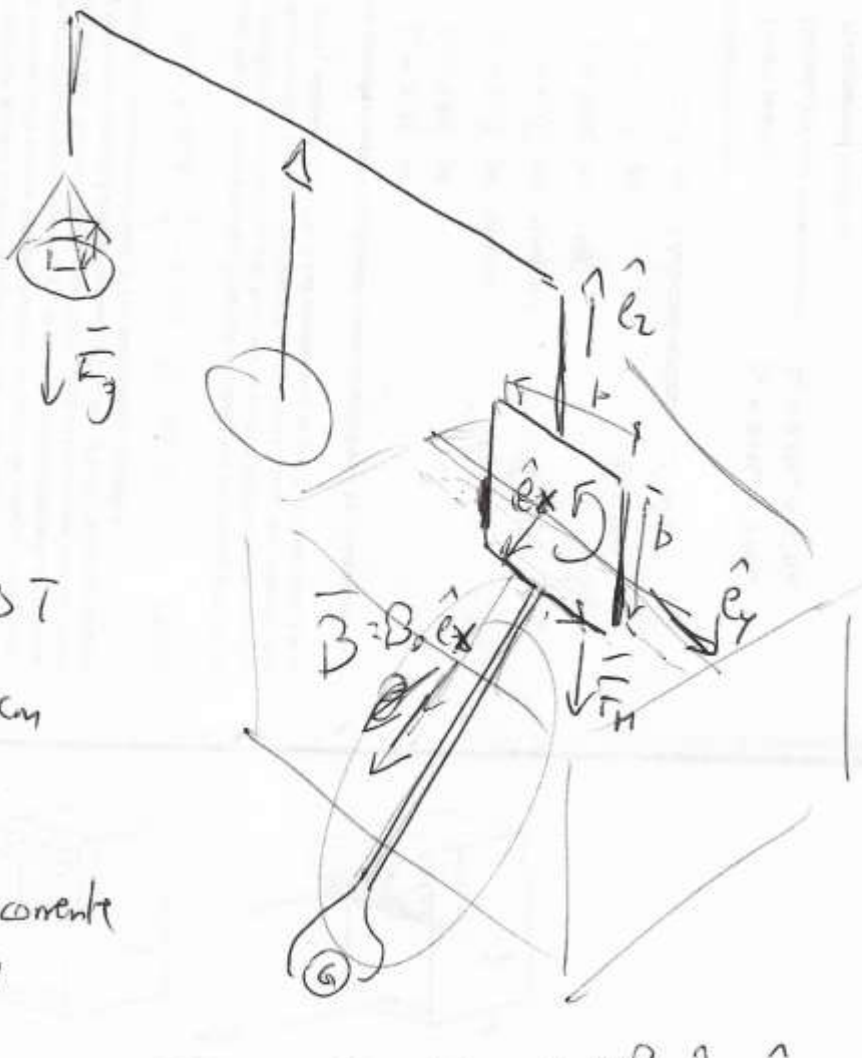
$$\bar{F} = F_r \hat{e}_r = \ominus$$

$$= \frac{\mu_0 I_1 I_2 b}{2\pi}\left(\frac{1}{r_1+a} - \frac{1}{r_1}\right)\hat{e}_r =$$

$$= - \frac{\mu_0 I_1 I_2 a b}{2\pi r_1 (r_1+a)}\hat{e}_r$$

$$|\bar{F}_r| = 4 \cdot 10^{-6}\,N$$

Bilanciamento magnetico



$B_0 = 0.3\,T$

$b = 10\,cm$

$I = 1\,A$

Verso corrente

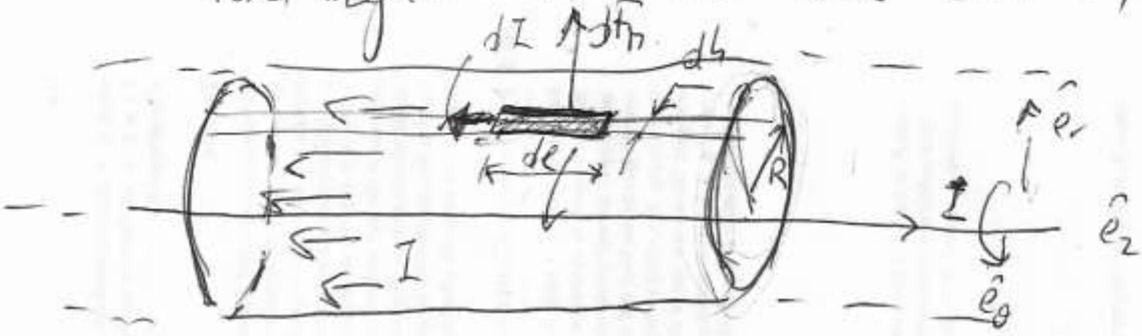
$m = ?$

$$d\bar{F} = I\,d\bar{\ell} \times \bar{B} = I\,d\ell\,B_0\,\hat{e}_y \times \hat{e}_x = -I\,d\ell\,B_0\,\hat{e}_z$$

I antiorario $\quad \bar{F} = \int_{\phi}^{b} -I B_0\,d\ell\,\hat{e}_z = -I B_0 b\,\hat{e}_z$

$$I b B_0 = mg \Rightarrow m = IB_0 b/g = 3\,g$$

Forza magnetica in coax cable (rettilineo)



$$d\bar{F}_M \sim \hat{e}_r$$

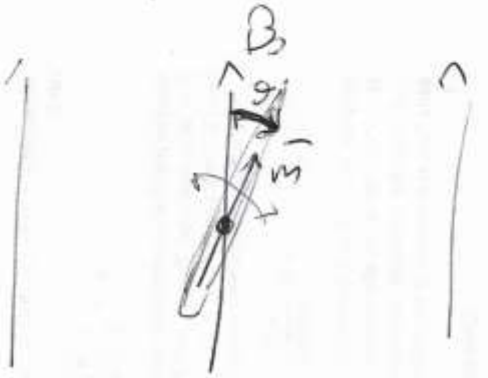
$$dI = I\frac{dh}{2\pi R}$$

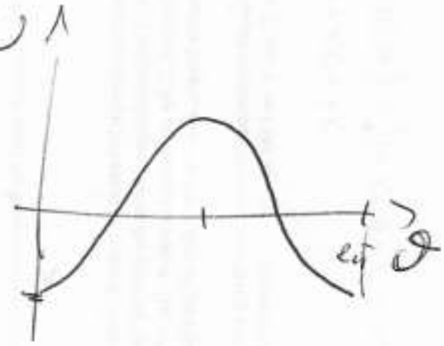
$$B_{o\theta} = \frac{\mu_0 I}{2\pi R}$$

$$dF = dI\, dl\, B_{o\theta}(R) = \frac{I\,dh}{2\pi R}\, dl\, \frac{\mu_0 I}{2\pi R}$$

$$p_M = \frac{dF}{dh\, dl} = \mu_0\left(\frac{I}{2\pi R}\right)^2$$

FORZA / MOMENTO MAGNETICA SU $\bar{m}$





$$U = -\bar{m}\cdot\bar{B} = -m B_0 \cos\vartheta$$

$$\bar{M} = \bar{m}\times\bar{B}$$

$$M = m B_0 \sin\vartheta \quad (\simeq m B_0 \vartheta)$$

$$\Rightarrow \quad M = -m B_0 \sin\vartheta \simeq -m B_0 \vartheta$$

Ago: $\rightarrow$ I mom. di inerzia

$$M = I \frac{d\omega}{dt} = I \frac{d^2\vartheta}{dt^2}$$

$$\omega = \dot{\vartheta} = d\vartheta/dt$$

$$I \frac{d^2\vartheta}{dt^2} = -m B_0 \sin\vartheta \simeq -m B_0 \vartheta$$

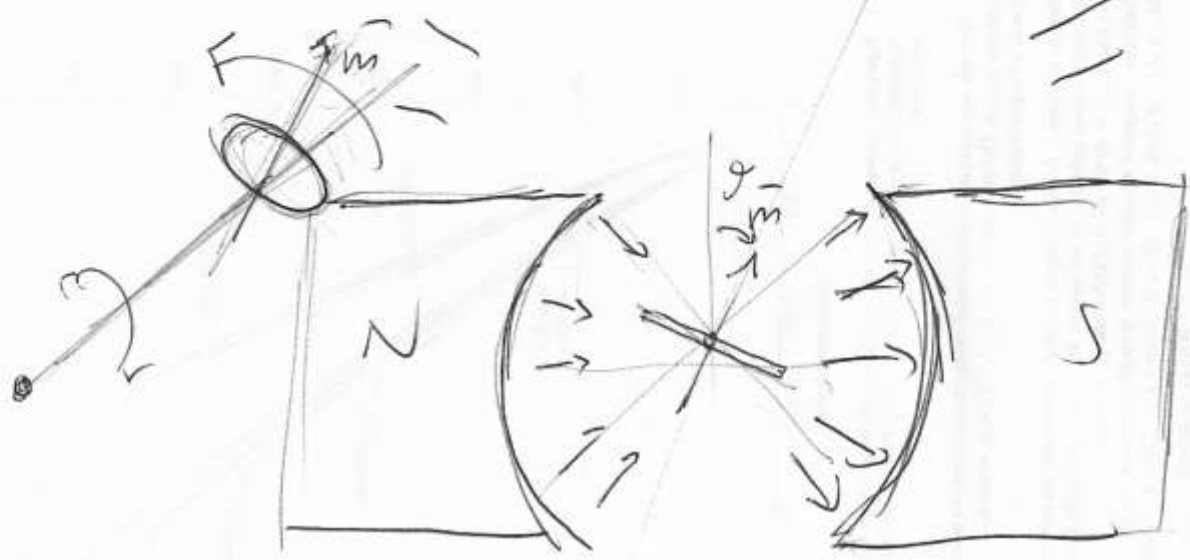
$$\frac{d^2\vartheta}{dt^2} = -\omega_0^2 \vartheta \qquad \omega_0 \doteq \sqrt{m B_0 / I}$$

$$\vartheta(t) = \vartheta_{max} \sin(\omega_0 t + \varphi)$$

$$M = -\frac{dU}{d\vartheta}$$

Galvanometro di D'Arsonval [A]

↳ misura corrente

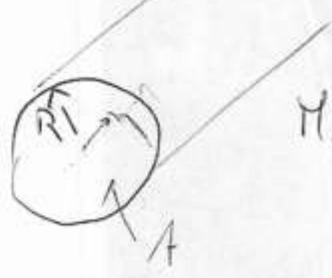


N spire quadrate di lato d, I

$m = NIS = NId^2$

$M = |\bar{m} \times \bar{B}| = mB = NId^2B$

Mom. di torsione del filo $\sim k\vartheta$



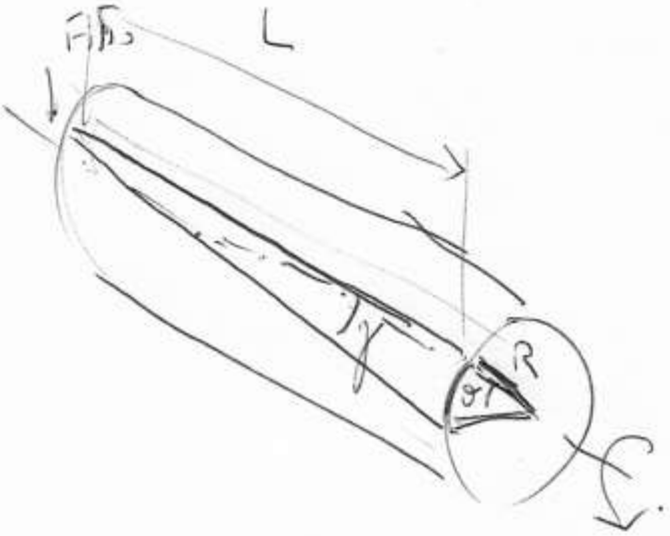
$$M_t = \int r\tau_t \, dA = \frac{\tau_{t\,max}}{R} \underbrace{\int r^2 dA}_{J_z} =$$

J_z II momento dell'area

$$\tau_t(r) = \tau_{t\,max} \, r/R$$

$$J_z = \int r^2 dA = \int_0^{2\pi}\int_0^R r^2 \, r \, dr \, d\vartheta = 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2}$$

Relazione costitutiva $\tau_t = G\gamma$ G modulo di elasticità torsionale



$$L\gamma = R\vartheta$$

$$\tau_t(r) = G\gamma = G\vartheta\, r/L$$

$$\tau_{t\,max} = G\vartheta R/L$$

$$M_t = \frac{\tau_{t\,max} J_2}{R} = \frac{G\vartheta R}{L}\cdot\frac{1}{R}\,\frac{\pi R^4}{2} = \frac{\pi}{2}\,\frac{GR^4\vartheta}{L}$$

$$M_t = M_m$$

$$\frac{\pi}{2}\,\frac{GR^4\vartheta}{L} = NId^2B$$

$$\vartheta = \frac{2NId^2BL}{\pi G R^4} = 2.75 \text{ mrad} \simeq 0.16^\circ$$

$N = 20$

$d = 15$ mm

$B = 0.2$ T

$L = 150$ mm

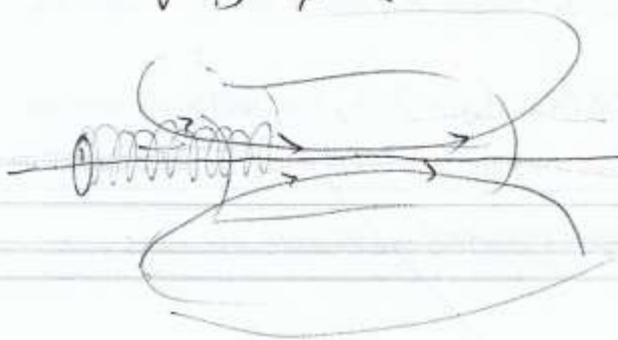
$R = 50\ \mu$m

$G = 5\cdot 10^8$ N/m^2

$I = 0.1\ \mu$A

Magnetic mirror / Magn. bottle

$\bar{\nabla}\cdot\bar{B} = 0$



$\frac{\partial B_z}{\partial z} \neq 0$

① Cyl. symmetry

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{\partial}{\partial r}(rB_r) = -r\frac{\partial B_z}{\partial z} \leadsto rB_r = -\frac{1}{2}r^2\frac{\partial B_z}{\partial z}$$

$$\boxed{B_r = -\frac{1}{2}r\frac{\partial B_z}{\partial z}}$$

$-e$

cyclotron

$\bar{v} \doteq v_\perp \hat{e}_\vartheta$

$$\bar{a} = \frac{-e}{m}v_\perp \hat{e}_\vartheta \times \bar{B}_r = -\frac{e}{m}v_\perp \hat{e}_\vartheta\left(-\frac{1}{2}r_c\frac{\partial B_z}{\partial z}\hat{e}_r\right) =$$

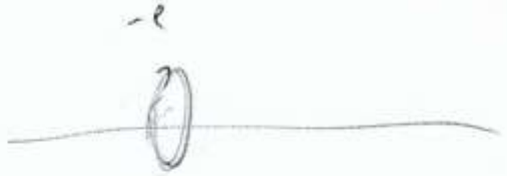
$$= -\frac{e v_\perp r_c}{2m}\frac{\partial B_z}{\partial z}\hat{e}_z\frac{B_0}{B_0} \qquad r_c = \frac{v_\perp}{\omega_c}$$

$B \quad < B'$

$$= -\frac{v_\perp \omega_c r_c}{2}\frac{1}{B_0}\frac{\partial B_z}{\partial z}\hat{e}_z =$$

$$= -\frac{v_\perp^2}{2}\underbrace{\frac{1}{B_0}\frac{\partial B_z}{\partial z}}_{\to 1/l_c}\hat{e}_z = -\frac{v_\perp^2}{2l_c}\hat{e}_z$$

$$F_z = m a_z = -\frac{m v_\perp^2}{2B_0}\frac{\partial B_z}{\partial z} = -\mu\frac{\partial B_z}{\partial z}$$



$$T = \frac{2\pi}{\omega_c} = \frac{2\pi r_c}{v_\perp}$$

$$I = \frac{-e v_\perp}{2\pi r_c}$$

$$m = IS = I \pi r_c^2 = -e \frac{v_\perp}{2\pi r_c} \pi r_c^2 = -\frac{e v_\perp}{2} r_c = -\frac{m v_\perp^2}{2B}$$

$$\bar F = \bar\nabla(\bar m \cdot \bar B) = \bar\nabla(m B_z) = m \bar\nabla B_z = m \frac{\partial B_z}{\partial z} = -\mu \frac{\partial B_z}{\partial z}$$

$$\bar m = -\frac{m v_\perp^2}{2B_0} \hat e_z = -\mu \hat e_z$$

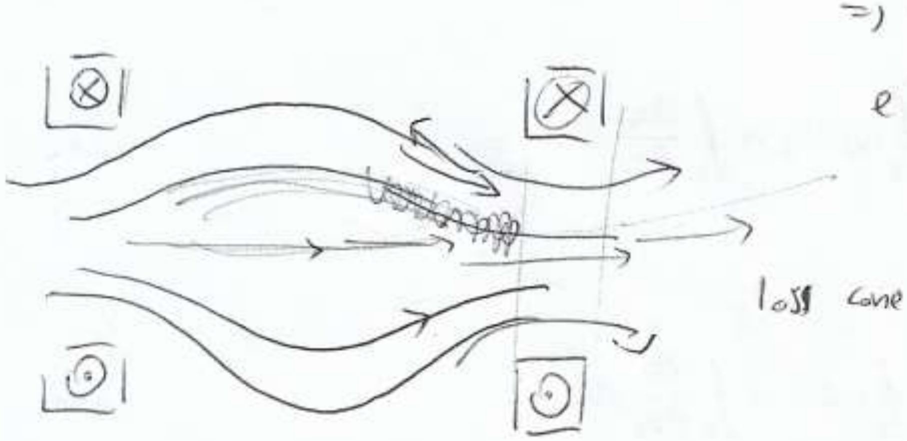
ignoring $\partial B_z / \partial r$

$v_{\perp 0} \quad v_{z0} \rightarrow \odot \downarrow$

$\frac{v_{z0}}{v_{\perp 0}}$ suff. piccolo

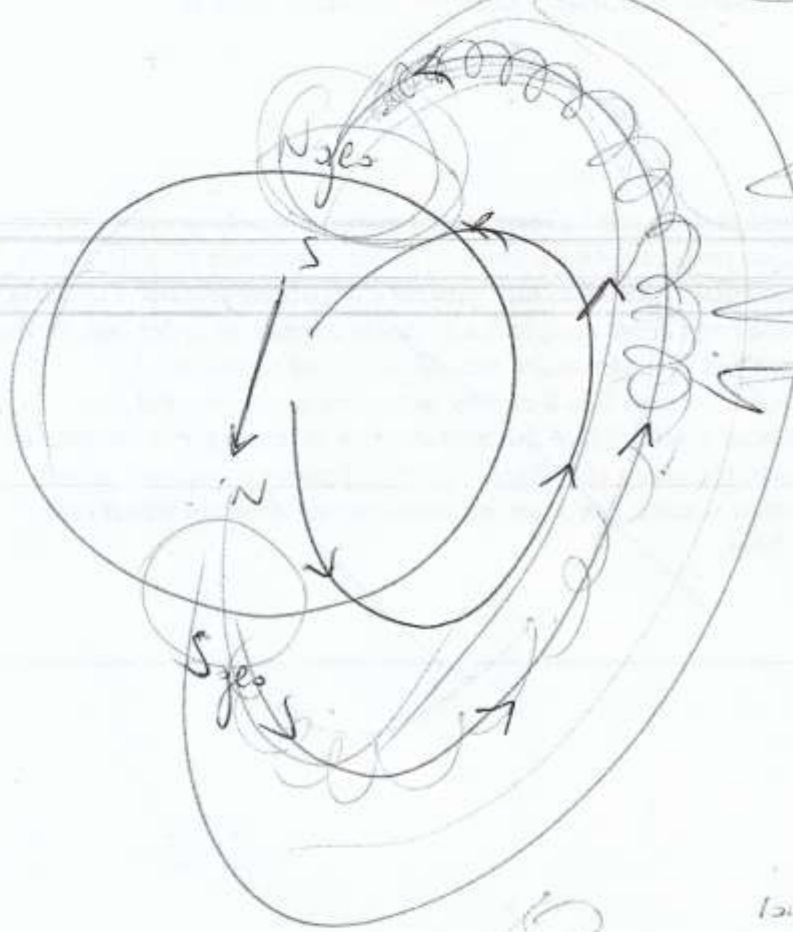
$\Rightarrow$ frenamento completo

e inversione



magn. bottle

Van Allen belt   NO VAN HALEN



Wind
Solar

magnetosphere

Northern / Southern lights
(auroras)

Ionosphere
telecommunications