ESERC. 06/IV/2020

$\bar{B} + \bar{E}$; $\bar{F}_M$; POT. VETTORE ; RELATIVITÀ

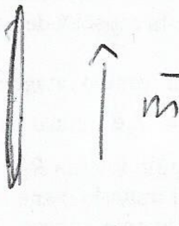
Misura $\bar{B}_g$ comp. orizzontale $\bar{B}_{Terra}$

(1) Magnete permanente a barretta

$\sim\!> \bar{\mu}$

I mom. di inerzia

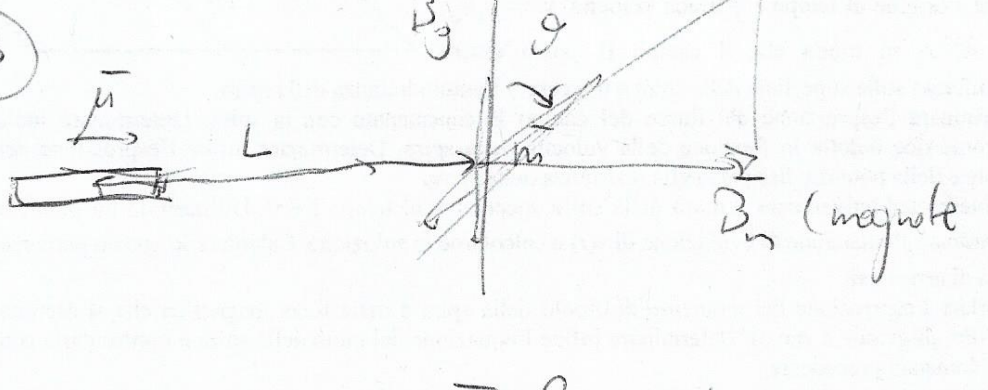
(2) Ago magnetico (ferromagnetico)



$\bar{m} \parallel \bar{B}_{ext}$ minimo di U_M

(A)

(B)



Quanto vale $\bar{\mu}$ e $\Rightarrow \bar{B}_m$? Det. da piccole oscillazioni in $\bar{B}_g$

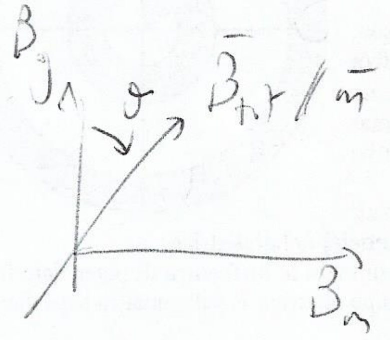
B_{ext} μ

$$\omega = \sqrt{\frac{\mu B_{ext}}{I}}$$

oriz.

=) per magnete a bussola

$$\mu = \omega^2 I / B_g$$



$$B_g = B_{tot} \cos\vartheta = \frac{B_m}{\sin\vartheta} \cos\vartheta = \frac{B_m}{tg\,\vartheta}$$

$$B_m = \frac{\mu_0}{2\pi} \frac{\mu}{L^3}$$

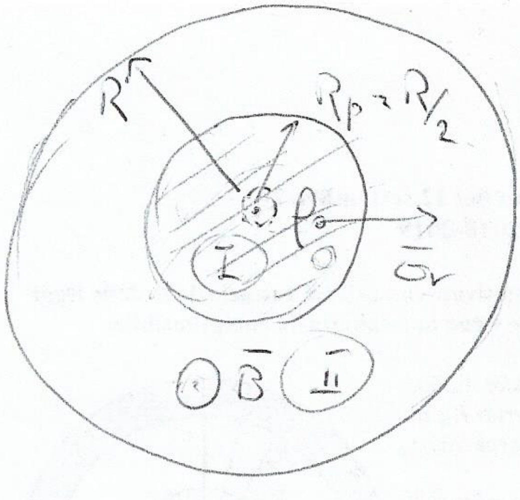
$$B_g = \frac{1}{tg\,\vartheta} \frac{\mu_0}{2\pi} \frac{\mu}{L^3} = \frac{1}{tg\,\vartheta} \frac{\mu_0}{2\pi L^3} \frac{\omega^2 I}{B_g}$$

$$\Rightarrow B_g = \frac{\mu_0 \omega^2 I}{2\pi L^3 tg\,\vartheta} = 0.5 \cdot 10^{-4}\ T \quad (0.5\ \text{Gauss})$$

$$I = 3.17 \cdot 10^{-3}\ kg\, m^2$$

$$\nu = \frac{\omega}{2\pi} = 0.1\ Hz$$

$$\vartheta = \pi/4 \quad (45°)$$



$\bar{B} = B\hat{e}_z$

(Trappola di Penning) ciclotrone

$E_r + B_z \Rightarrow \perp$

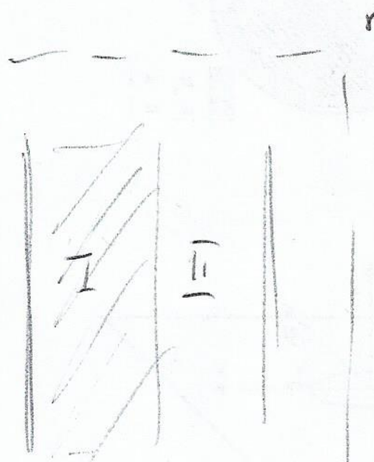
$\bar{v} = \frac{\bar{E} \times \bar{B}}{B^2}$

B.C. $\phi(r = R) = 0$

$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$ cord. cil. (invarianza in θ, z)

$\Rightarrow \phi = \phi(r)$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = -\frac{\rho(r)}{\varepsilon_0} \qquad \rho(r) = \begin{cases} \rho_0 & 0 < r < R_p = R/2 \\ 0 & R/2 < r < R \end{cases}$$



I $\quad \frac{d}{dr}\left(r\frac{d\phi}{dr}\right) = -\frac{\rho}{\varepsilon_0} r$

$$r\frac{d\phi}{dr} = -\frac{\rho r^2}{2\varepsilon_0} + C_1$$

$$(-E_r) = \frac{d\phi}{dr} = -\frac{\rho r}{2\varepsilon_0} + \frac{C_1}{r}$$

$r = 0 \Rightarrow C_1 = 0$

$$\boxed{\phi(r) = -\frac{\rho}{4\varepsilon_0} r^2 + C_2} \qquad r < R/2$$

(II) $\cancel{\frac{1}{r}}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)=0$

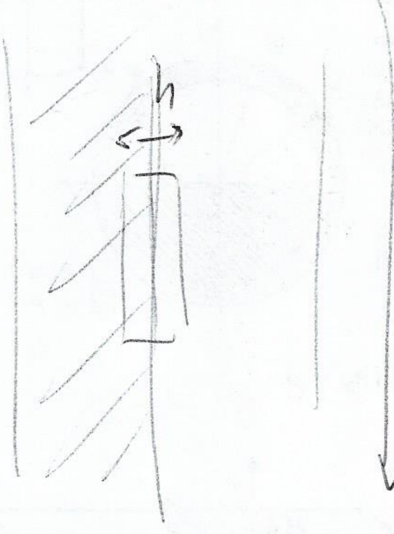
$$r\frac{d\phi}{dr}=C_3 \qquad \frac{d\phi}{dr}=\frac{C_3}{r}$$

$$\boxed{\phi(r)=C_3\log r+C_4}$$

$$\phi(r=R)=0 \qquad -C_3\log(R)=+C_4$$

$$\boxed{\phi(r)=C_3\log\left(\frac{r}{R}\right) \qquad \frac{R}{2}<r<R}$$

$r=\frac{R}{2}$



E discontinuo se $\exists\sigma$

$h\to 0$ $\begin{cases} Q=\rho V=\rho S h \to 0 & \text{CONT.} \\ Q=\sigma S & \text{rimane finito DISC.} \end{cases}$

Continui E_r, ϕ

$$E_r^{I}(r=R_2)=E_r^{II}(r=R/2)$$

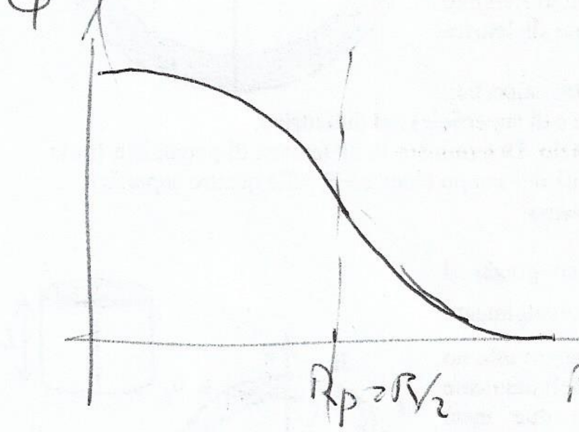
$$-\frac{\rho}{2\varepsilon_0}\frac{R}{2}=2\frac{C_3}{R} \Rightarrow C_3=-\frac{\rho R^2}{8\varepsilon_0}$$

$$\phi^{I}(R/2)=\phi^{II}(R/2)$$

$$-\frac{\rho}{4\varepsilon_0}\frac{R^2}{4}+C_2=-\frac{\rho R^2}{8\varepsilon_0}\log\left(\frac{1}{2}\right) \to -\log 2$$

$$C_2 = \frac{\rho R^2}{16\varepsilon_0}\left(1+\log 4\right) \qquad 2\log 2 = \log 4$$

$$\phi(r) = \begin{cases} \frac{\rho}{4\varepsilon_0}\left[r^2 - R^2\left(\frac{1+\log 4}{4}\right)\right] & r < R/2 \\ \frac{\rho R^2}{8\varepsilon_0}\log\left(\frac{r}{R}\right) & \frac{R}{2} < r < R \end{cases}$$

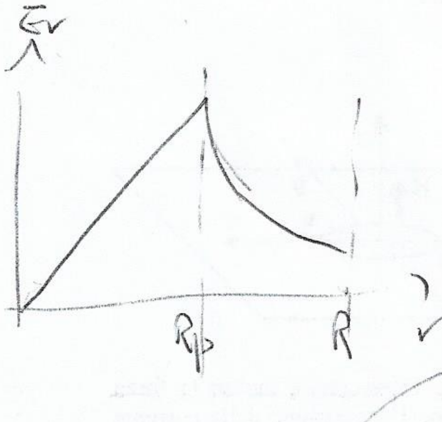


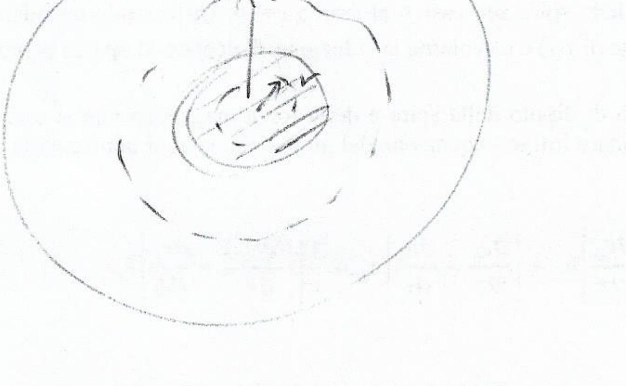
$$\rho = -en \quad m^{-3}$$

$$\omega_{pe}^2 = \frac{ne^2}{\varepsilon_0 m}$$

$$E_r = -\frac{d\phi}{dr}$$

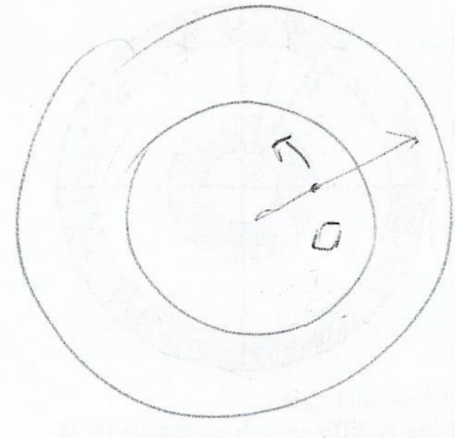
$$\begin{cases} \frac{\rho}{2\varepsilon_0} r = (-)\frac{m}{2e}\omega_{pe}^2 r \\ -\frac{\rho R^2}{8\varepsilon_0}\frac{1}{r} = (+) - \frac{m}{8e}\omega_{pe}^2 \frac{1}{r} \end{cases}$$





$$\bar{v} = \frac{\bar{E} \times \bar{B}}{B^2} = -\frac{\bar{E}_r}{B} = \frac{en}{2\varepsilon_0 B} r \, \hat{e}_\vartheta \quad \text{(elettroni)}$$

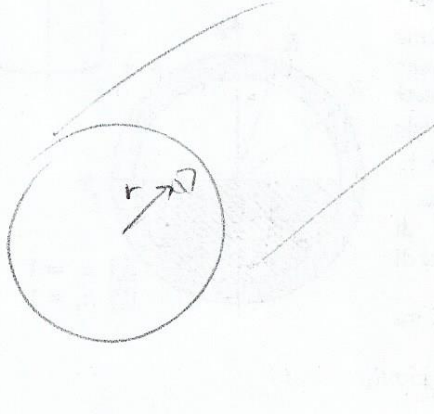
$$-\hat{e}_r \times \hat{e}_z = \hat{e}_\vartheta$$



$$\bar{v}(r) = v_\vartheta(r)\, \hat{e}_\vartheta$$

$$\omega = \frac{v_\vartheta}{r} = \frac{en}{2\varepsilon_0 B} \quad \left(= \frac{m}{2eB} \omega_{pe}^2 \right)$$

Radial force balance



$$\hat{e}_r : \; F_{\text{centrifuga}} + F_e + F_m = \emptyset$$

$$F_{\text{centripeta}} = F_e + F_L$$

$$\boxed{-\frac{m v_\vartheta^2}{r} = -eE_r - e\underbrace{v_\vartheta B}_{\bar{v} \times \bar{B}}}$$

$$\omega(r) = \frac{v_\vartheta(r)}{r}$$

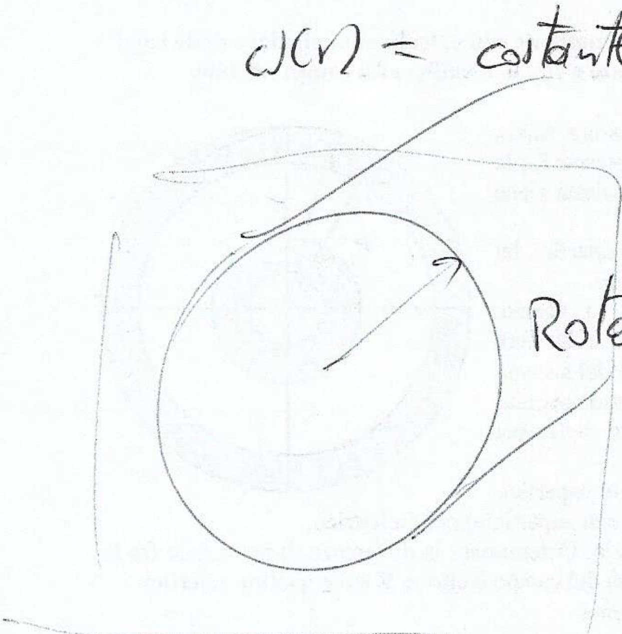
$$-m\omega^2(r) r = -e\left(-\frac{m}{2e}\omega_{pe}^2 r\right) - e\omega(r) r B$$

$$\frac{eB}{m} = \omega_c$$

$$\Rightarrow \boxed{\omega^2 + \omega_c \omega - \frac{1}{2}\omega_{pe}^2 = \emptyset}$$

$$\omega(r) = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_{pe}^2}{2}} = \frac{\omega_c}{2}\left\{1 \pm \left(1 - \frac{2\omega_{pe}^2}{\omega_c^2}\right)^{\frac{1}{2}}\right\}$$

$$\omega(r) = \text{costante}$$

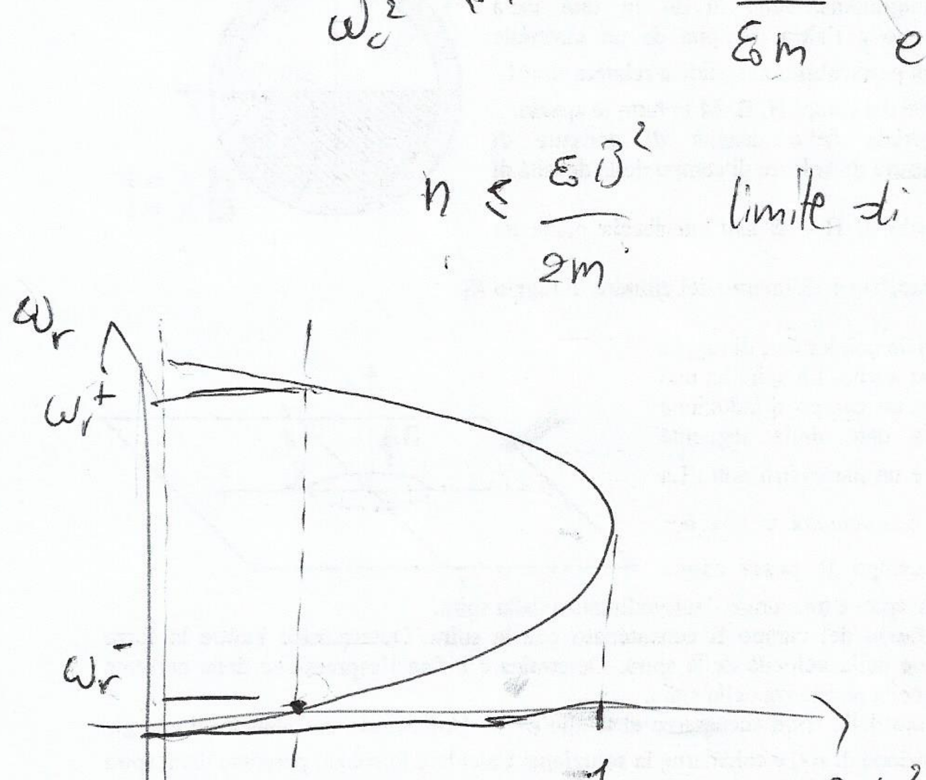


Rotazione rigida (eq. rotazionale)

Equilibrio nel modello $\bar{E} \times \bar{B}$

$$\frac{2\omega_{pe}^2}{\omega_c^2} \lesssim 1 \qquad 2\frac{ne^2}{\varepsilon_0 m}\,\frac{m^2}{e^2 B^2} = \frac{2nm}{\varepsilon_0 B^2} \lesssim 1$$

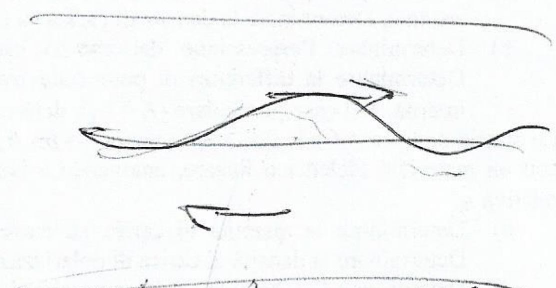
$$n \lesssim \frac{\varepsilon_0 B^2}{2m}$$ limite di Brillouin



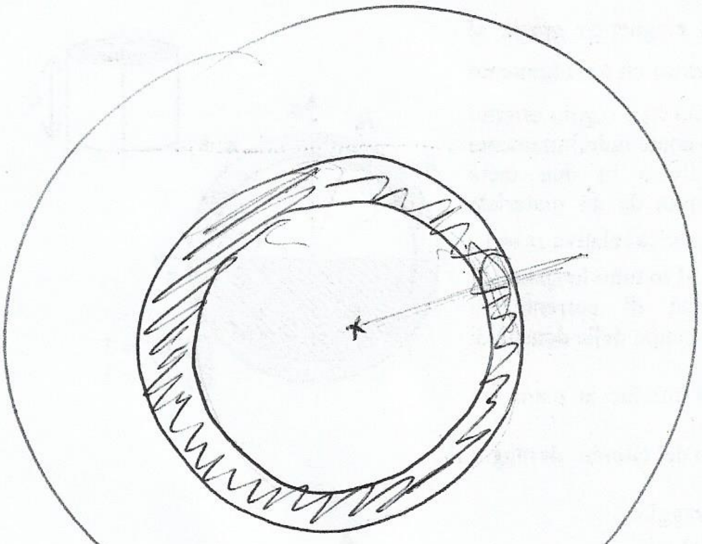
Trappola di Penning-Malmberg

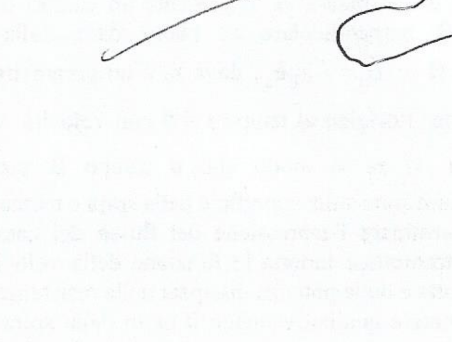
$n(r) \neq \text{cost.}$

$\omega(r) = f(r)$



instabilità di Kelvin-Helmholtz





$\bar{A}$ potenziale vettore

$\bar{A}$ def. $\quad \bar{B}_0 = \bar{\nabla} \times \bar{A}_0$

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} \quad F$$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{A}) = \mu_0 \bar{J}$$

$$-\nabla^2 \bar{A} + \cancel{\bar{\nabla}(\bar{\nabla} \cdot \bar{A})} = \mu_0 \bar{J}$$

$$\nabla^2 \bar{A} = -\mu_0 \bar{J}$$

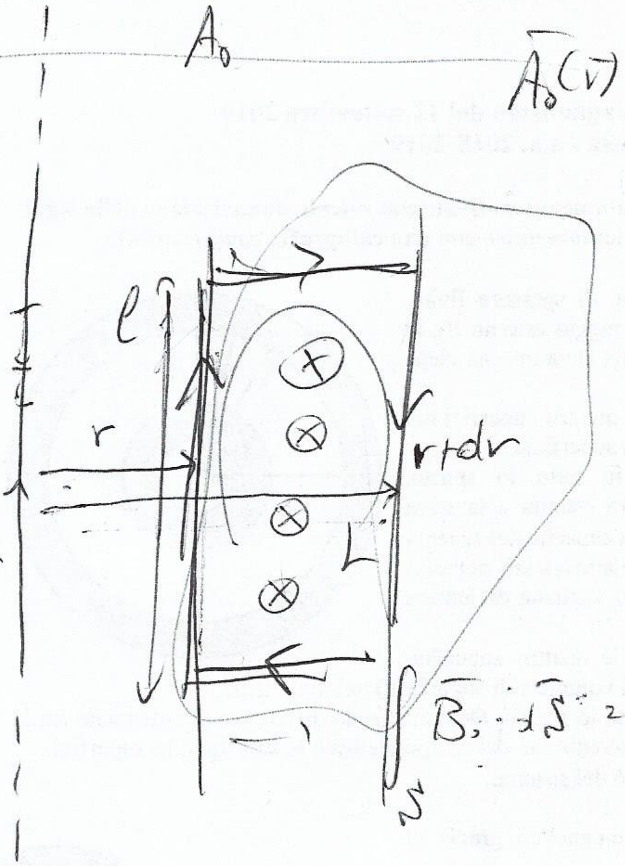
$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$$

$$\bar{A}_0(\bar{r}) = \frac{\mu_0}{4\pi} \int_\tau \frac{\bar{J}(\bar{r}')}{|\bar{r}-\bar{r}'|} \overset{dS'\,dl'}{d\tau'} = \frac{\mu_0}{4\pi} \oint \frac{I\, d\bar{l}'(\bar{r}')}{|\bar{r}-\bar{r}'|}$$

$$\phi(r) = \frac{\mu_0}{4\pi} \int_\tau \frac{\rho(r')}{|r-r'|}\, d\tau'$$

$$\bar{B}_0 = \bar{\nabla} \times \bar{A}_0 \quad F$$

$$\int_S \bar{B}_0 \cdot d\bar{S} = \int_S (\bar{\nabla} \times \bar{A}_0) \cdot d\bar{S} \underset{\text{Stokes}}{=} \oint_{C=\partial S} \bar{A}_0 \cdot d\bar{l}$$



$$\bar{A}_0(\bar{r}) = \frac{\mu_0}{4\pi}\int \frac{I\,d\bar{l}(\bar{r}')}{|\bar{r}-\bar{r}'|}$$

$$d\bar{A}_0 \propto I\,d\bar{l} \leftarrow \hat{e}_z$$

$$\bar{A}_0(\bar{r}) = A_{0z}(\bar{r})\,\hat{e}_z$$

$$(r, \vartheta, z) \Rightarrow \bar{A}_0 = A_{0z}(r)\,\hat{e}_z$$

$$\int_S \bar{B}_0 \cdot d\bar{S} = \oint_{C=\partial S} \bar{A}_0 \cdot d\bar{l}$$

$$B_0(r)\,l\,dr = A_{0z}(r)\,l - A_{0z}(r+dr)\,l =$$

$$= l\left[A_{0z}(r) - (A_{0z}(r) + dA_{0z})\right] = -l\,dA_{0z}$$

$$\frac{\mu_0 I}{2\pi r}\,l\,dr = -l\,dA_{0z}$$

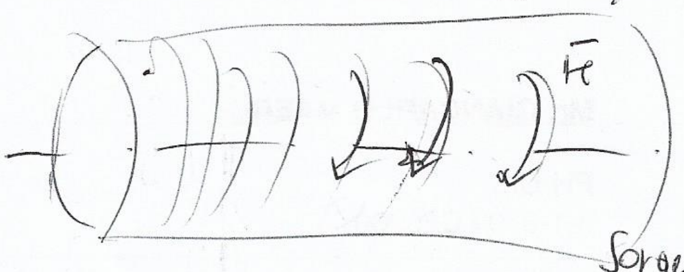
$$\Rightarrow dA_{0z}(r) = -\frac{\mu_0 I}{2\pi r}\,dr$$

$$A_{0z}(r) - \underbrace{A_{0z}(r_0)}_{0} = \int_{r_0}^{r} dA_{0z} = -\frac{\mu_0 I}{2\pi}\log\left(\frac{r}{r_0}\right)$$

$$\bar{B} = \bar{\nabla}\times\bar{A}$$

$\bar{A}_0$ di solenoide rettilineo ∞, I, R

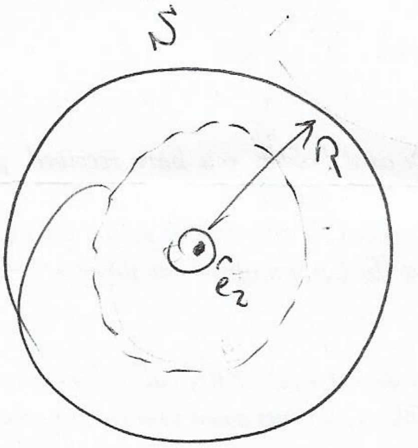
nI $\quad I, n \qquad H = nI = \frac{N}{L} I$



Sorgenti $\parallel \hat{e}_\theta \implies \bar{A}_0(\bar{r}) = A_{0\theta}\,\hat{e}_\theta$

Invarianza rot. θ
trasl. z $\implies \bar{A}_0(\bar{r}) = A_{0\theta}(r)\,\hat{e}_\theta$

$$\int_S \bar{B}_0 \cdot d\bar{S} = \oint_{C=\partial S} \bar{A}_0 \cdot d\bar{l}$$



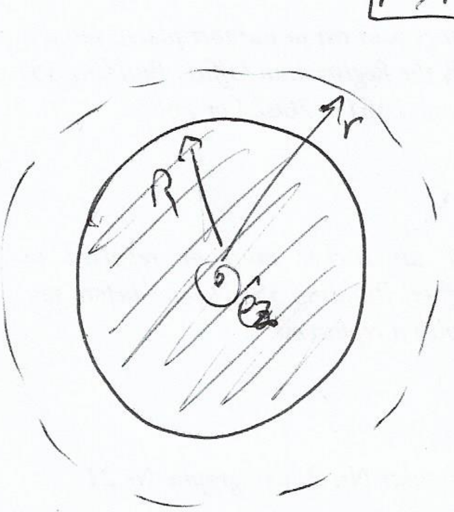
$\boxed{r<R}$

$$B_{0z}\,\pi r^2 = A_{0\theta}(r)\,2\pi r$$

$$\implies A_{0\theta}(r) = \frac{1}{2} B_{0z}\, r = \frac{1}{2}\mu_0 n I r$$

$B_{0z} = \mu_0 n I$

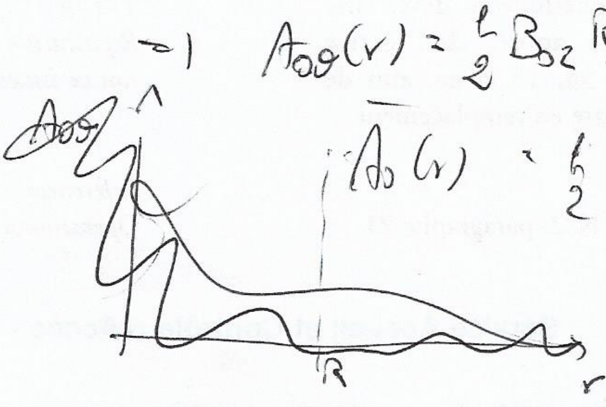
$\boxed{r>R}$



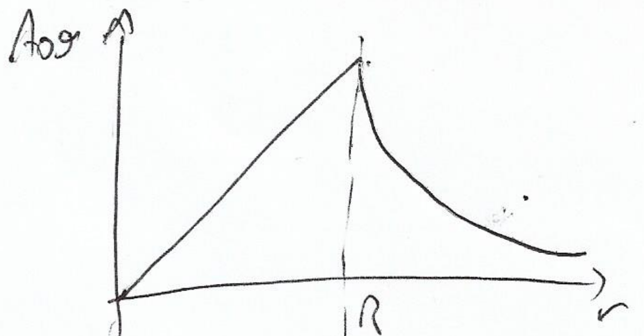
$$B_{0z}\,\pi R^2 = A_{0\theta}(r)\,2\pi r$$

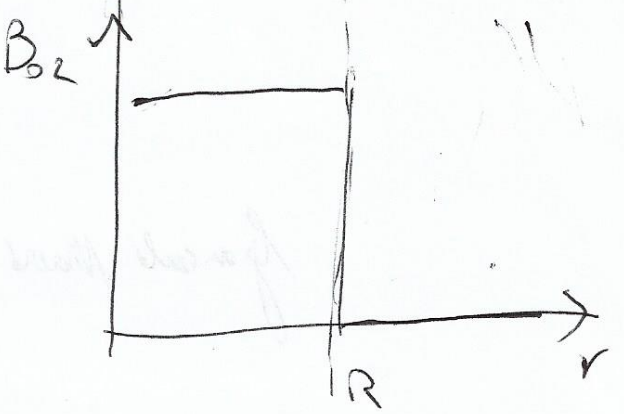
$$\implies A_{0\theta}(r) = \frac{1}{2} B_{0z} \frac{R^2}{r} = \frac{1}{2}\mu_0 n I R^2 \frac{1}{r}$$

$$\bar{A}_0(r) = \frac{1}{2}\mu_0 n I \frac{R^2}{r}\,\hat{e}_\theta$$

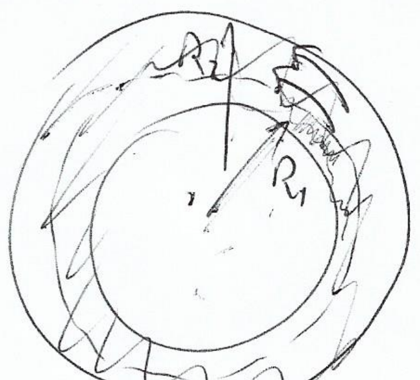


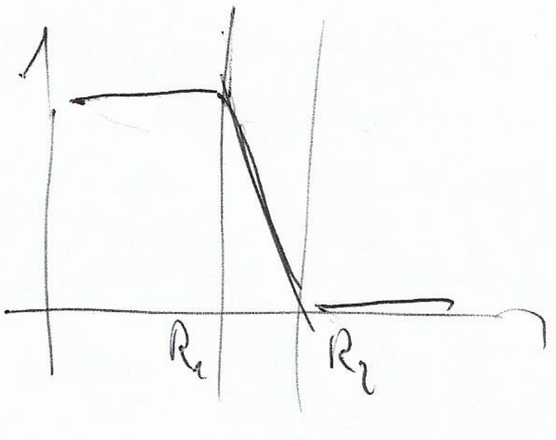
$\mathrm{rot}\,\bar{A}_0 = 0$
$\bar{B}_0 = 0$





Solenoide spesso ?





$A_{o\vartheta} = ?$