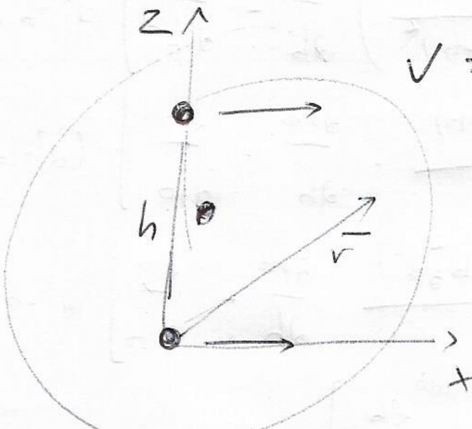
ESERC. 20/IV/2020

RELATIVITÀ; FARADAY E INDUZIONE ELETTROMAGNETICA



$v = \beta c \qquad \beta = \frac{v}{c} < 1$

$$E_x = \frac{e}{4\pi\varepsilon_0}\frac{\cos\vartheta}{r^2} = \frac{e}{4\pi\varepsilon_0}\frac{x}{(x^2+z^2)^{\frac{3}{2}}}$$

$$E_z = \frac{e}{4\pi\varepsilon_0}\frac{\sin\vartheta}{r^2} = \frac{e}{4\pi\varepsilon_0}\frac{z}{(x^2+z^2)^{\frac{3}{2}}}$$

$\left(\begin{array}{c}\text{radiale} \\ \frac{E_z}{E_x} = \frac{z}{x}\end{array}\right)$

$S' \qquad \bar{v} \parallel \hat{e}_x \qquad t = \phi$

$$\begin{cases} x' = \gamma x - \gamma\beta c t \\ ct' = \gamma c t - \gamma \beta x \end{cases} \qquad \begin{pmatrix} y' = y \\ z' = z \end{pmatrix}$$

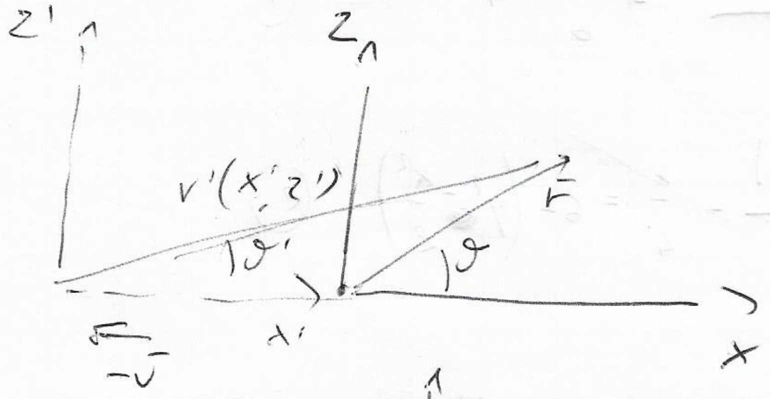
in realtà $S' \quad -\bar{v} \quad$ (LRF)

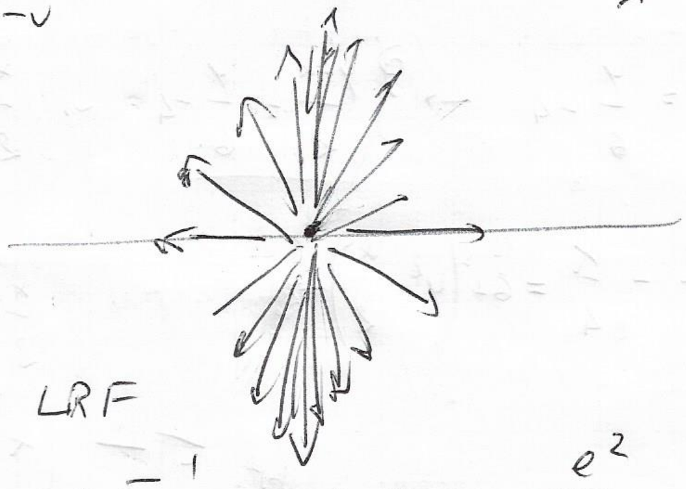
$\downarrow$

$$\begin{cases} x = \gamma x' - \gamma\beta c t' \\ z = \gamma c t' - \gamma\beta x' \end{cases}$$

In S'

$$E' = \frac{e}{4\pi\varepsilon_0}\,\frac{1}{r'^2}\,\frac{1-\beta^2}{\left(1-\beta^2\sin^2\vartheta'\right)^{3/2}}$$





$E'_x = E_x$

$E'_z = \gamma E_z$

LRF

$$F'_{ez} = eE'_z \rightarrow = \gamma\,\frac{e^2}{4\pi\varepsilon_0 h^2} = \gamma K$$

CoRF

$$F_{ez} = \frac{e^2}{4\pi\varepsilon h^2}$$

Trasformazioni di Lorentz per forza

$$F'_\perp = \frac{1}{\gamma} F_\perp \qquad \perp = \hat{e}_z,\ \hat{e}_y$$

$$F'_\parallel = F_\parallel$$

$$F'_z = \frac{1}{\gamma}\,\frac{e^2}{4\pi\varepsilon h^2} \doteq K$$

$$= \frac{1}{\gamma} K$$

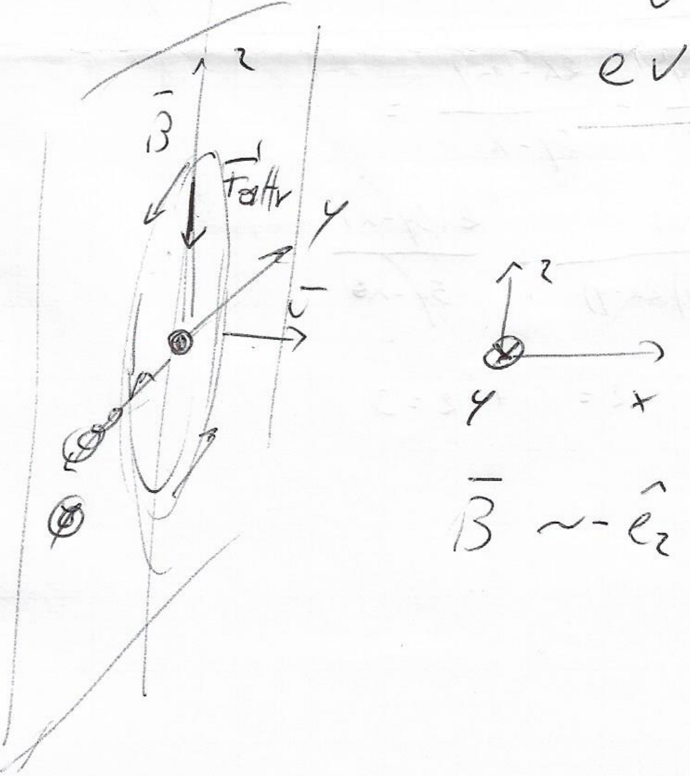
$$\underbrace{\frac{1}{\gamma} K}_{F'_{z\,tot}} = \underbrace{\gamma K}_{F'_{zE}} - F'_{attr}$$

$$F'_{attr} = \left(\gamma - \frac{1}{\gamma}\right) K = \beta^2 \gamma K$$

$$(1-\beta^2) \qquad \gamma^2 \quad \frac{1}{\sqrt{1-\beta^2}}$$

$$F'_{attr_z} = \beta^2 \gamma K = \gamma \beta^2 \frac{e^2}{4\pi\varepsilon_0 h^2} = \underbrace{e\beta c}_{ev} \underbrace{\left(\frac{\beta}{c} \frac{\gamma e}{4\pi\varepsilon_0 h^2} \right)}_{B}$$

$$e v B$$

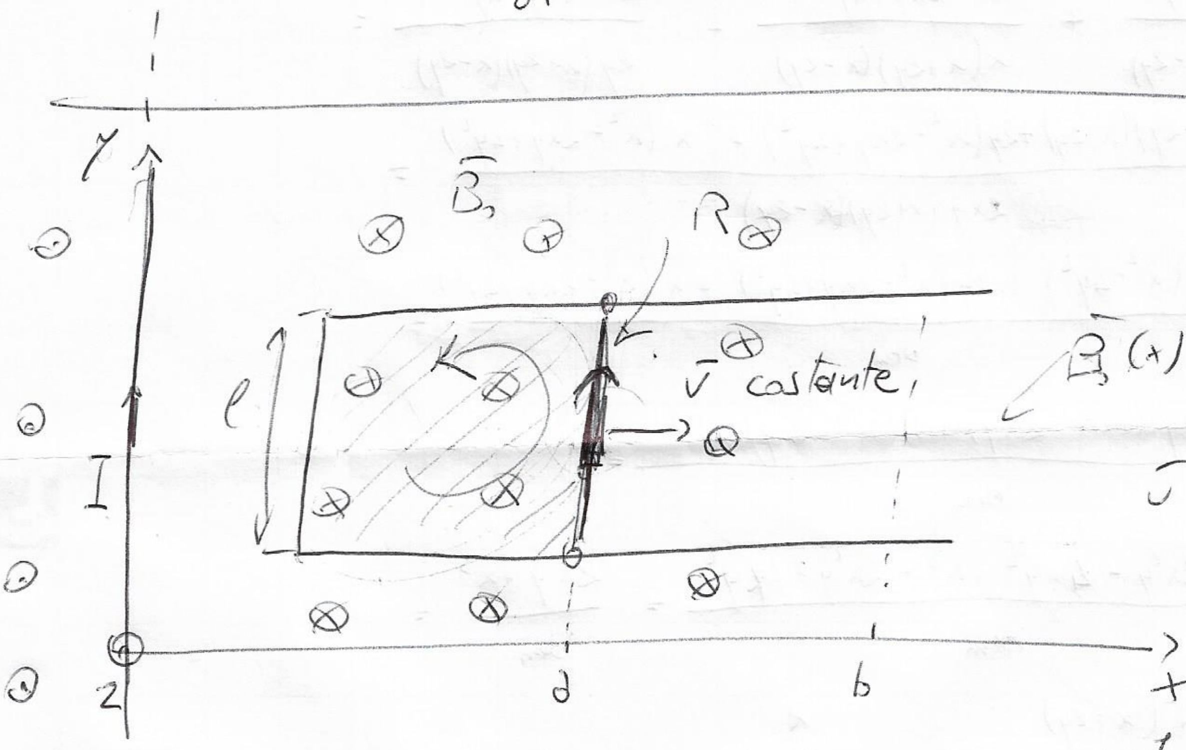


$$\bar{B} \sim -\hat{e}_z$$

$$f_i = \oint_{\gamma = \partial S} \bar{E}_i \cdot d\bar{l} = \cancel{-\frac{d}{dt}\Phi(B)}$$

$$= -\frac{d\Phi(\bar{B})}{dt} = -\frac{d}{dt}\int_S \bar{B}\cdot d\bar{S}$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$



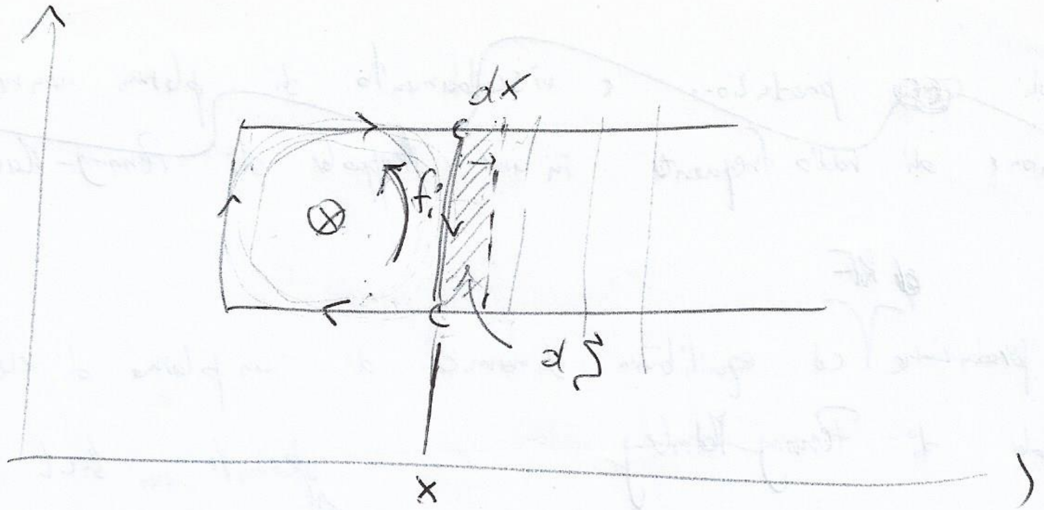
$$\bar{B}_0(x) = -\frac{\mu_0 I}{2\pi x}\hat{e}_z$$

$$\bar{v} = v\,\hat{e}_x$$

$$\bar{E}_i = \bar{v}\times\bar{B}_0 = \frac{\mu_0 I}{2\pi x} v\,\hat{e}_x\times(-\hat{e}_z) = \frac{\mu_0 I v}{2\pi x}\hat{e}_y$$

$$f_i = \int_l \bar{E}_i\cdot d\bar{l} = \int (\bar{v}\times\bar{B}_0)\cdot d\bar{l} = vB_0 l = \frac{\mu_0 I v l}{2\pi x}$$

$$d\bar{l} = dl\,\hat{e}_y$$



$$\phi = \int \bar{B} \cdot d\bar{\Sigma}$$

$$f_i = -\frac{d}{dt}\int \bar{B} \cdot d\bar{\Sigma}$$

$$d\phi = B_0(x)\underbrace{l\,dx}_{d\Sigma} \Rightarrow \frac{d\phi}{dt} = B_0(x)\,l\frac{dx}{dt} = B_0 l v$$

$$f_i = -B_0 l v$$

$\boxed{5}$ i_{ind}

$$i_{ind} = \frac{f_i}{R} = \frac{\mu_0 I l v}{2\pi R x} \quad \text{antioraria}$$

$i_{ind} \sim \hat{e}_y$ su barretta

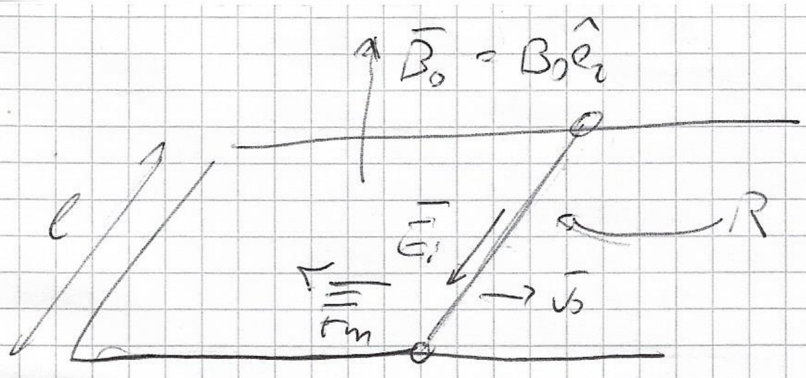
$$\bar{F}_m = i\bar{l} \times \bar{B}_0 = -i l B_0 \hat{e}_x = -\frac{B_0^2 l^2 v}{R}\hat{e}_x = -\left(\frac{\mu_0 I l}{2\pi x}\right)^2 \frac{v}{R}\hat{e}_x$$

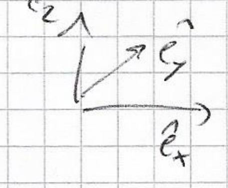
$$\hat{e}_y \times \hat{e}_z \Downarrow -\hat{e}_x \qquad \bar{F}_{ext} = -\bar{F}_m$$

$$d\mathcal{L} = \bar{F}_{ext} \cdot d\bar{l} = F_{ext}\,dx \Rightarrow \mathcal{L} = \int_a^b F_{ext}\,dx =$$

$$= \left(\frac{\mu_0 I l}{2\pi}\right)^2 \frac{v}{R}\int_a^b \frac{dx}{x^2} = \xrightarrow{\quad a \quad} = \left(\frac{\mu_0 I l}{2\pi}\right)^2 \frac{v}{R}\,\frac{b-a}{ab}$$

$$\hookrightarrow -\frac{1}{x}\Big|_a^b$$





$t=0 \rightarrow \bar{J} = m\bar{v}_0$

$$\bar{E}_i = \bar{v}\times\bar{B}_0 = vB_0\,\hat{e}_x\times\hat{e}_z = -vB_0\hat{e}_y \qquad v(t),\ v(x(t))$$

$$f_i = \int \bar{E}_i\cdot d\bar{\ell} = \bar{E}_i\cdot \ell\hat{e}_y = -vB_0\ell$$

$d\bar{\ell} \to d\ell\,\hat{e}_y$

$$i_{ind} = f_i/R = -vB_0\ell$$

$$\bar{F} = \underbrace{i_{ind}\ell\hat{e}_y}_{-\hat{e}_y}\times\bar{B}_0 = -\frac{vB_0^2\ell^2}{R}\hat{e}_x$$

luego $\hat{e}_x$: $F_x = m\dfrac{dv}{dt} \rightarrow$ $\boxed{-\dfrac{B_0^2\ell^2}{R}v = m\dfrac{dv}{dt}}$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B_0^2\ell^2}{mR}\int_{\phi}^{t}dt' \rightarrow v(t) = v_0 e^{-t/\tau} \qquad \tau \doteq mR/B_0^2\ell^2$$

$$v = dx/dt \qquad \underbrace{\int_{x_0}^{x}dx'}_{x-x_0} = \int_{\phi}^{t}v(t')dt' = v_0\int_{\phi}^{t}e^{-t'/\tau}dt' = v_0\tau\left[1-e^{-t/\tau}\right]$$

$$x = x_0 + v_0\tau\left[1-e^{-t/\tau}\right]$$

$$x(t\to+\infty) = x_0 + v_0\tau = x_0 + mRv_0/B_0^2\ell^2$$

$$E_J = \int_{\phi}^{+\infty}P_J\,dt = \int_{\phi}^{+\infty}R\,i_{ind}^2\,dt = $$

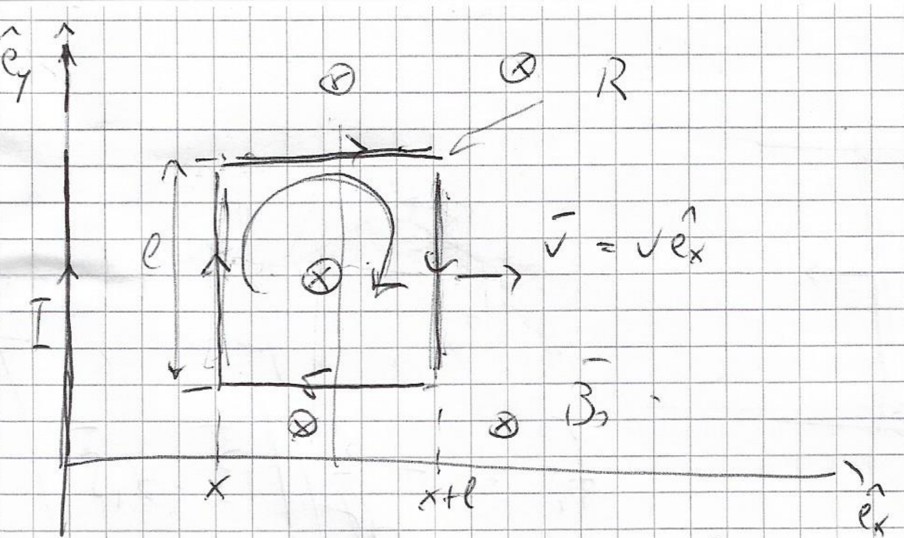
$$= \frac{B_0^2\ell^2v_0^2}{R}\int_{\phi}^{+\infty}e^{-2t/\tau}dt =$$

$$= -\frac{B_0^2 \ell^2 v_0^2}{R} \underbrace{\frac{1}{2} \frac{mR}{B_0^2 \ell^2}}_{\tau} \underbrace{\left[e^{-2t/\tau} \right]_0^{+\infty}}_{-1} = \frac{1}{2} m v_0^2$$

$$E_J(t \to +\infty) = \frac{1}{2} m v_0^2 = E_K(t=0)$$

conservazione dell'energia del sistema

($\nexists$ attrito, $\vec{B}$ non fa lavoro)



(A) $\bar{E}_i = \bar{v} \times \bar{B}_0 \quad \uparrow \hat{e}_y$

$f_i = \int \bar{E}_i \cdot d\bar{\ell}$?

$$f_i = f_{i,sx} + f_{i,dx} = [\underbrace{\bar{v} \times \bar{B}_0(x)}_{\sim \hat{e}_y}] \cdot \ell\hat{e}_y + [\underbrace{\bar{v} \times \bar{B}_0(x+\ell)}_{\sim \hat{e}_y}] \cdot \ell(-\hat{e}_y) =$$

$$= v\ell \frac{\mu_0 I}{2\pi x} - v\ell \frac{\mu_0 I}{2\pi(\ell+x)} = \frac{\mu_0 I \ell^2 v}{2\pi x(\ell+x)} \qquad i_{ind} = \frac{f_i}{R}$$

$d\bar{F} = i\,d\bar{\ell} \times \bar{B}_0$

$$\bar{F} = \bar{F}_{sx} + \bar{F}_{dx} = i\ell\hat{e}_y \times \bar{B}_0(x) + i\ell \overset{(-1)}{\hat{e}_y} \times B_0(x+\ell) =$$

$$= i\ell[-B_0(x) + B_0(x+\ell)]\hat{e}_x \qquad F_x < 0$$

(B) Faraday - Neumann

$$f_i = -\frac{d}{dt}\Phi(\bar{B}_0) = -\frac{d}{dt}\int_x^{\ell+x} B_0(x')\overbrace{\ell dx'}^{dS} \overset{?}{=} \longrightarrow = -\frac{\mu_0 I \ell}{2\pi}\frac{d}{dt}\left[\log\left(\frac{x+\ell}{x}\right)\right]$$

$$B_0(x') = \frac{\mu_0 I}{2\pi x'}$$

$dS = \ell dx$ e

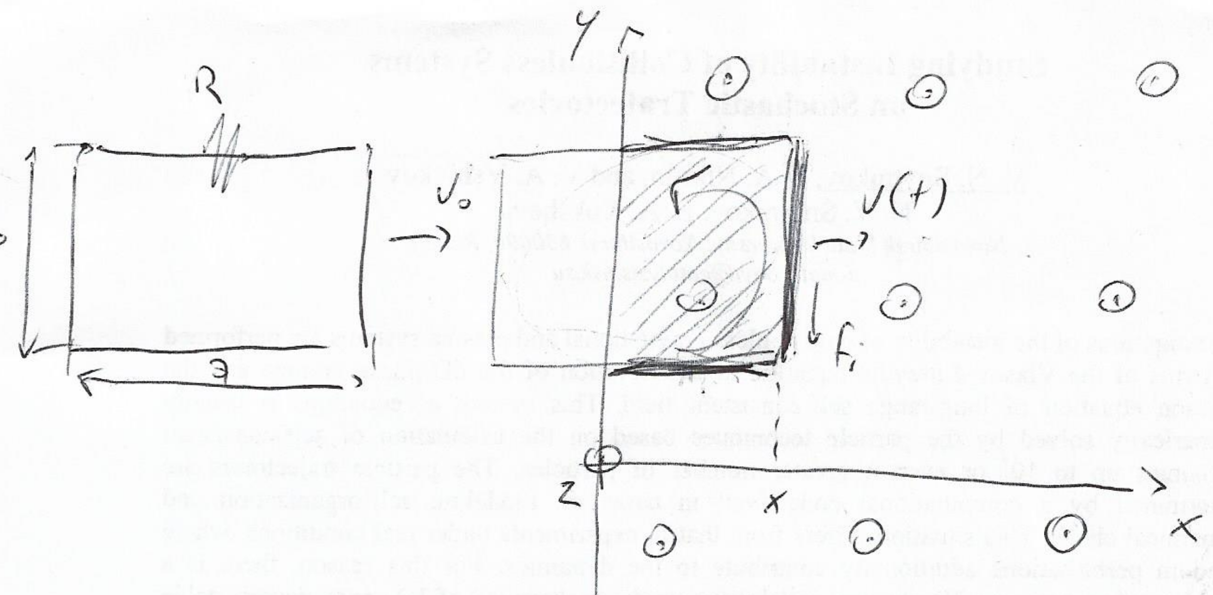
corrispondente $B_0(x)$

$B(x)dS = d\Phi$

$$\frac{d}{dt} = \frac{d}{dx} \overset{= v}{\frac{dx}{dt}} = v \frac{d}{dx}$$

$$f_1 = -\frac{\mu_0 I l v}{2\pi} \frac{d}{dx} \underbrace{\left[\log\left(\frac{x+l}{x} \right) \right]}_{= -\frac{l}{(x+l)x}} = \frac{\mu_0 I l^2 v}{2\pi x (l+x)}$$

$$P_1 = \frac{f_1^2}{R} = \frac{1}{R} \left[\frac{\mu_0 I l^2 v}{2\pi x (x+l)} \right]^2$$



$x > \phi \Rightarrow \bar{B}_0 = B_0 \hat{e}_z$

$v(x)$ velocità $f(x$ penetrazione$)$

$\phi(x) = B_0\, b\, x(t) = B_0 S(t)$

$f_i = -\frac{d}{dt}\phi(B_0) = -\frac{d}{dt}\left[B_0 S(t)\right] = -B_0 b \frac{dx(t)}{dt} = -B_0 b\, v(t)$

$i_{ind} = \frac{f_i}{R} = -B_0 b v / R$

$d\bar{F} = i\, d\bar{l} \times \bar{B} = \frac{B_0 b v}{R}\, \hat{e}_y \times B_0 \hat{e}_z = B_0^2 b$

$\hat{e}_y$

$-\hat{e}_y \sim i\, d\bar{l}$

$= \left(-\frac{B_0 b v}{R}\right) dl\, \hat{e}_y \times B_0 \hat{e}_z$

$\bar{F} = -\frac{B_0^2 b^2}{R} v(t)\, \hat{e}_x$

eq. moto $\hat{e}_x \sim \quad F_x = m a_x = m \frac{dv(t)}{dt}$

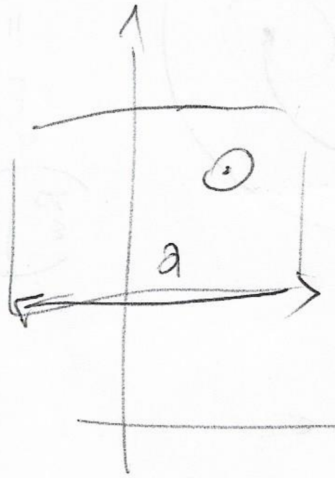
$-\frac{B_0^2 b^2}{R} v(t) = m \frac{dv(t)}{dt} \qquad v(x)$

$dx = v\, dt$

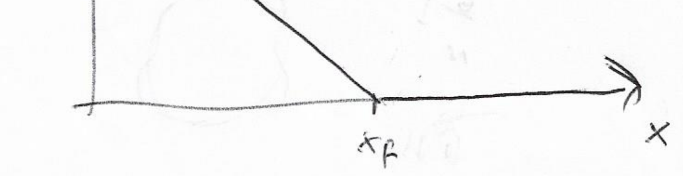
$$-\frac{B_0^2 b^2}{R}\,dx = m\,dv$$

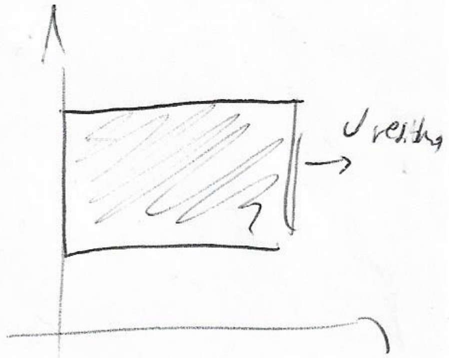
$$dv = -\frac{B_0^2 b^2}{mR}\,dx$$

$$\int_{v_0}^{v} dv' = -\frac{B_0^2 b^2}{mR}\int_{0}^{x} dx' \Rightarrow \boxed{v(x) = v_0 - \frac{B_0^2 b^2}{mR}\,x}$$



$$v = 0 \iff x_{finale} = \frac{mRv_0}{B_0^2 b^2} \leq a$$





$x_{finale} \geq a$

$x = a$

$$v_{res} = v_0 - \frac{B_0^2 b^2}{mR}\,a$$