ESERC. 27/11/2020

INDUZIONE ELETTROMAGNETICA, AUTOINDUZIONE (INDUTTANZA)

Solenoide ~~retti~~ rettilineo ∞ sottile

$i(t) = I_0 \sin(\omega t)$ $\qquad$ n densità di spire, vuoto

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

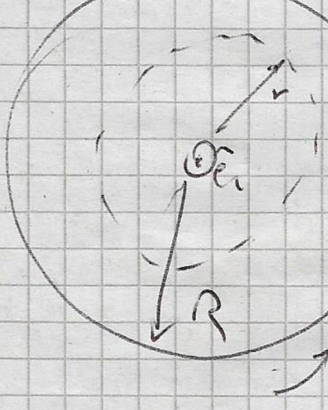
$\bar{E} \perp \hat{e}_z \quad \exists\, E_r, E_\theta \quad (\nexists\, E_z)$

Invarianza per rotazioni $\quad$ no dip. da θ

$\Rightarrow \quad \exists!\ \bar{E}_\theta(r)$

$$\oint \bar{E} \cdot d\bar{l} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} \qquad \left(-\frac{d}{dt} \int \bar{B} \cdot d\bar{S} \right)$$

$\boxed{r < R}$



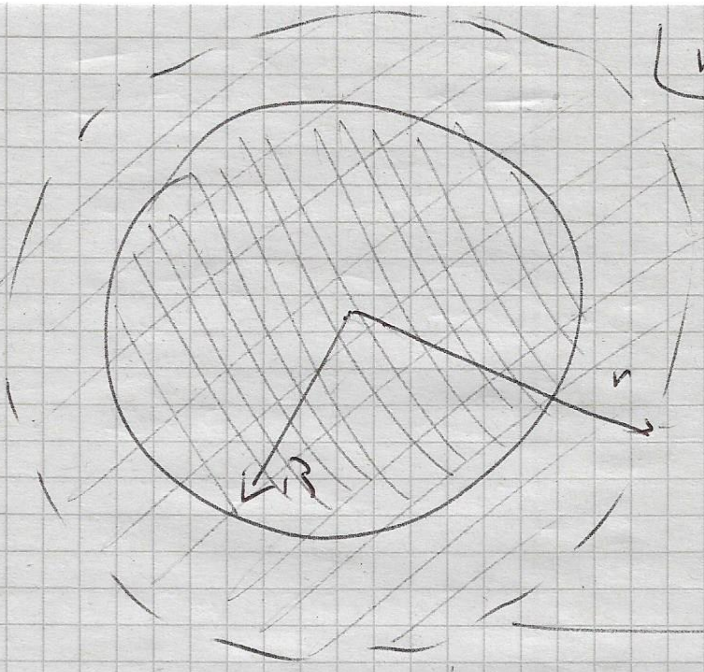
$$\int_0^{2\pi} \bar{E} \cdot r\, d\theta\, \hat{e}_\theta = \int_0^{2\pi} E_\theta(r)\, r\, d\theta = 2\pi r E_\theta(r)$$

$$\bar{B}_0(t) = \mu_0 n\, i(t) \hat{e}_z = \mu_0 n I_0 \sin(\omega t) \hat{e}_z$$

$$-\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S} = -\frac{\partial B_{0z}}{\partial t} \int_{S_{cer}} dS = -\mu_0 n I_0 \omega \cos(\omega t) \pi r^2$$

$$\boxed{\bar{E}_\theta(r) = -\frac{1}{2} \mu_0 n I_0 \omega \cos(\omega t)} \qquad r < R$$

$\boxed{r > R}$



$$\oint \bar{E} \cdot d\bar{l} = 2\pi r E_\theta(r)$$

$$-\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$ sup. attraverso cui si trova flusso $\neq 0$ è πR^2

$$2\pi r E_\theta(r) = -\frac{\partial B_0}{\partial t} \pi R^2$$

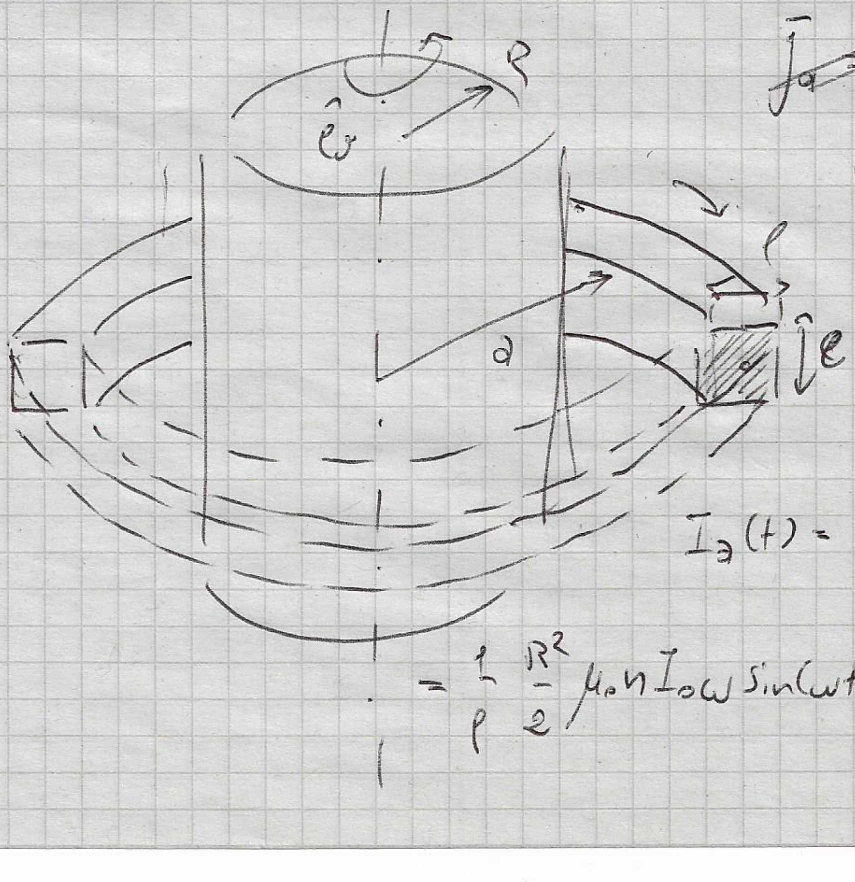
$$\Rightarrow \boxed{E_\theta(r) = -\frac{R^2}{2r} \mu_0 n I_0 \omega \cos(\omega t)}$$ $r > R$

$i(t) \sim \sin(\omega t)$

$$\bar{E} \propto \frac{1}{r} \hat{e}_\theta$$

$$\text{rot}\, \bar{E} = 0 \qquad -\frac{\partial \bar{B}}{\partial t} = 0 \text{ perché } \bar{B} = 0$$

Atto II, scena I $\qquad \rho, \quad \sigma = \frac{1}{\rho}$



$\cancel{\bar{J}_a = \sigma \bar{E}}$

$$\bar{J}_a(r,t) = \sigma \bar{E}(r,t) = \frac{1}{\rho} \bar{E}(r,t)$$

$$dS(r) = \ell\, dr$$

$$I_a(t) = \iint \bar{J}_a(r,t) \cdot d\bar{S} =$$

$$= \frac{1}{\rho} \frac{R^2}{2} \mu_0 n I_0 \omega \sin(\omega t)\, \ell \int_a^{a+\ell} \frac{dr}{r} =$$

$$\bar{i}_a(t) = \frac{R^2}{2\rho}\,\mu_0 n I_0 \ell \log\left(\frac{a+\ell}{a}\right)\omega\cos(\omega t)$$

Atto II, scena II

R: $V = RI$

$P = RI^2 = VI = V^2/R$

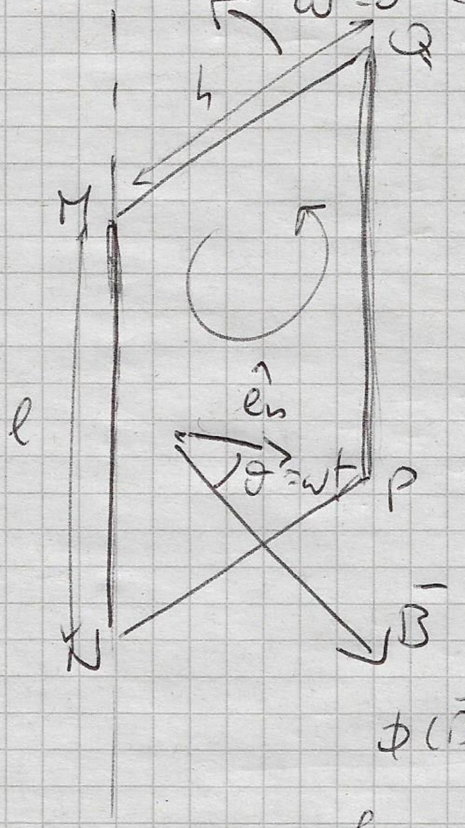
$$\frac{d\mathcal{L}}{dt\,d\tau} = \bar{E}\cdot\bar{J}$$

$$P_J = \int_{\tau_{anello}} \bar{E}\cdot\bar{J}\,d\tau$$

$\ell \ll R, a$ $\rightarrow$ $P_J^{(t)} = R_a I_a^2$

$$R_a = \frac{\rho\, 2\pi (a+\ell/2)}{\ell^2}$$

$\omega = \dot{\vartheta} = 150$ rad/s



MN fisso, con rotazione

$l = 10$ cm ; $h = 5$ cm

$B = 0.1$ T

$t = 0$ $\bar{B} \parallel \hat{e}_n$ spira

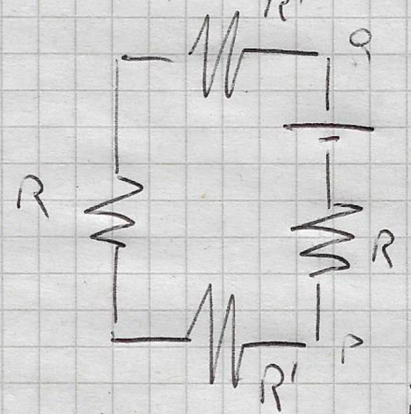
$\rho = 1.68 \cdot 10^{-8}\ \Omega \cdot m$

$\Sigma = 1\ mm^2$ $f_{ind} = ?$ $i_{ind} = ?$

$V_P - V_Q = ?$

$$\Phi(\bar{B}) = \bar{B} \cdot \bar{S}(t) = B l h \cos(\omega t)$$

$$f_i = -\frac{d\Phi}{dt} = B l h \omega \sin(\omega t)$$



$R' = \rho h / \Sigma$

$R = \rho l / \Sigma$

$$R_{tot} = 2R' + 2R = \frac{2\rho}{\Sigma}(l + h)$$

$$i_{ind}(t) = \frac{f_{ind}(t)}{R_{tot}} = B l h \omega \sin(\omega t) \cdot \Sigma / 2\rho(l+h)$$

$i_{ind,max} = 29.76$ A

$V_P - V_Q$ Legge di Ohm generalizzata:

$$V_A - V_B + \sum_{A \to B} f_{el} = I \sum_{A \to B} R_k$$

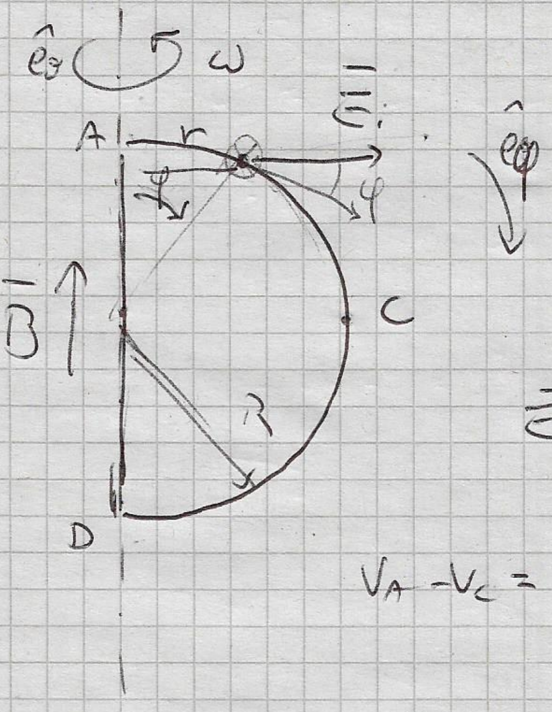
$$V_P - V_Q + f_{ind}(t) = i_{ind}(t) R = i_{ind}(t) \frac{\rho l}{\Sigma}$$

$$V_p - V_2 = -Blh\omega \sin(\omega t) + Blh\omega \sin(\omega t) \frac{\Sigma}{2\rho(l+h)} \cdot \frac{\rho l}{\Sigma} =$$

$$= Blh\omega \sin(\omega t) \left[\frac{l}{2(l+h)} - 1 \right]$$

$$V_p - V_2 = -Blh\omega \frac{2h+l}{2(h+l)} \sin(\omega t)$$

$$(V_p - V_2)_{max} = 0.1\ V$$



$$\bar{E}_i = \bar{v} \times \bar{B}$$

$$\bar{\omega} \times \bar{r}$$

$$\bar{v} = (\bar{\omega} \times \bar{r}) = r\omega\, \hat{e}_\vartheta$$

$$v = r\omega = R\sin\varphi\, \omega$$

$$\bar{E}_i = R\sin\varphi\, \omega B\, \hat{e}_{ext}$$

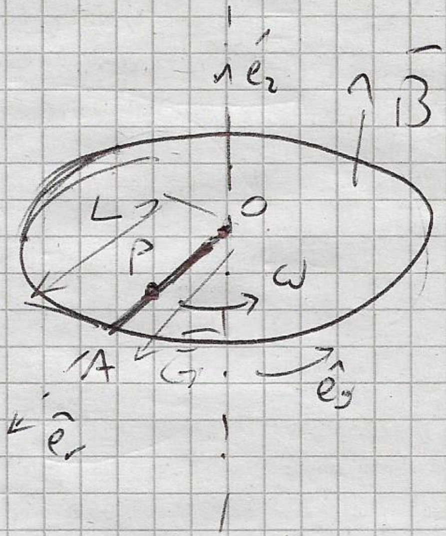
$$V_A - V_C = \int_A^C \bar{E}_i \cdot d\bar{l} = \int_0^{\pi/2} \underbrace{\omega B R \sin\varphi}_{E_i}\, \underbrace{R\,d\varphi}_{dl}\, \cos\varphi$$

$$d\bar{l} = R\,d\varphi\, \hat{e}_\varphi$$

$$V_A - V_C = \omega B R^2 \int_0^{2\pi} \sin\varphi \cos\varphi\, d\varphi = \omega B R^2 \left(\frac{\sin^2\varphi}{2} \right)_0^{\pi/2} = \frac{\omega B R^2}{2}$$

$$V_D - V_C = V_A - V_C \qquad V_D = V_A$$

Sbarra rotante / disco di Barlow

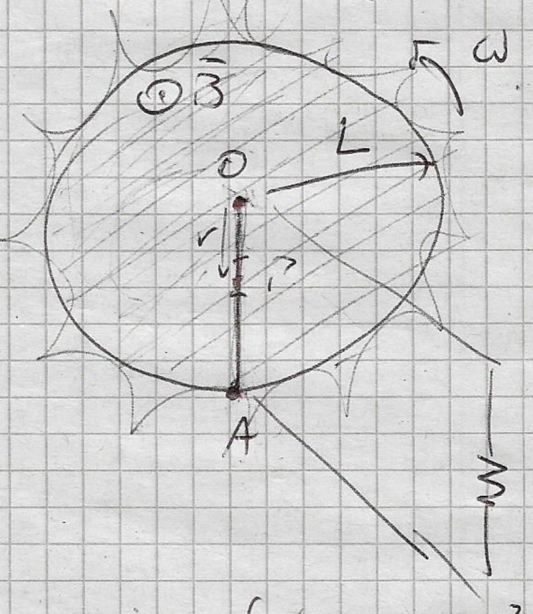


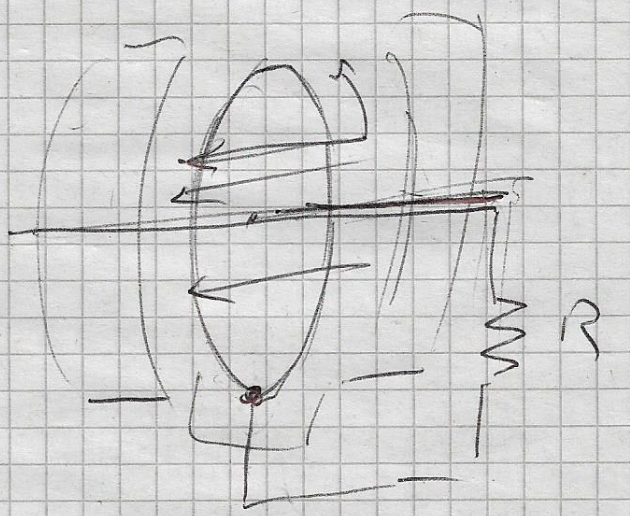
ω costante

$$\bar{E}_i = \bar{v} \times \bar{B} = (\bar{\omega} \times \bar{r}) \times \bar{B} = \omega r B \hat{e}_r$$

$$f_i = \int \bar{E}_i \cdot d\bar{l} = \int_0^L \omega r B \, dr = \frac{\omega B L^2}{2}$$

Barlow wheel





$$i_{ind} = f_i / R = \omega B L^2 / 2R$$

Forza magnetica $d\bar{F}_m = i \, d\bar{r} \times \bar{B} = -iB \, dr \, \hat{e}_\theta$

$$d\bar{M}_m = \bar{r} \times d\bar{F} = -iBr \, dr \, \hat{e}_r \times \hat{e}_\theta = -iBr \, dr \, \hat{e}_z$$

$$\bar{M}_m = -iB \int_0^L r \, dr \, \hat{e}_z = -iB \frac{L^2}{2} \hat{e}_z = \frac{\omega B^2 L^4}{4R} \hat{e}_z$$

$$\bar{M}_{ext} = -\bar{M}_m = \frac{\omega B^2 L^4}{4R} \hat{e}_z \qquad P_{ext} = \omega M_{ext} = \frac{\omega^2 B^2 L^4}{4R}$$

$$P_J = R i^2 = \omega^2 B^2 L^4 / 4R$$

Autoinduzione / induttanza

Quasi - stazionarietà

$$\bar{B}_0(\bar{r}) = \frac{\mu_0}{4\pi}\int_{l'} I \frac{d\bar{l}' \times \Delta\bar{r}}{|\Delta r|^3} \qquad \bar{B}_0 \propto I$$

$$\phi = \int \bar{B} \cdot d\bar{S}$$

$$d\Phi(\bar{B}) \propto B_0 \propto I$$

$$\bar{\Phi}(\bar{B}_0) = L\, I$$ coeff. di autoinduzione / induttanza

$$-\frac{d\Phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

$$L = \frac{\Phi}{I} \qquad [L] = \frac{[Wb]}{[A]} = \frac{[V \cdot s]}{[A]} = [\Omega \cdot s] = [H] \quad \text{Henry}$$

$$\bar{\Phi} = L I \qquad \left(U = \frac{1}{2} L I^2 \right)$$

L solenoide R, $l \gg R$, N coils

autoflusso

$$\bar{B}_0 = \mu_0 \frac{N}{l} I \hat{e}_z$$

$$\phi_{coil}(\bar{B}_0) = \bar{B}_0 \cdot \bar{S} = B_0 \pi R^2 \Rightarrow \Phi = N\Phi_c = N B_0 \pi R^2 = \mu_0 \frac{N^2 I}{l} \pi R^2$$

$$L_0 = \frac{\Phi}{I} = \mu_0 \frac{N^2}{l} \pi R^2 = \mu_0 n^2 l \pi R^2$$

$$n = \frac{N}{l}$$

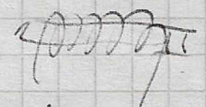
$l = 10$ cm

$R = 1$ cm

$N = 1000$

$$L_0 = 3.95 \text{ mH}$$

core



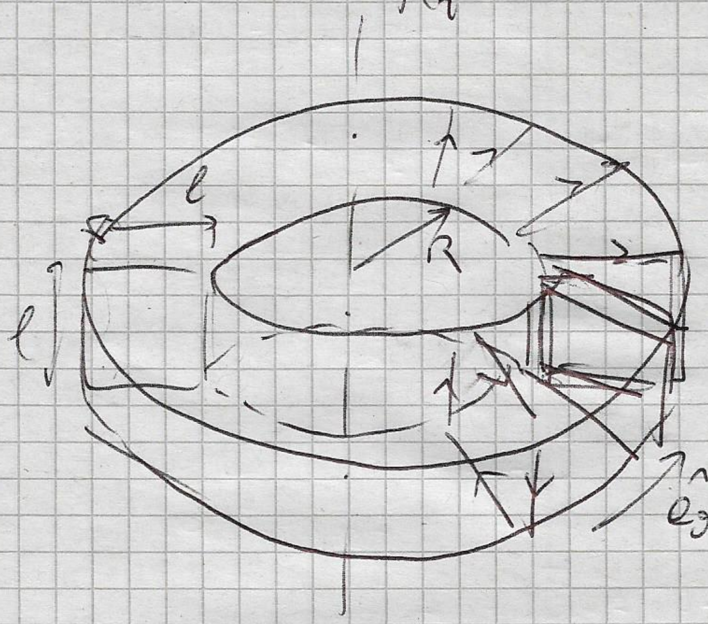
μ_r

$$B = \mu_r B_0$$

$$L = \mu_r L_0$$

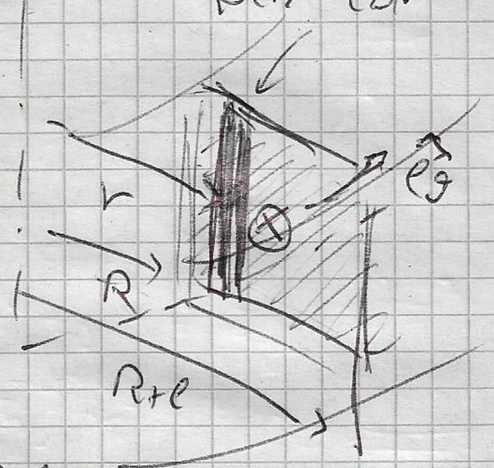
L toroide

N coil, ℓ = lato quadrato
raggio int R



$$\bar{B}_0(r) = \frac{\mu_0 N I}{2\pi r}\hat{e}_\vartheta$$

$\vec{S}(r) = \ell dr$



$$\Phi(\bar{B}_0) = N\,\Phi_c(\bar{B}_0) =$$

$$= N\int \bar{B}_0(r)\cdot \ell dr\,\hat{e}_\vartheta = N\int_R^{R+\ell} \frac{\mu_0 N I}{2\pi r}\ell dr = \frac{\mu_0 N^2 I \ell}{2\pi}\log\left(\frac{R+\ell}{R}\right)$$

$$L_0 = \Phi(\bar{B}_0)/I = \frac{\mu_0 N^2 \ell}{2\pi}\log\left(R+\ell/R\right)$$

$R = 5\,mm$
$\ell = 5\,mm$
$N = 200$

$L_0 = 2.77\cdot 10^{-5}\,H$

Wah pedal blabbering

(fasel) ~ 500 mH

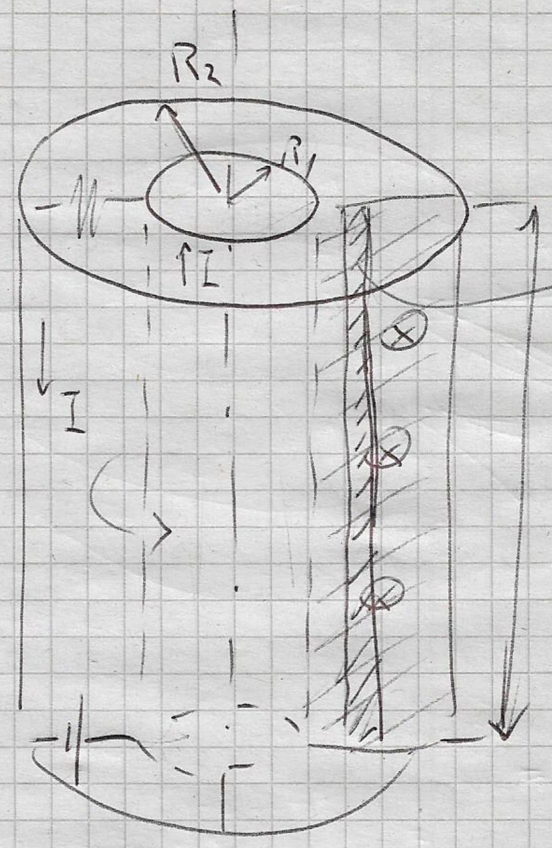
[→ www.electrosmash.com/vox-v847-analysis]

RLC

Wah pedal

[→ Youtube: 'Cry Baby - The Pedal that Rocks the World']

Induttanza coax



$$B_0(r) = \frac{\mu_0 I}{2\pi r}\hat{e}_\vartheta \qquad R_1 < r < R_2$$

$$dS = \ell dr$$

$$\Phi(\bar{B}_0) = \int_{R_1}^{R_2} \frac{\mu_0 I}{2\pi r}\ell dr = \frac{\mu_0 I \ell}{2\pi}\log\left(\frac{R_2}{R_1}\right)$$

$$L_{0,1} = \frac{\Phi}{\ell I} = \frac{\mu_0}{2\pi}\log\left(\frac{R_2}{R_1}\right)$$

ind. esterna

$$d\Phi'_{in} = \underbrace{B_0(r)}\ell dr = \underbrace{\frac{\mu_0 I}{2\pi R_1^2} r}\ell dr$$

$$I' = I\left(\frac{r}{R_1}\right)^2$$

$$d\Phi_{in} = d\Phi'_{in}\left(\frac{r}{R_1}\right)^2 = \frac{\mu_0 I}{2\pi R_1^4} r^3 \ell dr$$

$$\Phi_{in}(B) = \int_0^{R_1} \frac{\mu_0 I \ell}{2\pi R_1^4} \underbrace{r^3 dr}_{\frac{R_1^4}{4}} = \frac{\mu_0 I \ell}{8\pi}$$

$$L_{0,1} = \frac{\Phi_{in}}{\ell I} + \frac{\Phi_{out}}{\ell I} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log\left(\frac{R_2}{R_1}\right)\right]$$

~1.8

$R_1 = 0.5$ mm

$R_2 = 3$ mm

$L_{0,1} = 4.1 \cdot 10^{-7}$ H

altrimenti

$$U_m = \frac{1}{2}LI^2$$