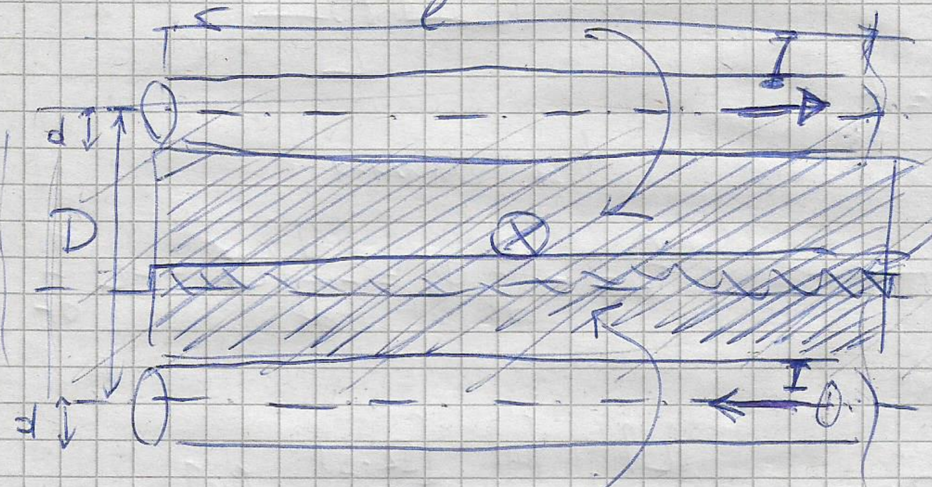
ESERC. 04/V/2020
AUTO- E MUTUA INDUTTANZA
ENERGIA MAGNETICA

Linea elettrica bifilare

"induttanza esterna"



$d << D$

$x \in [d, D-d]$

$$B_0(x) = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi (D-x)}$$

$$d_t S = \ell dx \qquad d\phi(B_0) = B_0(x)\, d_t S(x) = B_0(x)\,\ell\, dx$$

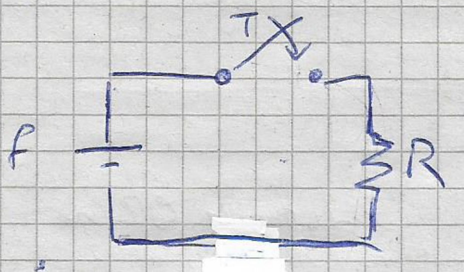
$$\Phi(B_0) = \frac{\mu_0 I}{2\pi}\int_d^{D-d}\left[\frac{1}{x}+\frac{1}{D-x}\right]\ell dx = \frac{\mu_0 I \ell}{2\pi}\Big[\log(x) - \log(D-x)\Big]_d^{D-d} =$$

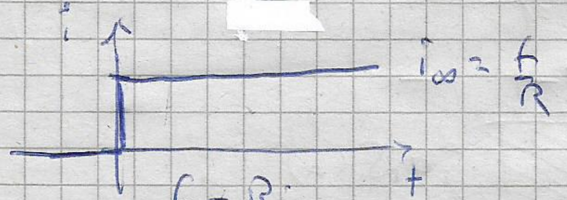
$$= \frac{\mu_0 I \ell}{\pi}\log\left(\frac{D-d}{d}\right) \simeq \frac{\mu_0 I \ell}{\pi}\log\left(\frac{D}{d}\right)!$$

$$\tilde{L}_0 = L_0/\ell = \frac{1}{\ell}\frac{\phi(B_0)}{I} = \frac{\mu_0}{\pi}\log\left(\frac{D-d}{d}\right)$$

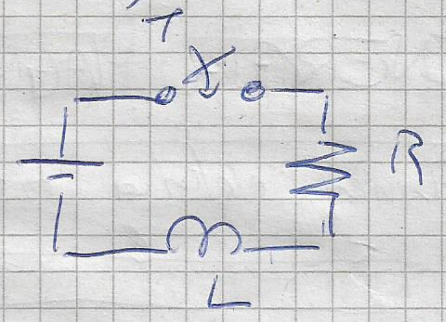
L come elemento circuitale

Chiusura circuito RL (transitorio)





$f = Ri$



t=0 chiusura

$$f_a = -\frac{d}{dt}\Phi_a = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$$

$$f + f_a = Ri$$

$$f - L\frac{di}{dt} = Ri$$

$$\frac{[L]}{[R]} = \frac{[H]}{[\Omega]} = \frac{[\Omega \cdot s]}{[\Omega]} = [s]$$

$$\tau \doteq L/R \qquad I_\infty = f/R$$

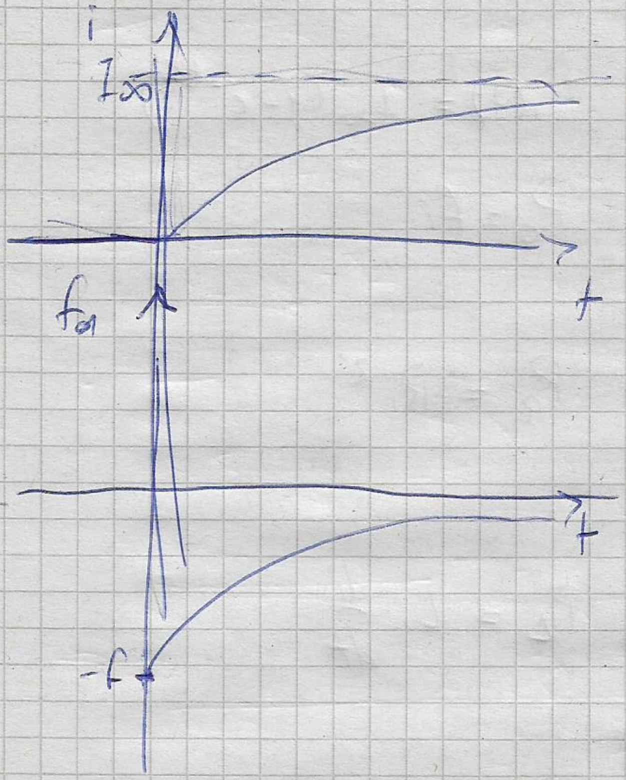
$$I_\infty = i + \tau\frac{di}{dt} \rightsquigarrow$$

$$\frac{di}{i - I_\infty} = -\frac{dt}{\tau} \qquad d(i - I_\infty) = di$$

$$\log(i - I_\infty) = -t/\tau + \log k \rightsquigarrow i(t) - I_\infty = k e^{-t/\tau}$$

Cond. iniziale $t=0 \rightarrow i(0) = 0 \Rightarrow -I_\infty = k$

$$i(t) = I_\infty(1 - e^{-t/\tau}) = \frac{f}{R}(1 - e^{-t/\tau})$$



$$\leftarrow I_\infty\left(1-e^{-t/\tau}\right)$$

$$I_\infty \to f/R \qquad \tau = L/R$$

$$f_a = -L\frac{di}{dt} = -fe^{-t/\tau}$$

$$f + f_a = Ri$$

$$t=0 \quad \phi \Rightarrow Ri(0) = f$$

Potenza / energia

$$d\mathcal{L}_G = f\,dQ = f\,i\,dt$$

$$f = Ri + L\frac{di}{dt} \leftarrow \times \; dQ = i\,dt$$

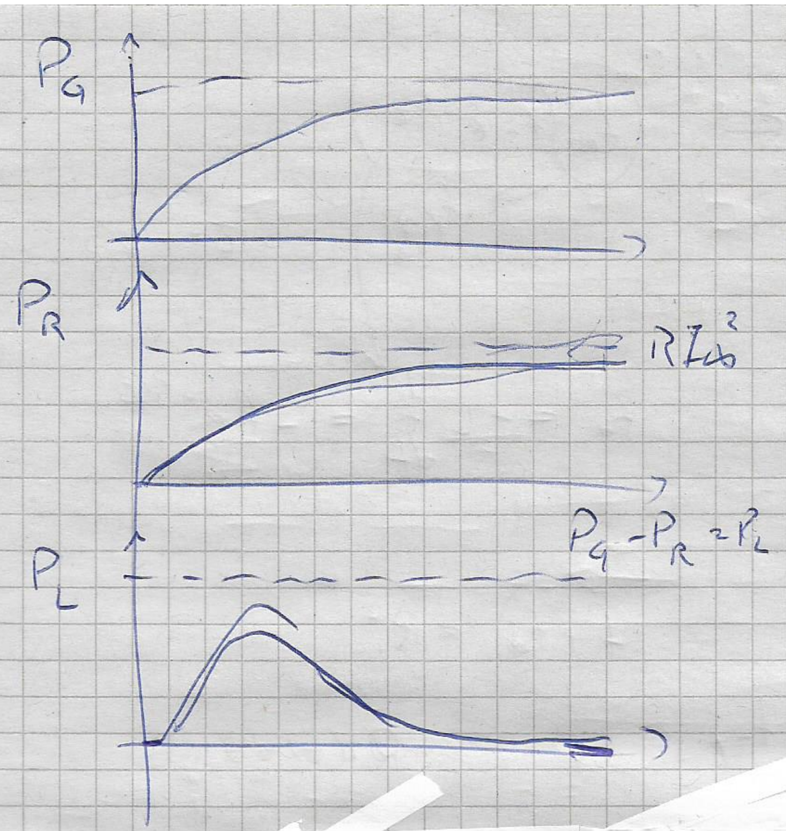
$$\underbrace{f\,i\,dt}_{d(\text{en. erogata})} = Ri^2dt + L\,i\,di = \underbrace{dU_R}_{\text{en. dissipata}} + \underbrace{dU_L}_{\text{en. accumulata nell'induttore}}$$

$$U_M = U_L = \int dU_L = L\int_0^{I} i\,di = \frac{1}{2}LI^2$$

$$U_C = \frac{1}{2}\frac{Q^2}{C} = U_E$$

$$u_m = \frac{1}{2\mu_0}\bar{B}\cdot\bar{B} = \frac{1}{2\mu_0}B^2$$

$$U_m = \int_\tau u_m\,d\tau$$

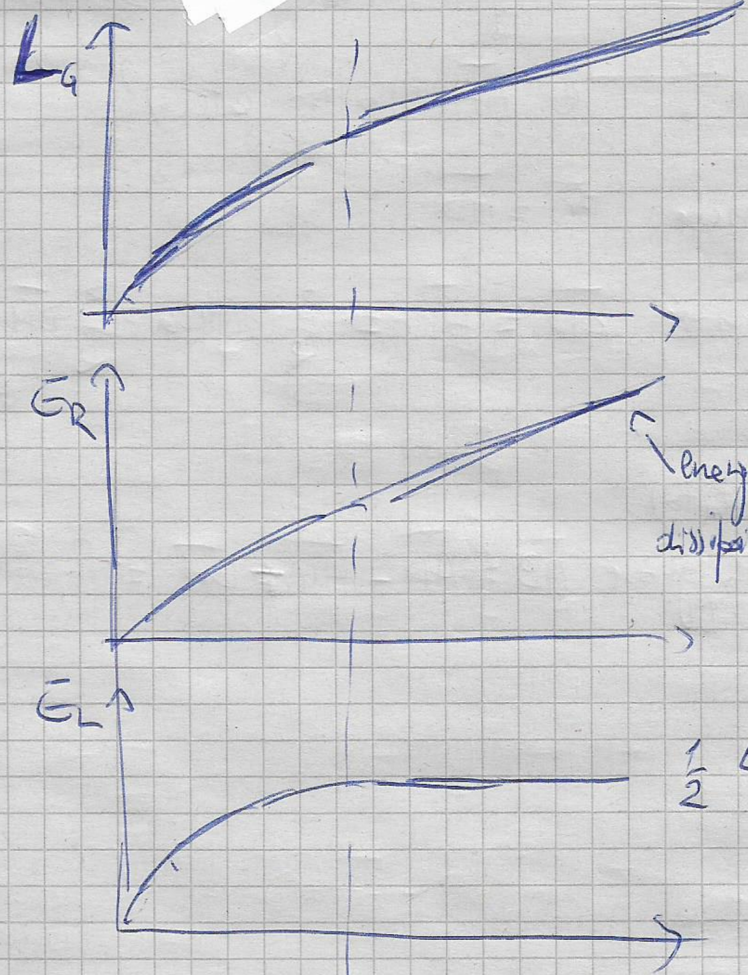


$i(t) = I_\infty (1 - e^{-t/\tau})$

$P_G = fi$

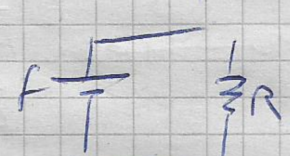
$P_R = Ri^2$

$P_L = Li\frac{di}{dt}$



$L_g = \int P_G \, dt$

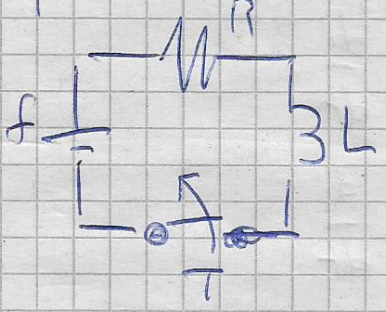
$t >> 0$



continuamente (infinita, per tempo infinito)

l'accumulo termina col transitorio

Apertura di circuito RL



$T = 0$ apertura (dopo cond. staz.)

$$f_a = -L\frac{di}{dt} \to \infty$$

$R = R(t)$ nel transitorio

Hyp: R nel transitorio fissa $= R' \gg R$

$$f - L\frac{di}{dt} = R' i \qquad \tau = L/R'$$

$$\frac{di}{i - f/R'} = -\frac{dt}{\tau} \rightarrow \log\left(\frac{i - f/R'}{K}\right) = -t/\tau$$

$$\Rightarrow i(t) = \frac{f}{R'} - K e^{-t/\tau}$$

cond. iniz.: $i(0) = \frac{f}{R} = I_0 = \frac{f}{R'} - K$

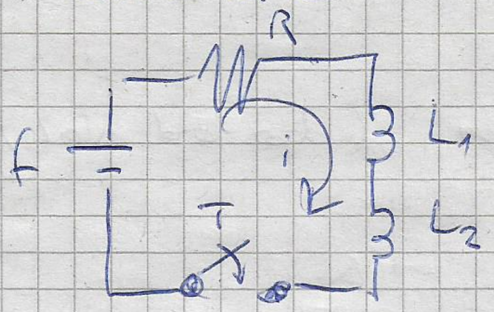
$$\Rightarrow K = f\left(\frac{1}{R'} - \frac{1}{R}\right) = \frac{f}{R'} - I_0$$

$$i(t) = \frac{f}{R'} - \frac{f}{R'}e^{-t/\tau} + I_0 e^{-t/\tau} = \frac{f}{R'}\left(1 - e^{-t/\tau}\right) + \overset{f/R}{I_0} e^{-t/\tau} \simeq I_0 e^{-t/\tau}$$

$$i(t) \simeq I_0 e^{-t/\tau} \simeq \frac{f}{R} e^{-t/\tau} \qquad R' \gg R$$

$R'(t)$

Serie di L



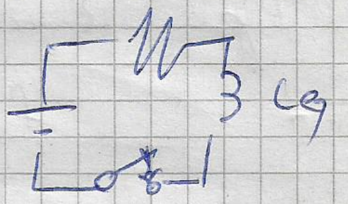
$$f_{a1} = -L_1 \frac{di}{dt}$$

$$f_{a2} = -L_2 \frac{di}{dt}$$

$$f - L_1 \frac{di}{dt} - L_2 \frac{di}{dt} = Ri$$

$L_{eq} \doteq L_1 + L_2$

$$f - L_{eq} \frac{di}{dt} = Ri$$



Serie: $L_{eq} = \sum_j L_j$

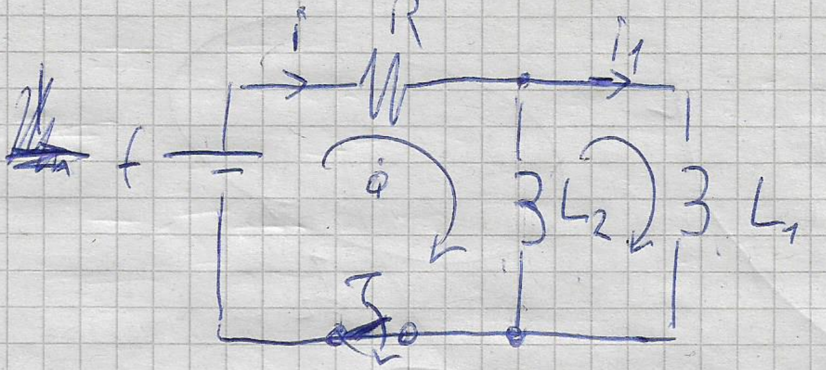
$$i(t) = \frac{f}{R}\left(1 - e^{-t/\tau}\right) \quad \text{con} \quad \tau \doteq \frac{L_{eq}}{R} = \frac{L_1 + L_2}{R}$$

$$f_{a1} = -L_1 \frac{di}{dt} = -f \frac{L_1}{L_1 + L_2} e^{-t/\tau}$$

$$f_{a2} = -L_2 \frac{di}{dt} = -f \frac{L_2}{L_1 + L_2} e^{-t/\tau}$$

$$f_{a1} + f_{a2} \begin{cases} \nearrow -f & t = 0 \\ \searrow 0 & t = \infty \end{cases}$$

Parallelo di L



$$\begin{cases} f - L_2 \dfrac{d}{dt}(i - i_1) = Ri \\ (\phi) - L_1 \dfrac{di_1}{dt} - L_2 \dfrac{d}{dt}(i_1 - i) = \phi \end{cases}$$

eq. 2 : $(L_1 + L_2) \dfrac{di_1}{dt} = L_2 \dfrac{di}{dt} \Rightarrow \dfrac{di_1}{dt} = \dfrac{L_2}{L_1 + L_2} \dfrac{di}{dt} \rightarrow$ in eq. 1

$$f - L_2 \frac{di}{dt} + \frac{L_2^2}{L_1 + L_2} \frac{di}{dt} = Ri$$

$$f = \left(\frac{L_1 L_2}{L_1 + L_2}\right) \frac{di}{dt} + Ri \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \qquad \boxed{\frac{1}{L_{eq//}} = \sum_j \frac{1}{L_j}}$$

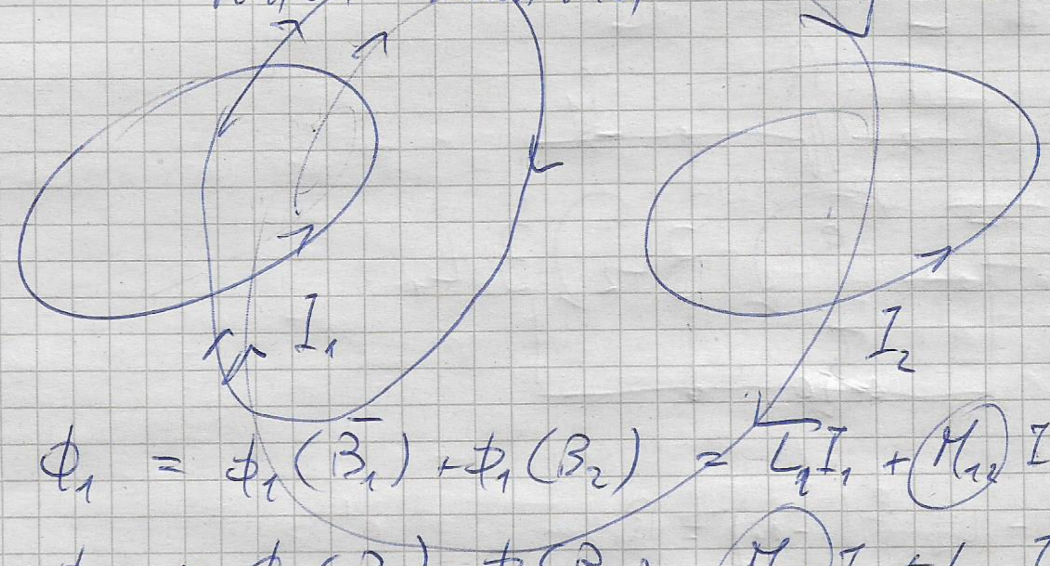
$$f = Ri + L_{eq} \frac{di}{dt} \rightarrow i(t) = \frac{f}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = L_{eq}/R$$

$$\frac{di_1}{dt} = \frac{L_2}{L_1 + L_2} \frac{di}{dt} \rightarrow i_1(t) = \frac{L_2}{L_1 + L_2} i(t) + K$$

in L_1: $i_1(t) = \dfrac{f}{R} \dfrac{L_2}{L_1 + L_2} \left(1 - e^{-t/\tau}\right) + K \rightarrow$ Cond. iniz. $\Rightarrow K = \phi$

in L_2: $i(t) - i_1(t) = \dfrac{f}{R} \dfrac{L_1}{L_1 + L_2} \left(1 - e^{-t/\tau}\right)$

Mutua Induttanza



$$\Phi_1 = \Phi_1(\bar{B}_1) + \Phi_1(B_2) = L_1 I_1 + M_{12} I_2$$

$$\Phi_2 = \Phi_2(B_1) + \Phi_2(B_2) = M_{21} I_1 + L_2 I_2$$

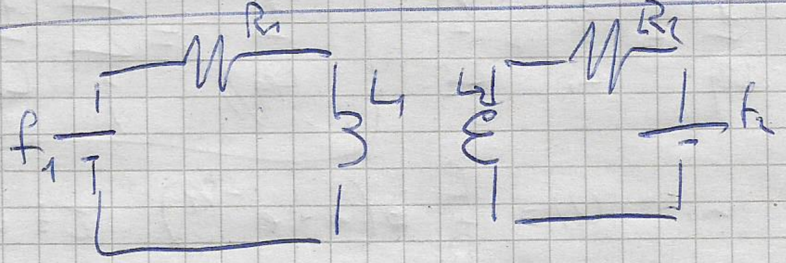
$$M = M_{12} = M_{21}$$

$$|M| \leq \sqrt{L_1 L_2} = k\sqrt{L_1 L_2}$$

$$0 \leq k \leq 1$$

$$\Phi_1(B_1) \geq \Phi_2(B_1)$$

$$L_1 I_1 \geq M I_1$$



accoppiati

$$\begin{cases} f_1 - L_1 \dfrac{dI_1}{dt} - M\dfrac{dI_2}{dt} = R_1 I_1 & dQ_1 = I_1\,dt \\ f_2 - M\dfrac{dI_1}{dt} - L_2\dfrac{dI_2}{dt} = R_2 I_2 & dQ_2 = I_2\,dt \end{cases}$$

$$f_1 I_1 dt + f_2 I_2 dt = \underbrace{(R_1 I_1^2 + R_2 I_2^2)dt}_{\to dU_R} +$$

$$+ \underbrace{L_1 I_1 dI_1 + L_2 I_2 dI_2 + M\overbrace{(I_1 dI_2 + I_2 dI_1)}^{d(I_1 I_2)}}_{dU_M}$$

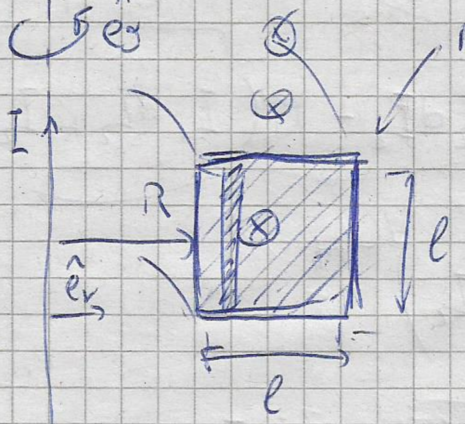
$$dU_M = d\left(\frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2\right) \circ$$

$$U_M = \int_{\emptyset}^{I_1, I_2} dU_M = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

$$U_M = \frac{1}{2} \sum_{i,j} M_{ij} I_i I_j \qquad M_{ii} = L_i$$

$$\ldots \qquad U_M = \frac{1}{2} \sum_k I_k \Phi_k$$

M filo – toroide (in moto)



(1) $\Phi_t(\bar{B}_f)$

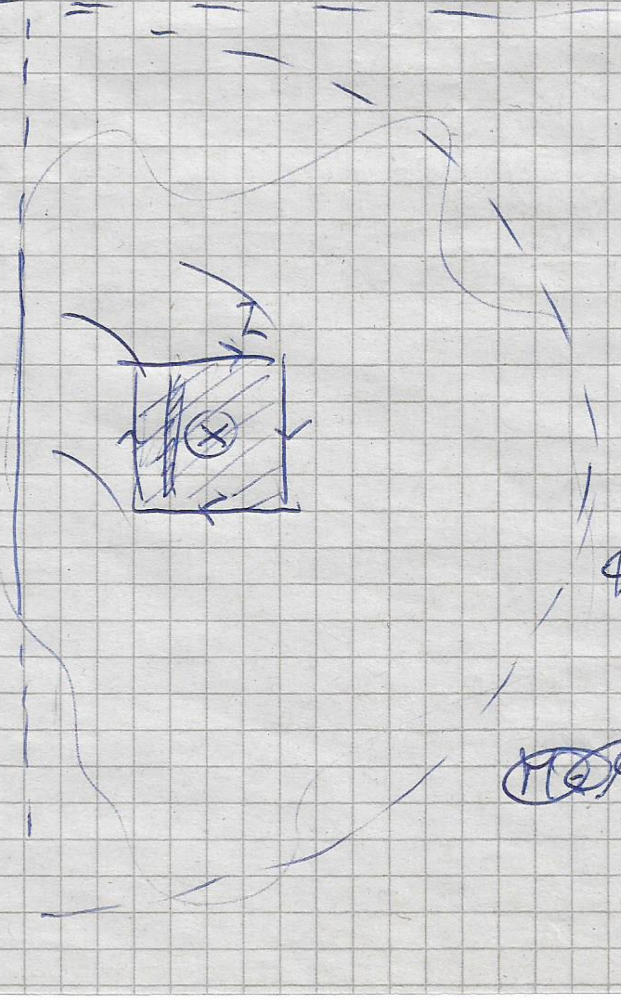
$$\bar{B}_{0f}(r) = \frac{\mu_0 I}{2\pi r}\,\hat{e}_\vartheta$$

$dS = l\,dr$

$$\Phi_t(\bar{B}_{0f}) = N\,\Phi_{sp}(\bar{B}_{0f}) = N\int_R^{R+l} \frac{\mu_0 I}{2\pi r}\,l\,dr = \frac{\mu_0 N I l}{2\pi}\log\left(\frac{R+l}{R}\right)$$

$$\boxed{M = \frac{\Phi_t}{I_f} = \frac{\mu_0 N l}{2\pi}\log\left(\frac{R+l}{R}\right)}$$

(2)



$$\bar{B}_{0t}(r) = \frac{\mu_0 N I}{2\pi r}\,\hat{e}_\vartheta$$

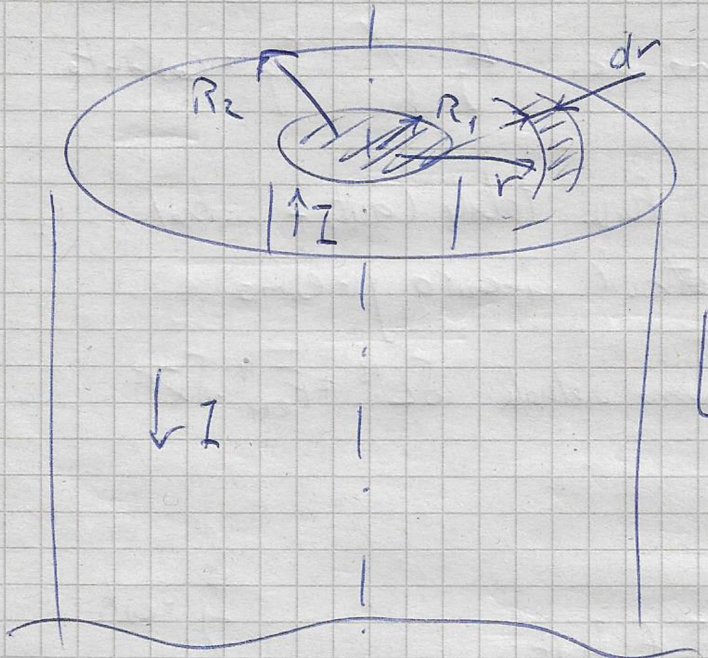
dentro il toroide

(fuori $\bar{B}_{0t} = \emptyset$)

$$\Phi_f(\bar{B}_{0t}) = \int_{R+l}^{R} \frac{\mu_0 N I}{2\pi}\,l\,dr = \frac{\mu_0 N I l}{2\pi}\log\left(\frac{R+l}{R}\right)$$

$$M = \frac{\Phi_f}{I_t} = \frac{\mu_0 N l}{2\pi}\log\left[(R+l)/R\right]$$

$\underline{L}$ cavo coassiale (metodo U_M)



$$U_M = \frac{1}{2} L I^2$$

$$\boxed{R_1 < r < R_2} \quad \bar{B}_0(r) = \frac{\mu_0 I}{2\pi r}\hat{e}_\theta$$

$$u_m = \frac{B^2}{2\mu_0}$$

$$dU_m^{ext} = u_m \, d\tau = u_m \, \ell 2\pi r dr$$

$$U_m^{ext} = \int_{R_1}^{R_2} \frac{B^2}{2\mu_0} \ell 2\pi r dr = \int_{R_1}^{R_2} \frac{\mu_0^2 I^2}{4\pi^2 r^2} \frac{2\pi \ell}{2\mu_0} r dr = \frac{\mu_0 I^2 \ell}{4\pi} \log\left(\frac{R_2}{R_1}\right)$$

$$\tilde{L}_{ext} = \frac{2U_m^{ext}}{\ell I^2} = \frac{\mu_0}{2\pi}\log\left(\frac{R_2}{R_1}\right)$$

$0 \leq r \leq R_1$ magneticamente trasparente (diamagnetico) $\Rightarrow$ come il vuoto

$$\bar{B}_0(r) = \frac{\mu_0 I}{2\pi R_1^2} r \hat{e}_\theta$$

$$U_m^{int} = \int_0^{R_1} \frac{B^2}{2\mu_0} \ell 2\pi r dr = \int_0^{R_1} \frac{\mu_0 I^2 r^2}{4\pi^2 R_1^4} \frac{1}{2\mu_0} \ell 2\pi r dr = \frac{\mu_0 I^2 \ell}{16\pi}$$

$$\tilde{L}_{int} = \frac{L_{int}}{\ell} = \frac{2U_m^{int}}{\ell I^2} = \frac{\mu_0}{8\pi}$$

$$\tilde{L} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log\left(\frac{R_2}{R_1}\right)\right]$$

Forze su circuiti rigidi (a corrente costante)

N circuiti rigidi → j-esimo circ. f_j / I_j = costante

$$U_m = \frac{1}{2}\sum_{j=1}^{N} I_j \Phi_j$$ Φ_j flusso totale (auto + mutuo) attr. il circuito j-esimo

forza virtuale $\bar{F}_{ext}^{k}$ ⇝ Lavoro virtuale $\delta L^k = \bar{F}_{ext}^{k} \cdot \delta \bar{x}^{k}$

$$\delta L^k + \delta L_g + \delta L_R = dU_m$$

$$dL_g = -dU_g = \sum_j f_j \, dq_j = \sum_j f_j I_j \, dt$$ I_j = cost.

$$\delta L_R = -\sum_j R_j I_j^2 \, dt \quad ; \quad dU_m = d\left(\frac{1}{2}\sum_j I_j \Phi_j\right) = \frac{1}{2}\sum I_j \, d\Phi_j$$

$$\delta L_k = dU_m - dL_g - dL_R = \frac{1}{2}\sum_j I_j \, d\Phi_j - \sum_j f_j I_j \, dt + \sum_j R_j I_j^2 \, dt$$

$$\forall j \quad f_j = R_j I_j + \frac{d\Phi_j}{dt}$$

$$\delta L_k = \frac{1}{2}\sum_j I_j \, d\Phi_j - \sum R_j I_j^2 \, dt - \sum I_j \, d\Phi_j + \sum R_j I_j^2 \, dt =$$

$$= -\frac{1}{2}\sum I_j \, d\Phi_j = -dU_m$$

$$\delta L_k = -dU_m \qquad \bar{f}^k = -\bar{F}_{ext}^{k}$$

$$\bar{f}^k \cdot \delta \bar{x}^k = -\bar{F}_{ext}^{k} \cdot \delta \bar{x}^k = -\delta L_k = dU_m$$

$$\boxed{\bar{f}_k = \bar{\nabla} \cdot U_m \Big|_{I_j = \text{cost.}}}$$