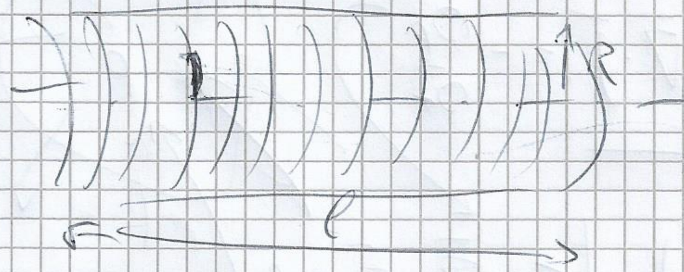
ESERC. 11/V/2020

FORZE MAGNETICHE SU CIRCUITI

CORRENTE DI SPOSTAMENTO

TEOREMA DI POYNTING (CONSERVAZIONE DI E_{elm})



Solenoide

$\ell \gg R$, N $n = \frac{N}{\ell}$

I stazionaria

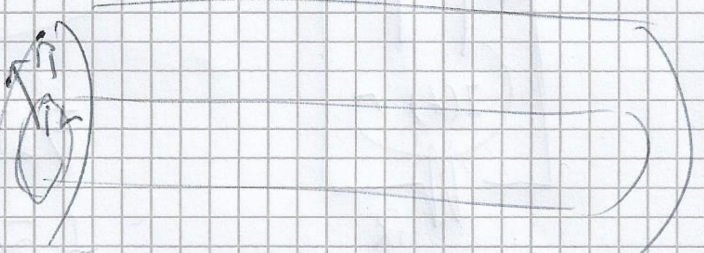
$$\overline{F} = \nabla U_m \big|_{I=cost}$$

$$\odot \quad \overline{B} = \mu_0 \frac{N}{\ell} I \hat{e}_z \qquad u_m = \frac{B^2}{2\mu_0} \quad \text{uniforme}$$

$$\underline{\underline{U_m}} = u_m \tau_{sol} = \frac{B^2}{2\mu_0} \pi R^2 \ell = \frac{1}{2}\mu_0 \frac{N^2 \pi R^2}{\ell} I^2$$

$$\overline{F} = \nabla U_m \Big|_{I=cost}$$

$R \to r$



$$U_m(r) = \frac{1}{2}\mu_0 \frac{N^2 \pi r^2}{\ell} I^2$$

$$F_r = \frac{\partial U_m}{\partial r}\Big|_{r=R,\ I=cost} = \frac{\partial}{\partial r}\left(\frac{1}{2}\mu_0 \frac{N^2 \pi I^2}{\ell} r^2\right)\Big|_{\substack{I=cost \\ r=R}} = \underline{\underline{\mu_0 \frac{N^2 I^2}{\ell} \pi R}}$$

$$p = \frac{dF}{dA} = \frac{F}{S} = \frac{1}{2}\mu_0 \frac{N^2}{\ell^2} I^2 =$$

$S = 2\pi R \ell$

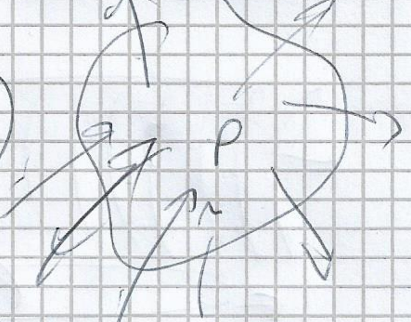
$$= \frac{1}{2}\mu_0 n^2 I^2 = \frac{B^2}{2\mu_0} = u_m$$

Corrente di spostamento

$$\vec\nabla \times \vec B = \mu_0 \vec J \quad \left(\leftrightarrow \quad \vec\nabla \times \vec H = \vec J\right)$$

$$\vec\nabla \cdot (\vec\nabla \times \vec B) = 0 \qquad \vec\nabla \cdot \vec J = 0$$

$$\vec\nabla \cdot \vec J = -\frac{\partial \rho}{\partial t}$$

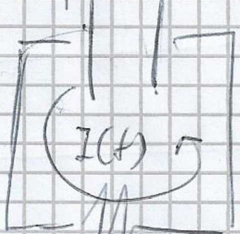


$$\boxed{\vec\nabla \cdot \left(\vec J + \varepsilon_0 \frac{\partial \vec E}{\partial t}\right) = 0} \leftarrow \vec\nabla \cdot \vec E = \rho/\varepsilon_0$$

$$\vec\nabla \times \vec B = \mu_0 \vec J + \mu_0 \varepsilon_0 \frac{\partial \vec E}{\partial t}$$

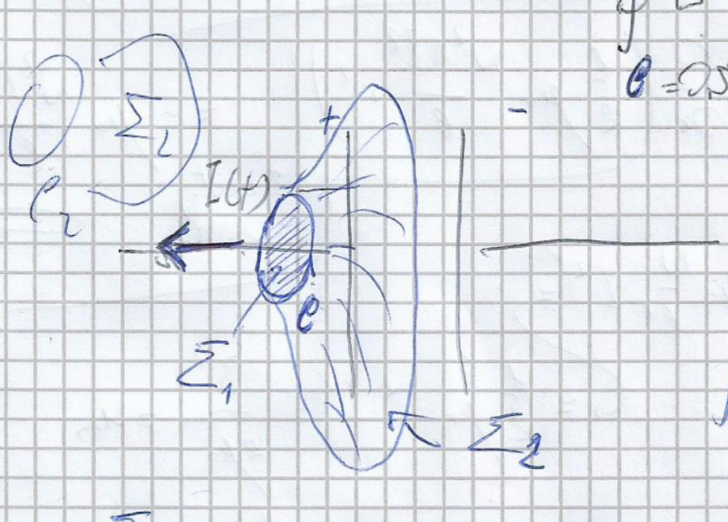
$$\vec J_s = \vec J_d = \varepsilon_0 \frac{\partial \vec E}{\partial t}$$ densità di corr. di spostamento

Es. RC



$$\vec\nabla \times \vec B = \mu_0 \vec J_c$$

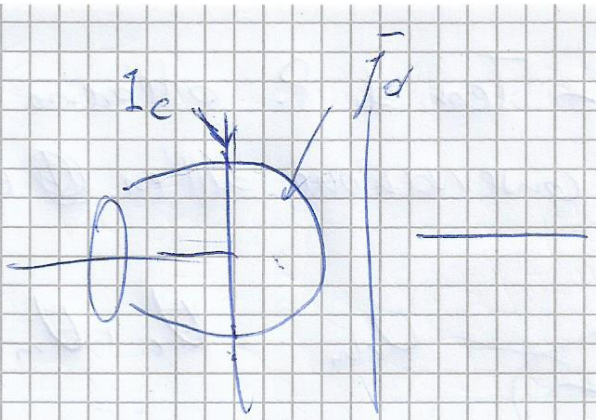
$$\oint_{\ell = \partial S} \vec B \cdot d\vec\ell = \mu_0 \iint_S \vec J_c \cdot d\vec S$$

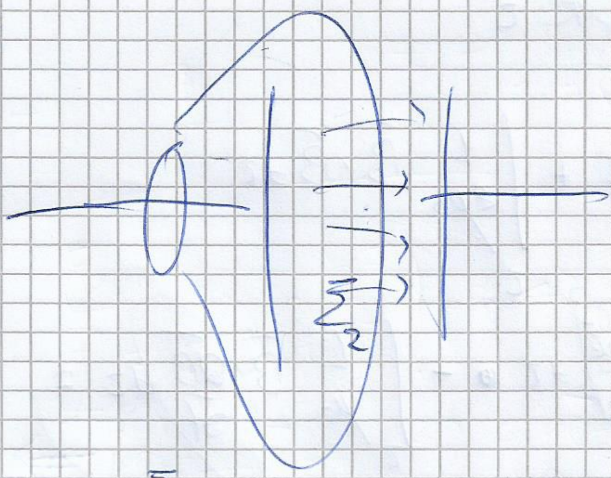


$$\iint_{\Sigma_1} \vec J_c \cdot d\vec S = I_c(t)$$

Σ_2 NON TOCCA MAI UN ELEMENTO CONDUTTORE

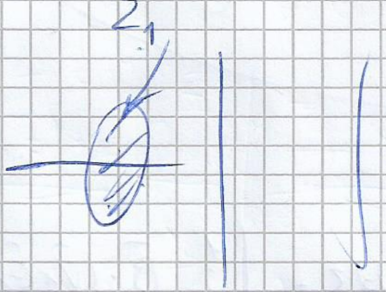
$$\iint_{\Sigma_2} \vec J_c \cdot d\vec S = 0$$



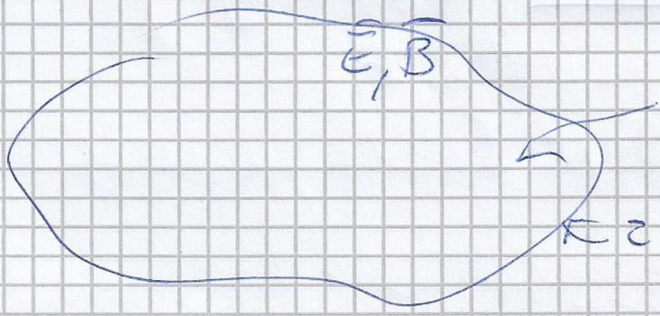


$$\oint \vec B \cdot d\vec l \neq 0$$

$$\iint_{\Sigma_2} \vec J_d \cdot d\vec S = I_c$$



Vettore di Poynting – Teor. di P. o teorema di conservazione dell'en. elettrom.



$$U_{elm} = U_e + U_m$$

$$U_{elm} = \int_\tau \frac{1}{2} \varepsilon_0 \vec{E}\cdot\vec{E}\, d\tau + \int_\tau \frac{1}{2\mu_0} \vec{B}\cdot\vec{B}\, d\tau$$

$$-\frac{dU_{elm}}{dt} = -\int_\tau \varepsilon_0 \vec{E}\cdot\frac{\partial \vec{E}}{\partial t}\, d\tau - \int_\tau \frac{1}{\mu_0} \vec{B}\cdot\frac{\partial \vec{B}}{\partial t}\, d\tau =$$

$$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \varepsilon_0\mu_0\, \partial\vec{E}/\partial t$$

$$\int_\tau \vec{E}\cdot\vec{J}\, d\tau - \int \frac{1}{\mu_0} \vec{E}\cdot(\vec{\nabla}\times\vec{B})\, d\tau + \int \frac{1}{\mu_0} \vec{B}\cdot(\vec{\nabla}\times\vec{E})\, d\tau$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{B})$$

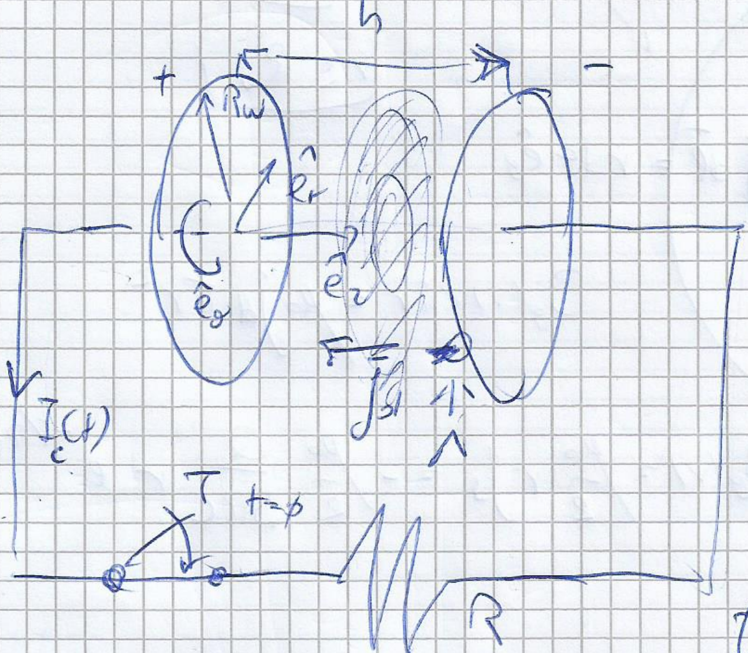
$$\boxed{-\frac{dU_{elm}}{dt}} = \int_\tau \vec{\nabla}\cdot\left(\frac{\vec{E}\times\vec{B}}{\mu_0}\right) d\tau + \int_\tau \vec{E}\cdot\vec{J}\, d\tau =$$

$$\boxed{= \oint_{S=\partial\tau} \frac{1}{\mu_0}(\vec{E}\times\vec{B})\cdot d\vec{S} + \int_\tau \vec{E}\cdot\vec{J}\, d\tau}$$ teor. di Poynting

$$\vec{I} = \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$ vettore di Poynting

$$[I] = \left[J/m^2 s\right]$$

$\bar{J}_d$ - cond. in scarica



$h << R_W$

$Q(t=\phi) = Q_0$

$$Q(t) = Q_0 e^{-t/RC}$$

$$I_c(t) = \frac{dq}{dt} = -\frac{dQ}{dt} \Rightarrow$$

$$I_c(t) = \frac{Q_0}{RC} e^{-t/RC}$$

$$\bar{J}_d = \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\bar{E} = \frac{\sigma(t)}{\varepsilon_0} \hat{e}_z = \frac{Q(t)}{\varepsilon_0 S} = \frac{Q_0}{\varepsilon_0 S} e^{-t/RC} \hat{e}_z$$

$$\underline{\bar{J}_d} = \varepsilon_0 \frac{\partial \bar{E}}{\partial t} = \frac{1}{S} \frac{dQ}{dt} \hat{e}_z = -\frac{1}{\underset{\uparrow \pi R_W^2}{S}} I_c(t) \hat{e}_z = -\frac{Q_0}{SRC} e^{-t/RC} \hat{e}_z$$

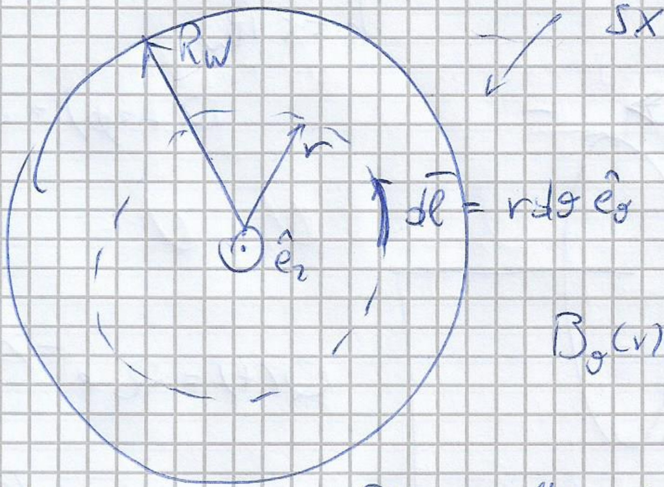
$$I_d = \int \bar{J}_d \cdot d\bar{S} = (-) I_c$$

$$\bar{\nabla} \times \bar{B} = \mu_0 (\bar{J}_c + \bar{J}_d) \underset{\uparrow}{=} \mu_0 \bar{J}_d = \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$\bar{J}_c = \phi$ nel conden.

$$\oint_{C = \partial \Sigma} \bar{B} \cdot d\bar{l} = \mu_0 \int_\Sigma \bar{J}_d \cdot d\bar{\Sigma}$$

$$\bar{B} = B_\theta(r) \hat{e}_\theta$$



$$\boxed{r \leq R_W}$$

$$B_\vartheta(r)\,2\pi r = \mu_0 J_d\,\pi r^2$$

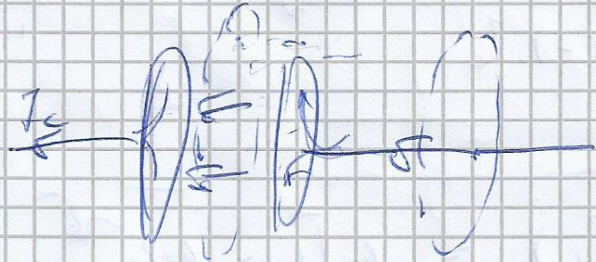
$$B_\vartheta(r) = \frac{\mu_0}{2} r J_d = -\frac{\mu_0}{2}\frac{Q}{SRC}\, r\, e^{-t/RC}$$

$r > R_W$

$$B_\vartheta(r)\,2\pi r = \mu_0 \iint_\Sigma \vec{J}_d \cdot d\vec{\Sigma} =$$

$$= \mu_0 \underbrace{\pi R_W^2 J_d}_{I_c}$$

$$B_\vartheta(r) = -\frac{\mu_0 I_c}{2\pi r}$$



$$U_e(t) = u_e \cdot Vol = \frac{1}{2}\varepsilon_0 E^2\, Vol = \frac{1}{2}\varepsilon_0 \frac{Q_0^2}{\varepsilon_0^2 S^2} e^{-2t/RC}\, Sh =$$

$$= \frac{Q_0^2 h}{2\varepsilon_0 S} e^{-2t/RC} = \boxed{\frac{1}{2}\frac{Q_0^2}{C}} e^{-2t/RC} = U_{e0}\, e^{-2t/RC}$$

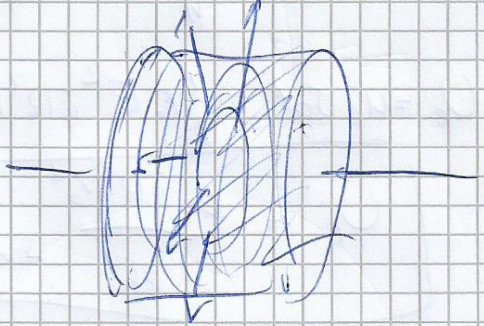
$$\frac{h}{\varepsilon_0 S} \rightarrow \frac{1}{C}$$

$$\frac{dU_e}{dt} = -\frac{2}{RC}\,\frac{1}{2}\,\frac{Q_0^2}{C} e^{-2t/RC} = -R\underbrace{\left(\frac{Q_0}{RC} e^{-t/RC}\right)^2}_{I_c^2(t)} = -R I_c^2(t) =$$

$$= -P_J$$

$$-\frac{dU_{elm}}{dt} = \int_{S=\partial\tau} \bar{I} \cdot d\bar{S} + \int_{\tau} \underbrace{\bar{E} \cdot \bar{J}_c}_{=\emptyset} d\tau$$

$\leftarrow$ applico al cond.



$$\bar{I} = \frac{1}{\mu_0} \bar{E} \times \bar{B} = (\bar{E} \times \bar{H}) =$$

$$= \underbrace{\frac{1}{\mu_0} \frac{Q_0}{\varepsilon_0 S} e^{-t/RC}}_{E_z(t)} \underbrace{\frac{\mu_0}{2} \frac{Q_0 r}{S RC} e^{-t/RC}}_{(-)B_\theta(r,t)} \underbrace{\hat{e}_z \times (-\hat{e}_\theta)}_{\hat{e}_r} \Rightarrow$$

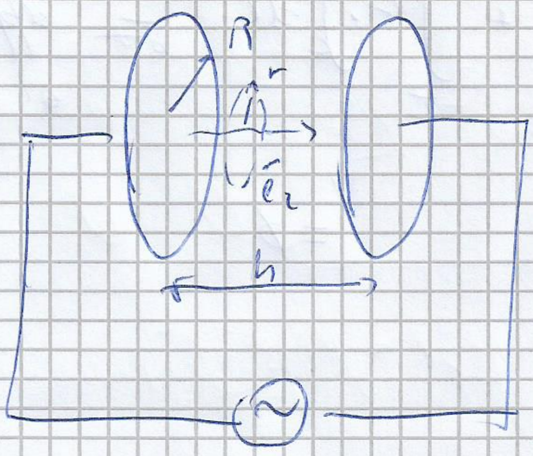
$$\bar{I}(r,t) = \frac{Q_0^2 r}{2\varepsilon_0 S^2 RC} e^{-2t/RC} \hat{e}_r$$

$$P_{out} = \int_{\Sigma_{lat}(r=R_W)} \bar{I} \cdot d\bar{\Sigma} = I(R_W, t) \underbrace{2\pi R_W h}_{\Sigma_{lat}} =$$

$$= \frac{Q_0^2 R_W}{2\varepsilon_0 S^2 RC} e^{-2t/RC} 2\pi R_W h \Rightarrow$$

$$S = \pi R_W^2$$

$$-\frac{dU_{elm}}{dt} = P_{out} = \frac{Q_0^2 h}{\varepsilon_0 S RC} e^{-2t/RC} = -\frac{dU_e}{dt}$$



$$\bar{E}(t) = \frac{V(t)}{h}\,\hat{e}_z = \frac{V_0}{h}\sin(\omega t)\,\hat{e}_z$$

$$U_e = u_e\,Vol = \frac{1}{2}\varepsilon_0 E^2 \pi R^2 h \Rightarrow$$

$$\boxed{U_e = \frac{\varepsilon_0 \pi R^2 V_0^2}{2h}\sin^2(\omega t)}$$

$$V(t) = V_0\sin(\omega t)$$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0\varepsilon_0 \frac{d}{dt}\int \bar{E}\cdot d\bar{S}$$

$r<R$ $$B_\theta(r)\,2\pi r = \mu_0\varepsilon_0 \frac{V_0}{h}\,\omega\cos(\omega t)\,\pi r^2$$

$$\Rightarrow \bar{B}(r,t) = \frac{\mu_0\varepsilon_0 V_0\omega}{2h}\, r\cos(\omega t)\,\hat{e}_\theta$$

$$u_m = \frac{B^2}{2\mu_0} = \frac{\mu_0\varepsilon_0^2 V_0^2\omega^2}{8h^2}\, r^2\cos^2(\omega t)$$

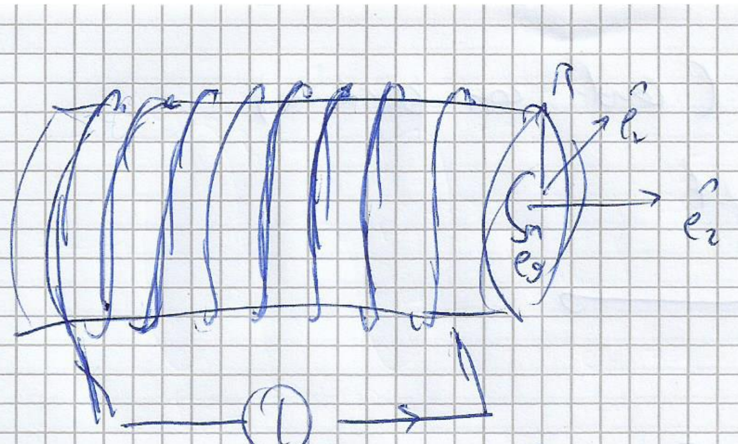
$$U_m = \int u_m\, d\tau = \int_0^R u_m \underbrace{h\,2\pi r\,dr}_{d\tau} = \frac{\mu_0\varepsilon_0^2 V_0^2\omega^2}{8h^2}\cos^2(\omega t)\,\frac{R^4}{4} \Rightarrow$$

$$\boxed{U_m = \frac{\pi\mu_0\varepsilon_0^2 V_0^2\omega^2 R^4}{16h}\cos^2(\omega t)}$$

$$\text{Ratio} = \frac{U_{e,max}}{U_{m,max}} = \frac{8}{\mu_0\varepsilon_0\omega^2 R^2} \gg 1$$

$\omega = 2\pi\cdot 50$ rad/s $\quad R = 10$ mm

$$\text{Ratio} \cong 4.6\cdot 10^{+17}$$



$$I(t) = I_0 \sin(\omega t)$$

$$\bar{B}(t) = \mu_0 n I(t) \hat{e}_z = \mu_0 n I_0 \sin(\omega t) \hat{e}_z$$

$$U_m = u_m \tau = \frac{B^2}{2\mu_0} \pi R^2 h = \frac{\pi \mu_0 n^2 I_0^2 h}{2} \sin^2(\omega t)$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\hookrightarrow \oint_{C = \partial\Sigma} \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_\Sigma \bar{B} \cdot d\bar{S}$$

$\hat{E}_\vartheta(r)$

$r<R \qquad E_\vartheta(r) 2\pi r = -\pi r^2 \frac{dB_z}{dt} = \cancel{\pi r^2 \mu_0 n I_0 \ldots(\omega t)}$

$$= -\pi r^2 n I_0 \omega \cos(\omega t)$$

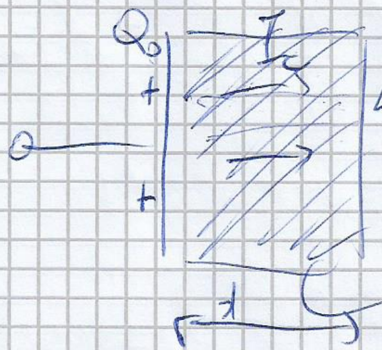
$$\bar{E}(r,t) = -\frac{\mu_0}{2} n I_0 \omega r \cos(\omega t) \hat{e}_\vartheta$$

$$U_e = \int u_e \, d\tau = \int_0^R \frac{1}{2} \varepsilon_0 E^2 h 2\pi r \, dr = \frac{\pi \varepsilon_0 \mu_0^2 n^2 I_0^2 \omega^2 h R^4}{16} \cos^2(\omega t)$$

$$\text{Ratio} = \frac{U_{e\,max}}{U_{m\,max}} = \frac{\varepsilon_0 \mu_0 \omega^2 R^2}{8} \simeq 2.2 \cdot 10^{-18}$$

$\omega = 2\pi \cdot 50 \text{ rad/s} \qquad R = 10 \text{ mm}$

Leaky capacitor (cond. con perdita)



$\rightarrow \varepsilon = \varepsilon_0 K$

Diel. ideale : $\rho \rightarrow \infty$

Diel. leaky : $\rho \uparrow$, ma finita

$$I_c = -\frac{dQ(t)}{dt} \qquad R = \frac{\rho d}{A}$$

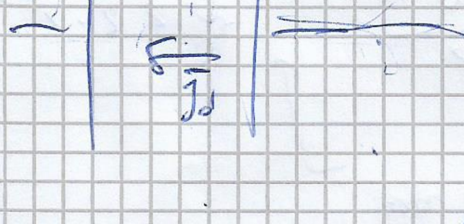
$$(=) \rightarrow I_c = \frac{V(t)}{R} = \frac{1}{R}\frac{Q(t)}{C}$$

$$-\frac{dQ(t)}{dt} = \frac{Q(t)}{RC} \rightsquigarrow Q(t) = Q_0 e^{-t/RC}$$

$$I_c(t) = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}$$

$$\bar{J}_d = \varepsilon \frac{\partial \bar{E}}{\partial t} \left(= \frac{\partial \bar{D}}{\partial t} \right) = \varepsilon \frac{d}{dt} \underbrace{\frac{Q(t)}{\varepsilon A}}_{\bar{E}(t)} \hat{e}_z = -\frac{1}{A}\frac{dQ}{dt}\hat{e}_z =$$

$$= -\frac{1}{A}\frac{Q_0}{RC} e^{-t/RC} \hat{e}_z$$



$$I_d = J_d A = -\frac{Q_0}{RC} e^{-t/RC} = -I_c$$

$$\bar{J}_c = -\bar{J}_d$$

$$\bar{B} = \emptyset$$