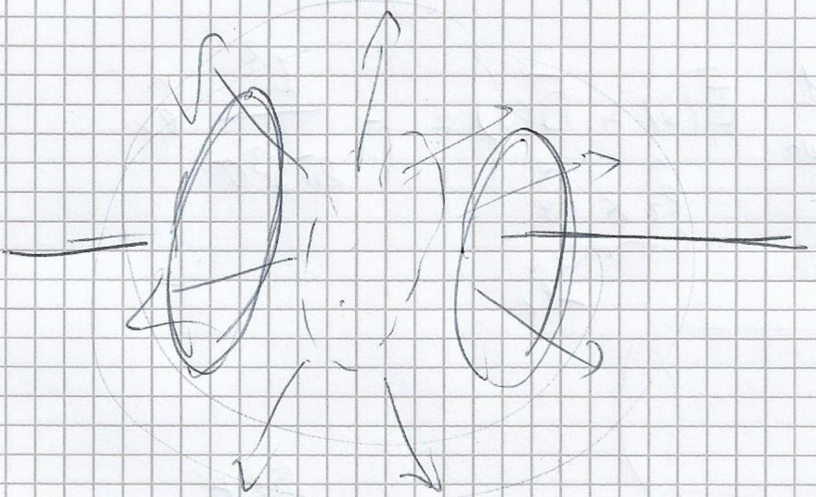
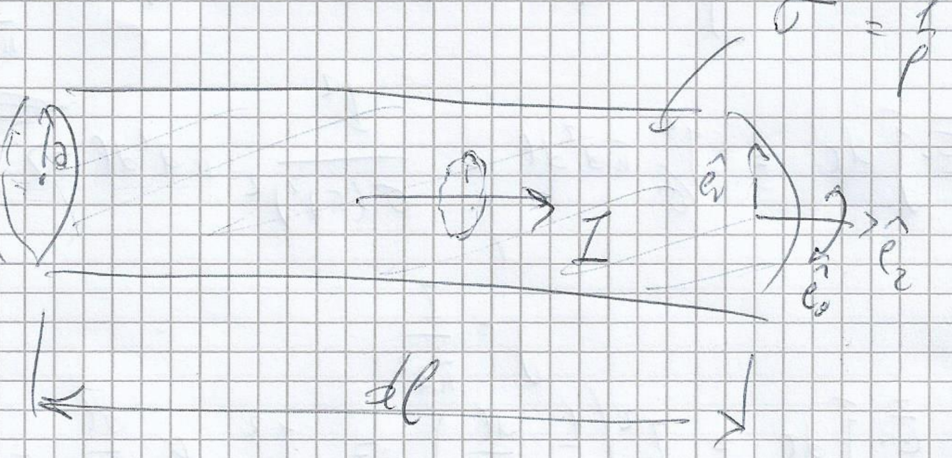
ESERC. 18/11/2020

POYNTING ; MAGNETISMO NELLA MATERIA





$$-\frac{dU_{elm}}{dt} = \int_{\Sigma_\tau} \bar{I}\cdot d\bar{S} + \int_\tau \bar{E}\cdot\bar{J}\,d\tau$$

$$\bar{I} = \frac{\bar{E}\times\bar{B}}{\mu_0}$$

$\bar{J} = \sigma\bar{E}$

$$\bar{E} = \bar{J}/\sigma = \frac{I}{\sigma\pi a^2}\,\hat{e}_z$$

$r \leq a \quad \vec{B}(r) = \frac{\mu_0 I}{2\pi a^2} r \hat{e}_\theta$

$\vec{B}(a) = \frac{\mu_0 I}{2\pi a} \hat{e}_\theta$

$r > a \quad \vec{B}(r) = \frac{\mu_0 I}{2\pi r} \hat{e}_\theta$

$$\vec{I}(r=a) = \frac{1}{\mu_0} \underbrace{\vec{E}(a) \times \vec{B}(a)}_{\hat{e}_z \times \hat{e}_\theta = -\hat{e}_r} = -\frac{I^2}{2\pi a^3 \sigma} \hat{e}_r$$

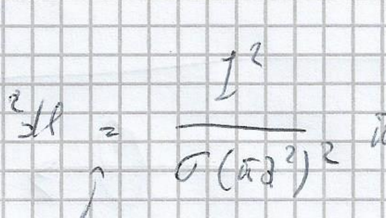
$$P = \int \vec{I} \cdot d\vec{S} = \vec{I}(a) \cdot \overbrace{2\pi a\, dl}^{S_{lat}}\, \hat{e}_r = -\frac{I^2}{\pi a^2 \sigma} dl$$

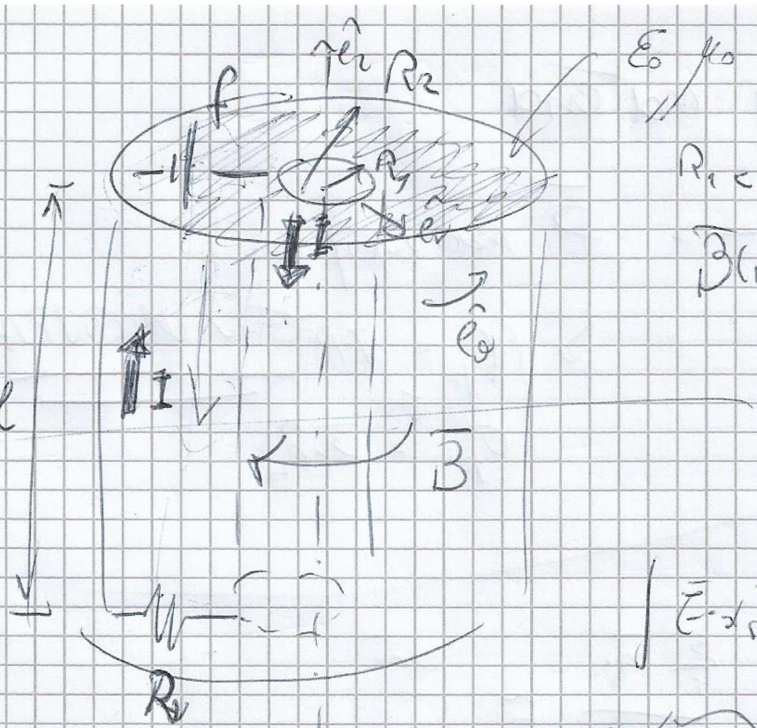
$$\int_\tau \vec{E} \cdot \vec{J}\, d\tau = \frac{J^2}{\sigma} \pi a^2 dl = \frac{I^2}{\sigma (\pi a^2)^2} \pi a^2 dl \Rightarrow$$

$J = \frac{I}{\pi a^2}$

$$\Rightarrow \int_\tau \vec{E} \cdot \vec{J}\, d\tau = I^2 \left(\frac{1}{\sigma} \frac{dl}{\pi a^2} \right) = I^2 \underbrace{\rho \frac{dl}{\pi a^2}}_{dR} = I^2 dR = P_J$$

$$\frac{dU_{elm}}{dt} = P$$





$\varepsilon_0 \quad \mu_0$

$R_1 < r < R_2$

$$\bar{B}(r) = -\frac{\mu_0 I}{2\pi r}\hat{e}_\theta = -\frac{\mu_0 f}{2\pi R_0}\frac{1}{r}\hat{e}_\theta$$

$f = R I$

$$\int \bar{E}\cdot d\bar{S} = \frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\bar{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{e}_r$$

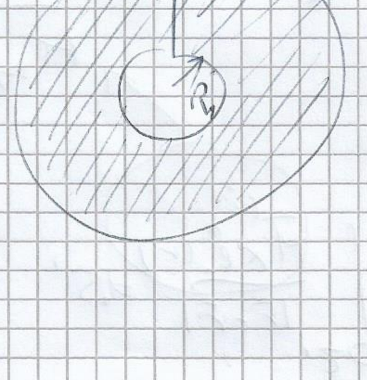
$$f = \int_{R_1}^{R_2} \bar{E}\cdot d\bar{r} = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{R_2}{R_1}\right) \rightsquigarrow \boxed{\lambda = \frac{2\pi\varepsilon_0 f}{\log(R_2/R_1)}}$$

$$\bar{E}(r) = \frac{f}{\log(R_2/R_1)}\frac{1}{r}\hat{e}_r$$

$$\bar{I} = \frac{1}{\mu_0}\bar{E}\times\bar{B} = -\frac{1}{\mu_0}E_r(r)\,B_\theta(r)\,\hat{e}_z = -\frac{f^2}{2\pi R_0\log(R_2/R_1)}\frac{1}{r^2}\hat{e}_z$$

$\hat{e}_r \times (-\hat{e}_\theta) = -\hat{e}_z$

$$P = \int \bar{I}\cdot d\bar{S}_{-\hat{e}_z} = \int_{R_1}^{R_2}\frac{f^2}{2\pi R_0\log(R_2/R_1)}\frac{1}{r^2}\,2\pi r\,dr = \frac{f^2}{R\log(R_2/R_1)}\cdot\log\left(\frac{R_2}{R_1}\right)$$



$$P_{\text{Poynting}} = \frac{f^2}{R_0} = P_J$$

Magnetismo nella materia

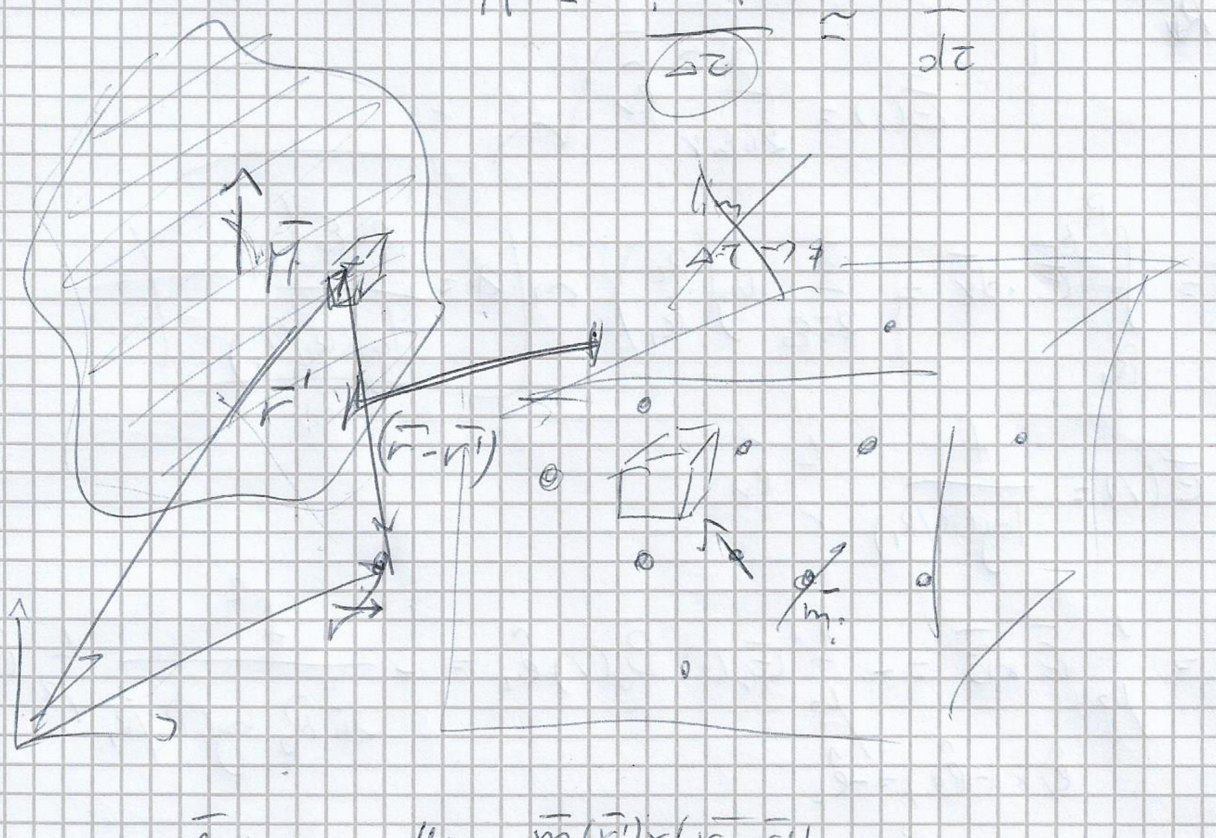
Elettrostatica $\bar p$ micro- $\longrightarrow$ $\bar P$ macroscopico

$$\begin{cases} \rho_p = -\bar\nabla\cdot\bar P \\ \sigma_p = \bar P\cdot\hat n \end{cases}$$ EQUIVALENZA

Magnetostatica

$\bar m$ micro-

$$\bar M = \frac{\sum_i \bar m_i}{\Delta\tau} \simeq \frac{d\bar m}{d\tau}$$



$$\bar A(\bar r) = \frac{\mu_0}{4\pi}\,\frac{\bar m(\bar r')\times(\bar r-\bar r')}{|\bar r-\bar r'|^3}$$

$d\bar m(\bar r') = \bar M(\bar r')\,d\tau$

$$\bar A(\bar r) = \int d\bar A(\bar r) = \int_{\tau'} \frac{\mu_0}{4\pi}\,\frac{d\bar m(\bar r')\times\ldots}{|\;\;|} =$$

τ' ← vol. del materiale

$\bar J_m$ $\qquad$ $\bar K_m$

nel vuoto $$= \frac{\mu_0}{4\pi}\int_{\tau'}\frac{\bar\nabla\times\bar M(\bar r')}{|\bar r-\bar r'|}\,d\tau' + \frac{\mu_0}{4\pi}\int_{\partial\tau'}\frac{\bar M(\bar r')\times\hat e_n(\bar r')}{|\bar r-\bar r'|}\,dS'$$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\bar{J}_c + \bar{J}_m)$$

$$\bar{J}_m \doteq \vec{\nabla} \times \bar{M}$$

$$\vec{\nabla} \times \left(\frac{\vec{B} - \mu_0 \bar{M}}{\mu_0} \right) = \bar{J}_c$$

$$\doteq \bar{H}$$

$$\boxed{\vec{\nabla} \times \bar{H} = \bar{J}_c} \quad \left(* \ \frac{\partial \vec{D}}{\partial t} \right)$$

$$\oint \bar{H} \cdot d\bar{l} = \sum I_{conc, cond.}$$

Rel. : $\bar{H}, \bar{B}, \bar{M}$

$$\bar{M} = \chi_m \bar{H}$$

suscettività magnetica

$(K = \varepsilon_r)$ $\quad \chi_m = f(\bar{H})$

$$\chi_m = f(\bar{r})$$

$\chi_m < 0 \quad \sim \quad 10^{-8} - 10^{-5}$

DIAMAGNETICI

Al, Cu, AISI 304, 316. stainless steel

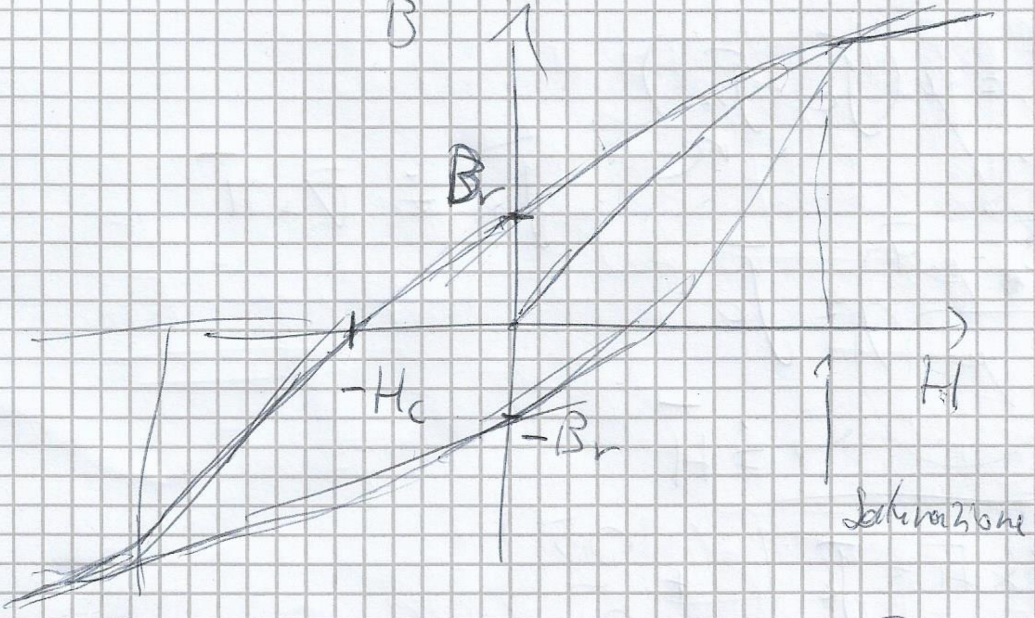
$$\bar{M} = \chi_m \bar{H}$$

$\chi_m > 0$ PARAMAGNETICI

$$\bar{B} = \mu_0 (\bar{H} + \bar{M}) = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r} \bar{H}$$

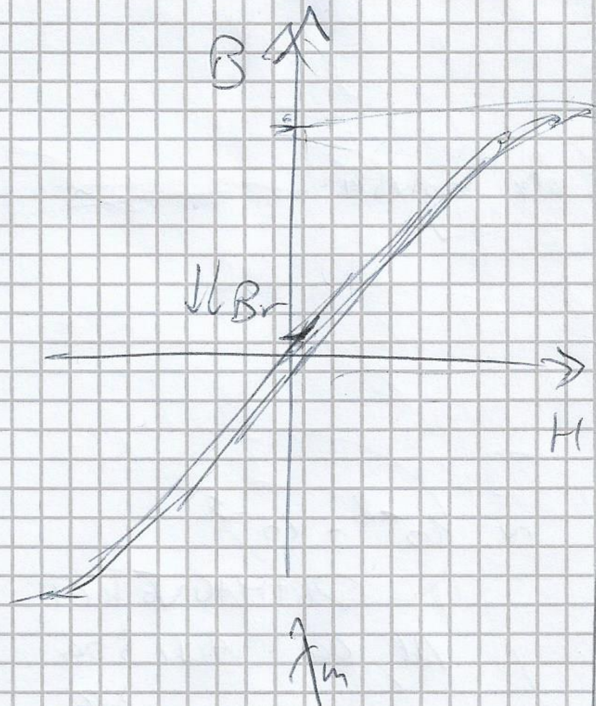
$\chi_m = f(H)$ FERRO-

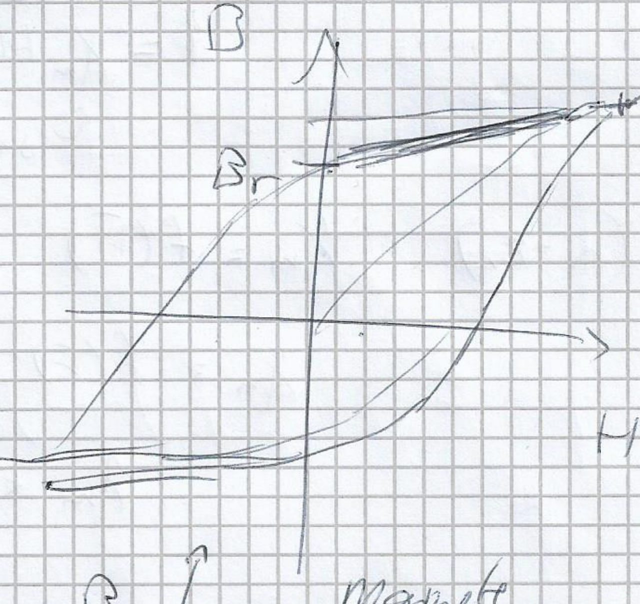
Fe, 400 stainless steel



B_r = residuo $\qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \vec{\nabla} \times \vec{M}_s$

soft ferro | hard ferro





χ_m

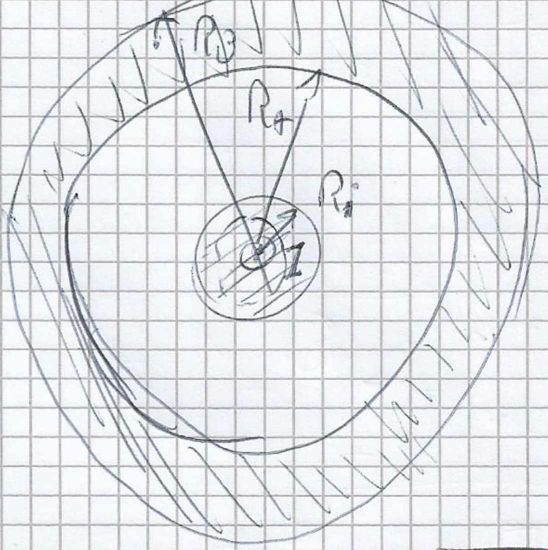
$B_r \uparrow$

Magnete permanente

$\mu_r = 1 + \chi_m \gg 1$

AlNiCo

Ceramic



μ_{r1} , μ_{r2} ≥ 1

$\bar{H}, \bar{B}, \bar{M}$

$\oint \bar{H} \cdot d\bar{l} = I_{conc}$

$\boxed{r < R_i}$ $\quad H_\theta 2\pi r = \frac{r^2}{R_i^2} I$

$$\begin{cases} \bar{H}(r) = \dfrac{I}{2\pi R_i^2} r \hat{e}_\theta \\ \bar{B}(r) = \mu_{r1}\mu_0 \bar{H}(\bar{r}) = \dfrac{\mu_0 \mu_{r1} I r}{2\pi R_i^2} \hat{e}_\theta \\ \bar{M}(\bar{r}) = \chi_{m1} \bar{H}(\bar{r}) = \dfrac{\chi_{m1} I r}{2\pi R_i^2} \hat{e}_\theta \\ (\chi_{m1} = \mu_{r1} - 1) \end{cases}$$

$R_i < r < R_A$ $\quad 2\pi r H_\theta = I$

$$\begin{cases} \bar{H}(r) = I/2\pi r \; e_\theta \\ \bar{B}(r) = \mu_0 \bar{H}(r) = \dfrac{\mu_0 I}{2\pi r} \hat{e}_\theta \\ \bar{M} = \emptyset \end{cases}$$

$R_A < r < R_B$ $\quad 2\pi H_\theta(r) = I$

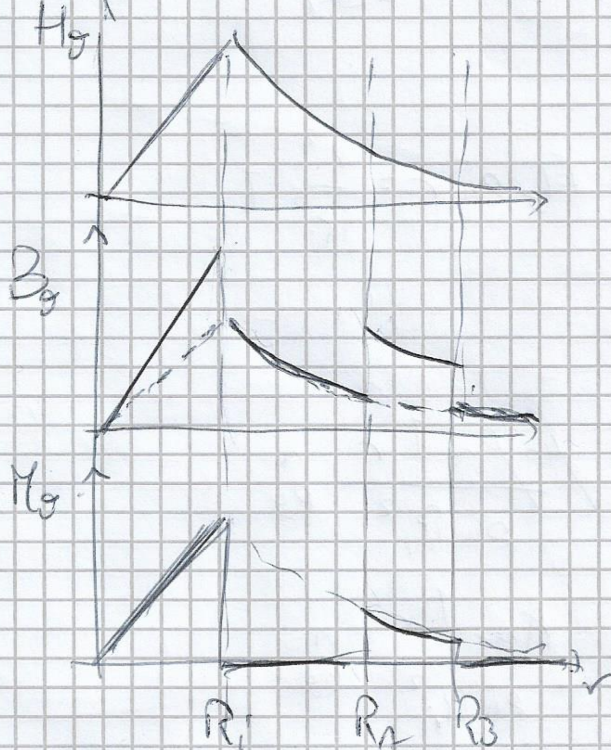
$$\begin{cases} \bar{H}(r) = I/2\pi r \; \hat{e}_\theta \\ \bar{B}(r) = \mu_0 \mu_{r2} \dfrac{I}{2\pi r} \hat{e}_\theta \\ \bar{M}(r) = \chi_{m2} \bar{H} = \dfrac{\chi_{m2} I}{2\pi r} e_\theta \end{cases}$$

$r > R_3$

$$\bar{H}(r) = \frac{I}{2\pi r}\,\hat{e}_\vartheta$$

$$\bar{B}(r) = \mu_0 \bar{H}$$

$$\bar{M} = \phi$$



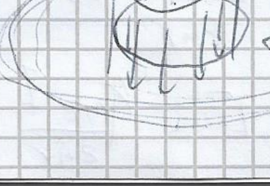
$r < R_1$

$$\bar{J}_m = \bar{\nabla} \times \bar{M} = \frac{1}{r}\left[\frac{\partial}{\partial r}(r M_\vartheta) - \cancel{\frac{\partial M_r}{\partial \vartheta}}\right]\hat{e}_z =$$

$$= \frac{\chi_{m1} I}{\pi R_1^2}\,\hat{e}_z = \chi_{m1} J_c\,\hat{e}_z$$

$$I_{mv} = J_m \cdot \pi R_1^2 = \chi_{m1} I$$

$$\bar{K}_m = \bar{M}(R_1) \times \hat{e}_n = M_\vartheta(R_1)\underbrace{\hat{e}_\vartheta \times \hat{e}_r}_{-\hat{e}_z} = -M(R_1)\,\hat{e}_z = -\frac{\chi_{m1} I}{2\pi R_1}\,\hat{e}_z$$



$\leftarrow 2\pi R_1$

$$I_{ms} = K_m\, 2\pi R_1 = -\chi_{m1} I$$

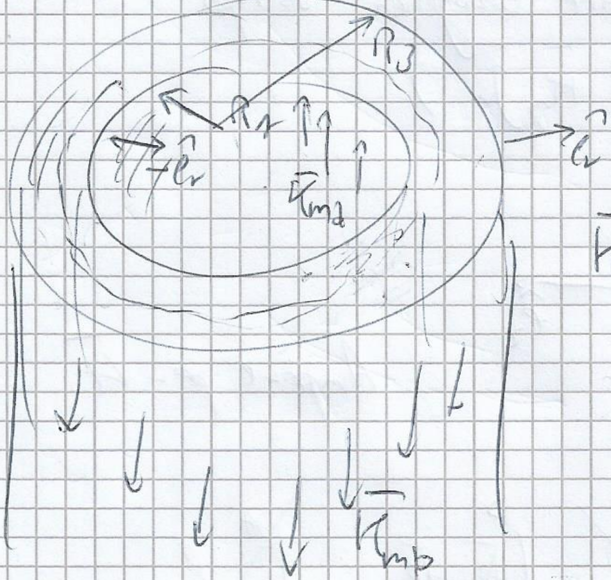
$R_A < r < R_B$

$$\bar{J}_m = \bar{\nabla} \times \bar{H}_m = \frac{\chi_{m2} I}{2\pi}\left\{\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{1}{r}\right) - \frac{\partial H_r}{\partial \theta}\right]\hat{e}_z = 0\right.$$

H_θ

$$\bar{H}_m(R_A) = \bar{M}(R_A) \times \hat{e}_n = \frac{\chi_{m2} I}{2\pi R_A}\hat{e}_z$$

$$\hat{e}_\theta \times (-\hat{e}_r) = \hat{e}_z$$

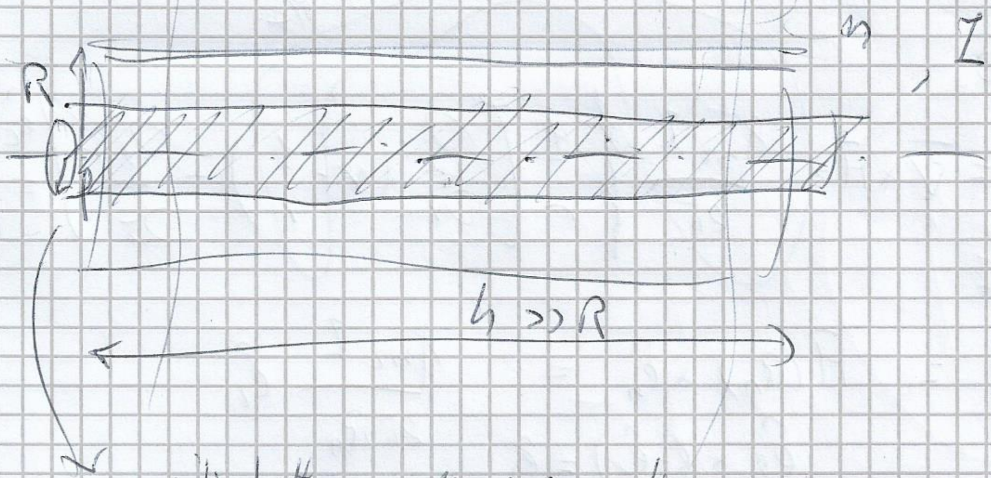


$$I_{m}(R_A) = H_m(R_A)\cdot 2\pi R_A = \chi_{m2} I$$

$$\bar{H}_m(R_B) = \bar{M}(R_B) \times \hat{e}_n = -\frac{\chi_{m2} I}{2\pi R_B}\hat{e}_z$$

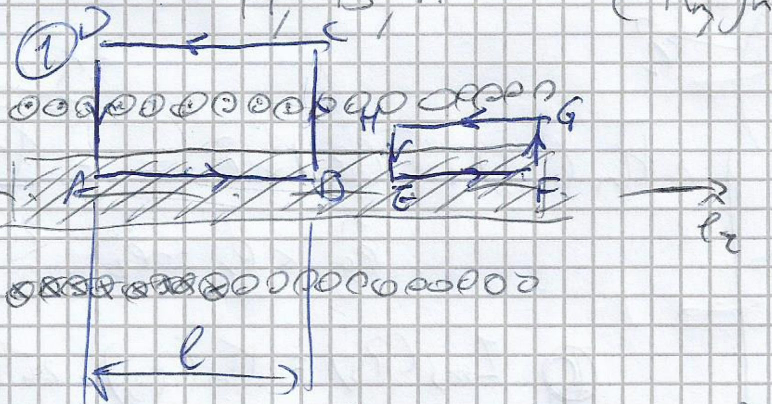
$$\hat{e}_\theta \times \hat{e}_r = -\hat{e}_z$$

$$I_{m}(R_B) = H_m(R_B)\cdot 2\pi R_B = -\chi_{m2} I$$



cilindretto $\mu_r > 1$ lineare omogeneo isotropo

$\vec{H}, \vec{B}, \vec{M}$ $(\vec{H}_n, \vec{J}_n)$



Ampère per $\vec{H}$

$$① \oint \vec{H}\cdot d\vec{l} = \int_A^B H_1\,dl + \cancel{\int_B^C}_{\perp} + \int_C^D \vec{H}_0\cdot d\vec{l} + \cancel{\int_D^A}_{\perp} =$$

$$= H_1 l = \underbrace{nl}_{\#\text{ spire}} I \qquad \underline{H_1 = nI}$$

$$② \oint \vec{H}\cdot d\vec{l} = \int_E^F H_1\,dl - \int_G^H H_2\,dl = (H_1 - H_2)\,l = 0$$

$$H_1 = H_2$$

$$\vec{H} = H\hat{e}_z = nI\hat{e}_z \quad \forall r < R$$

$$\vec{B}_1 = \mu_0\mu_r\vec{H} = \mu_0\mu_r nI\hat{e}_z \quad r < \rho$$

e $$\vec{B}_2 = \mu_0\vec{H} = \mu_0 nI\hat{e}_z \quad \rho < r < R$$

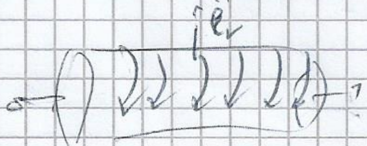
$$\vec{M} = \chi_m\vec{H} = \chi_m nI\hat{e}_z \quad r < \rho$$

$$\vec{M}_2 = 0 \quad \rho < r < R$$

$\bar{J}_m = \bar{\nabla} \times \bar{M} = \emptyset$

$\bar{K}_m (r=\rho) = \bar{M}(\rho) \times \hat{e}_n = \chi_m n I \hat{e}_\varphi$

$\hat{e}_z \times \hat{e}_r$

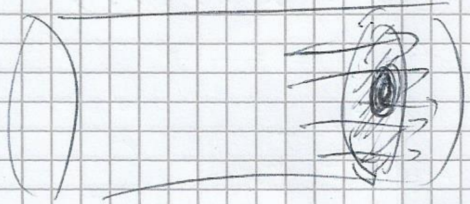


L

$N = nh$

$\Phi(\bar{B}) = LI = N \Phi_{spira} = N B_1 \pi \rho^2 + N B_2 \pi (R^2 - \rho^2) =$

$= \pi \mu_0 n^2 h I \left[\mu_r \rho^2 + (R^2 - \rho^2)\right]$



$L = \frac{\Phi}{I} = \mu_0 \pi n^2 h \left[\mu_r \rho^2 + (R^2 - \rho^2)\right]$

$\rho = R \quad L = \mu_0 \mu_r n^2 h \pi R^2 = \mu_r L_0$