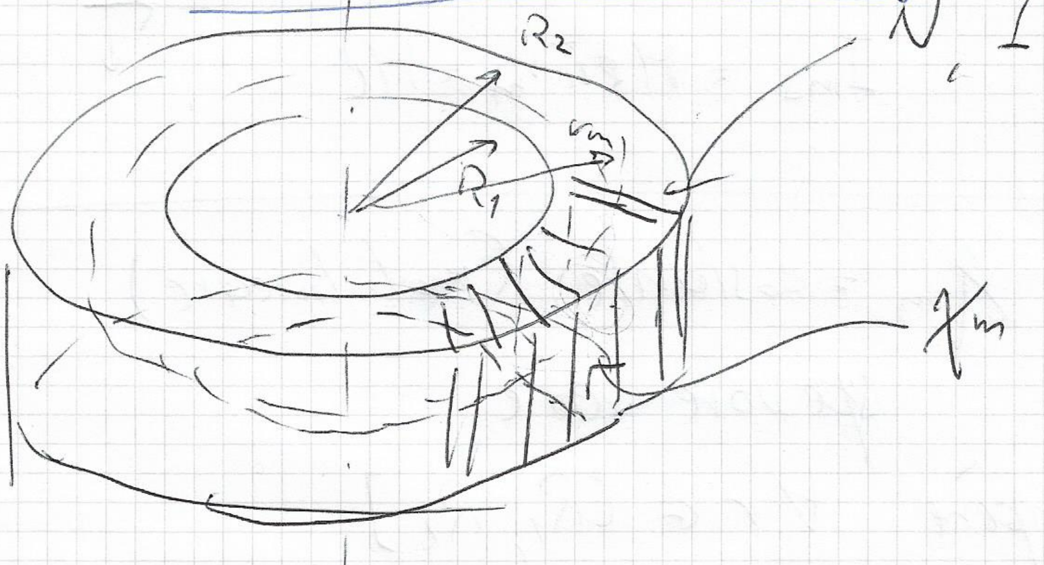
ESERC. 25/V/2020

MAGNETISMO NELLA MATERIA

Link audio : https://www.dropbox.com/s/75ft8xithxj55u3/Elm20200525audio.m4a?dl=0



1

$R_1 \simeq R_2 \simeq r_m$ valori medi per tutte le quantità

lineare

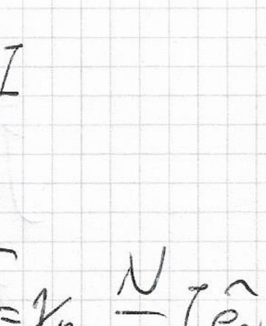
χ_m costante

$\mu_r = \chi_m + 1$

$$\oint \bar{H} \cdot d\bar{l} = \underbrace{2\pi r_m}_{l} H = lH = NI$$

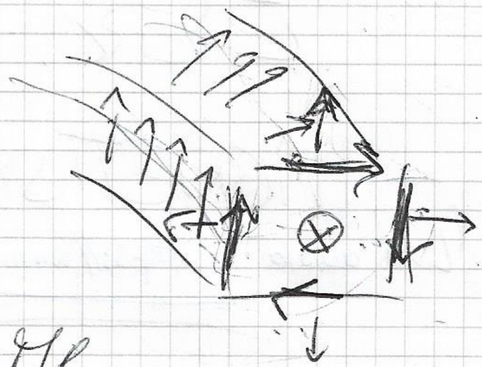
$$\bar{H} = \frac{N}{l} I \hat{e}_\theta \,;\quad \bar{B} = \mu_0 \mu_r \frac{N}{l} I \hat{e}_\theta \,;\quad \bar{M} = \chi_m \frac{N}{l} I \hat{e}_\theta$$

$$\bar{J}_m = \bar{\nabla} \times \bar{M} = 0$$



$$(\bar{\nabla} \times \bar{F})_{z,\,cyl} = \frac{1}{r}\left(\frac{\partial}{\partial r}(r F_\theta) - \frac{\partial F_r}{\partial \theta}\right)$$

$$\overline{K}_m = \overline{M} \times \hat{e}_n$$



$$|\overline{K}_m| = M$$

$$I_{ms} = M\, 2\pi r_m = M l$$

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(2) χ_m = costante (mat. lineare)

spessore reale

Ampère $\forall r \in [R_1, R_2]$

$$\oint \overline{H} \cdot d\overline{l} = 2\pi r H = NI$$

$$\overline{H} = \frac{NI}{2\pi r}\hat{e}_\theta \quad ; \quad \overline{B} = \mu_0 \mu_r \frac{NI}{2\pi r}\hat{e}_\theta ,$$

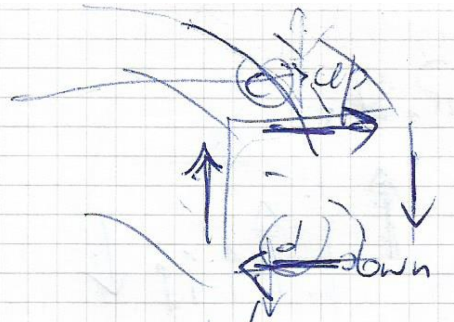
$$\overline{M} = \chi_m \frac{NI}{2\pi r}\hat{e}_\theta$$

$$\overline{J}_m = \overline{\nabla} \times \overline{M} = \emptyset$$

$$\overline{K}_m = \overline{M} \times \hat{e}_n = \chi_m \frac{NI}{2\pi r}\hat{e}_\theta \times \hat{e}_n$$

(a) $\overline{K}_m(R_1) = \chi_m \frac{NI}{2\pi R_1}\hat{e}_z \rightsquigarrow I_{ms}(R_1) = K_m(R_1) 2\pi R_1 = \chi_m NI$

$\overline{K}_m(R_2) = -\chi_m \frac{NI}{2\pi R_2}\hat{e}_z \rightsquigarrow I_{ms}(R_2) = -\chi_m NI$



$$\overline{K}_m^{\,up\,down}(r) = \pm \chi_m \frac{NI}{2\pi r}\,\hat{e}_r \rightarrow I_{ms}(r) = \pm\chi_m NI$$

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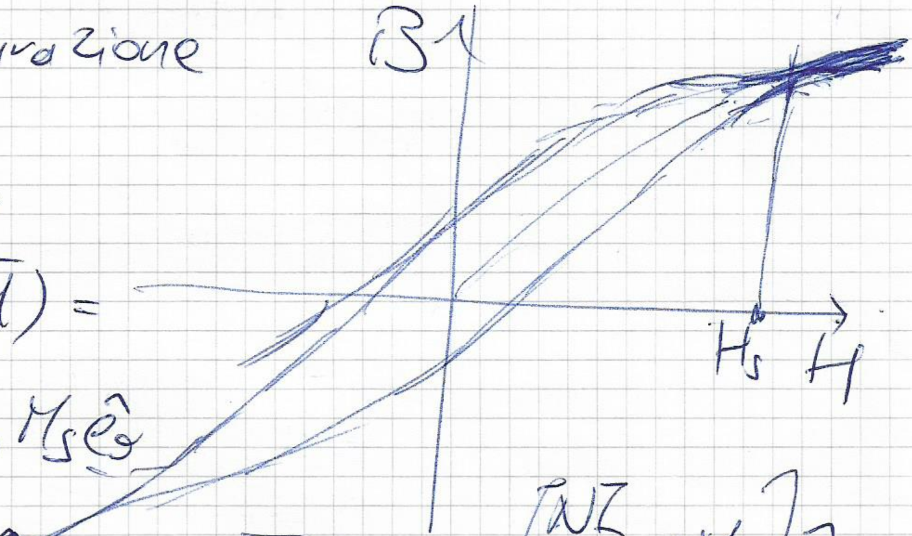
3 Mat. NON lineare (ferro-) a Saturazione



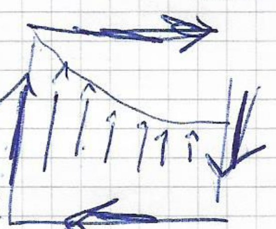
$$\overline{M} = M_s\,\hat{e}_\theta$$

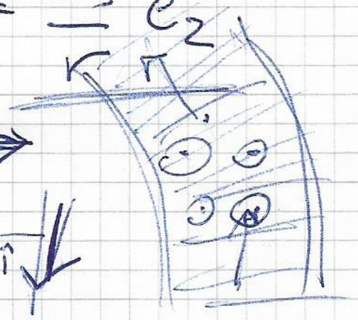
$$\overline{B} = \mu_0(\overline{H}+\overline{M}) = $$

$$= \mu_0 H\hat{e}_\theta + \mu_0 M_s \hat{e}_\theta$$

$$\overline{H} = \frac{NI}{2\pi r}\hat{e}_\theta \Rightarrow \overline{B}(r) = \mu_0\left[\frac{NI}{2\pi r} + M_s\right]\hat{e}_\theta$$

$$\overline{J}_m = \overline{\nabla}\times\overline{M} = \frac{1}{r}\left[\frac{\partial}{\partial r}(r M_s) - \frac{\partial \phi}{\partial z}\right] = \frac{M_s}{r}\hat{e}_z$$



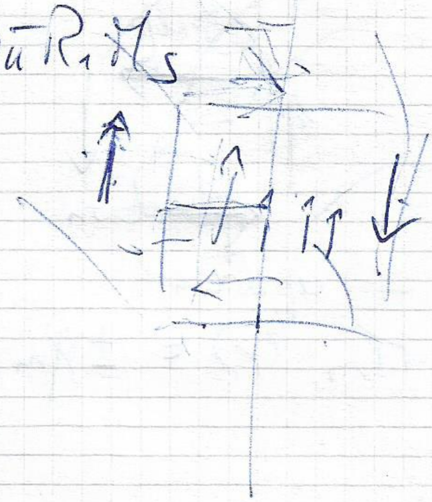


$$\overline{K}_m = \overline{M}\times\hat{e}_n = M_s\;\hat{e}_z\times\hat{e}_n$$

$$I_{mv} = \int_0^{2\pi}\int_{R_1}^{R_2}\frac{M_s}{r}\,r\,dr\,d\theta = 2\pi M_s\int_{R_1}^{R_2} dr$$

$$\Rightarrow I_{mv} = 2\pi M_s(R_2 - R_1)$$

$$I_{ms}(R_1) = 2\pi R_1 K_m(R_1) = 2\pi R_1 M_s$$



$$I_{ms}(R_2) = -2\pi R_2 M_s$$

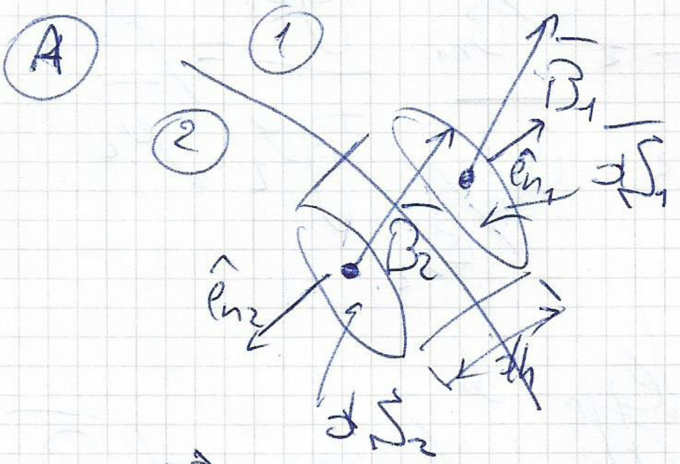
$$I_{mv} = 2\pi M_s (R_2 - R_1)$$

Circuiti magnetici

Legge di rifrazione delle linee di $\overline{B}$

$$\oint_{S_{chiusa}} \overline{B} \cdot d\overline{S} = 0 \quad ; \quad \oint \overline{H} \cdot d\overline{l} = 0$$

A



$$dh = o(\sqrt{S})$$

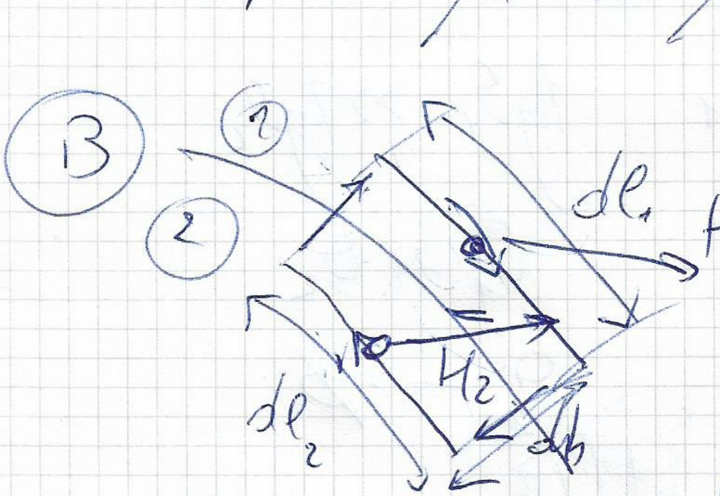
$$\Phi_{tot}(\overline{B}) = \overline{B}_1 \cdot d\overline{S}_1 + \overline{B}_2 \cdot d\overline{S}_2 + \cancel{\Phi_{lat}} = 0$$

$$dS_1 = dS_2 \qquad \Rightarrow (B_{n1} - B_{n2})\, dS = 0$$

$$B_{n1} = B_{n2}$$

omo-, iso- $\quad \mu_1 H_{n1} = \mu_2 H_{n2}$

B



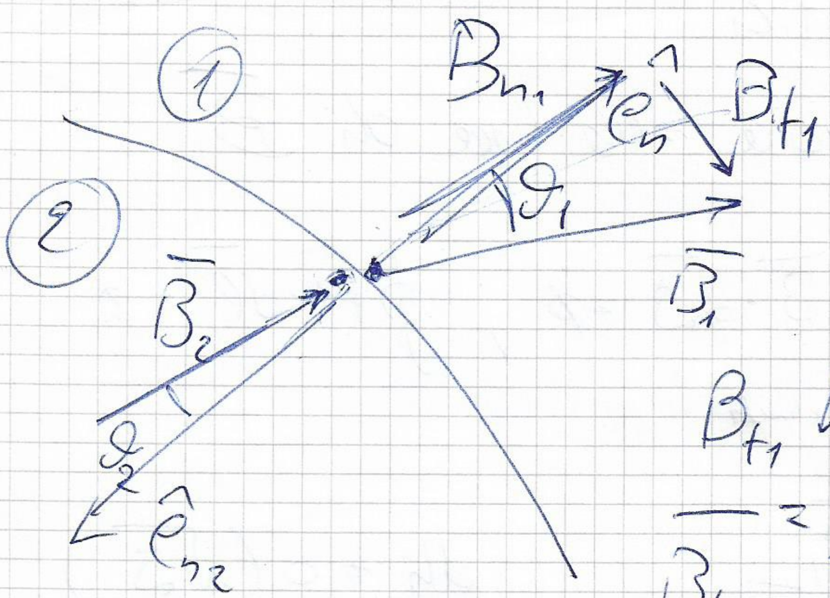
$$dh = o(dl)$$

$$dl_1 = dl_2$$

$$\overline{H}_1 \cdot d\overline{l}_1 + \overline{H}_2 \cdot d\overline{l}_2 = 0$$

$$(H_{t1} - H_{t2})\, dl = 0$$

$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2} \leftarrow H_{t1} = H_{t2}$$



$$B_{n1} = B_{n2}$$

$$\frac{B_{t1}}{B_{t2}} = \frac{B_{t1}/B_{n1}}{B_{t2}/B_{n2}} \qquad \frac{tg\theta_1}{tg\theta_2} = \frac{\mu_1}{\mu_2}$$

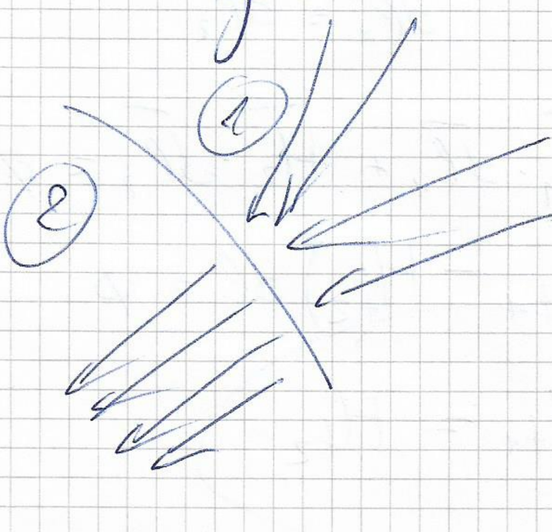
$$\boxed{\frac{tg\theta_1}{tg\theta_2} = \frac{\mu_1}{\mu_2}}$$ legge di rifrazione di $\bar{B}$

1. ferro
2. vuoto (app. aria) $\mu_{r2} = 1$

$$\frac{tg\theta_1}{tg\theta_2} = \mu_{r1} \gg 1 \qquad tg\theta_1 \gg tg\theta_2$$

$$\theta_1 \gg \theta_2$$

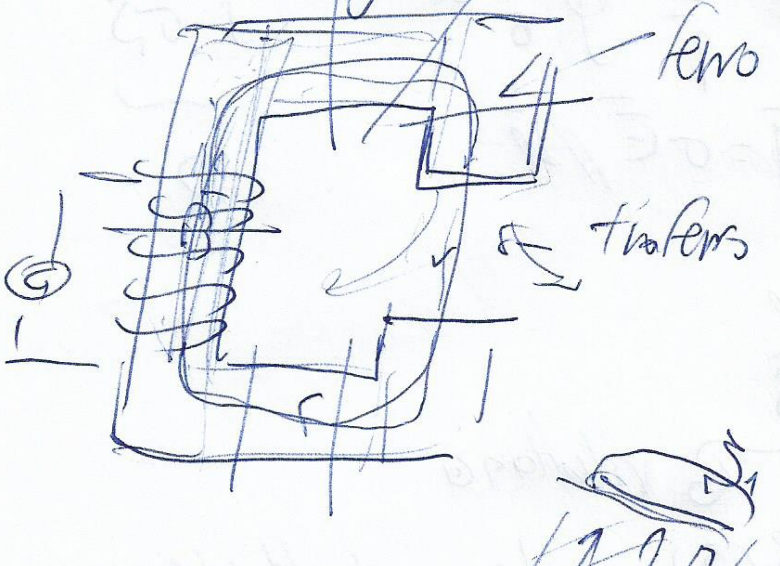
$$\theta_2 \to \emptyset$$



continuità di $\bar{B}$ attraverso i traferri

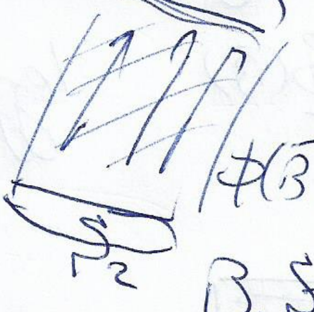
Circuito magn. – legge di Hopkinson

* Insieme di el. di mat. ferro-, di sezione S / $\sqrt{S} << l$,
* elementi sono disposti chiusa o quasi (al limite con traferri $\sqrt{S} >> e$)
* elementi possono concatenare I (avvolgimenti)



* $\Phi_{lat} = \phi$

NON C'È FLUSSO DISPERSO



$\Phi_{lat} = \phi$

$\Phi(B) = B_1 S_1 - B_2 S_2 = \phi$

$B_1 S_1 = B_2 S_2$

Φ (sez $\forall$) = costante

$\Phi = BS = $ cost. $\quad B = \Phi / S$

omo-, iso- $\bar{B} = \mu \bar{H}$ (alt. $= \mu(\bar{H})\,\bar{H}$)

$$NI = \oint \bar{H}\cdot d\bar{l} = \oint \frac{\bar{B}}{\mu}\cdot d\bar{l} = \int \frac{\Phi}{\mu S}\, dl \quad \text{alt.}$$

$\bar{B} \,//\, d\bar{l}$

$$\Rightarrow NI = \Phi \oint \frac{dl}{\mu S}$$

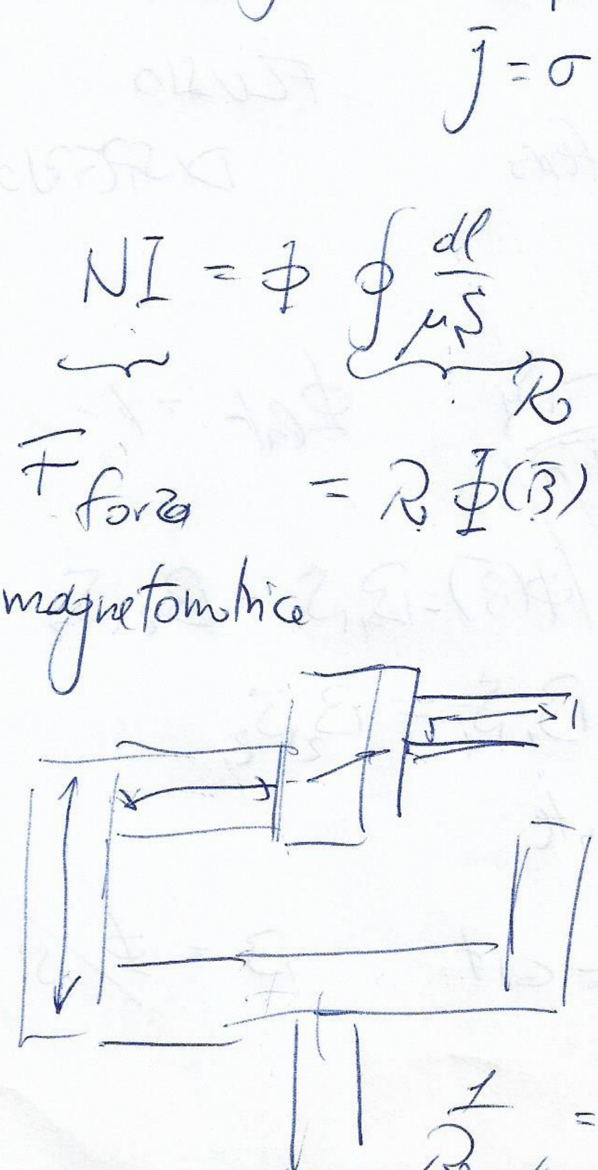
$J = I/S$

$$\text{fem} = \oint \bar{E}\cdot d\bar{l} \,\hat{=}\, \oint \frac{J}{\sigma}\, dl = I \underbrace{\oint \frac{dl}{\sigma S}}_{R}$$

$\bar{J} = \sigma \bar{E} \,//\, d\bar{l}$

$$\underbrace{NI} = \Phi \underbrace{\oint \frac{dl}{\mu S}}_{\mathcal{R}\ \text{riluttanza}}$$

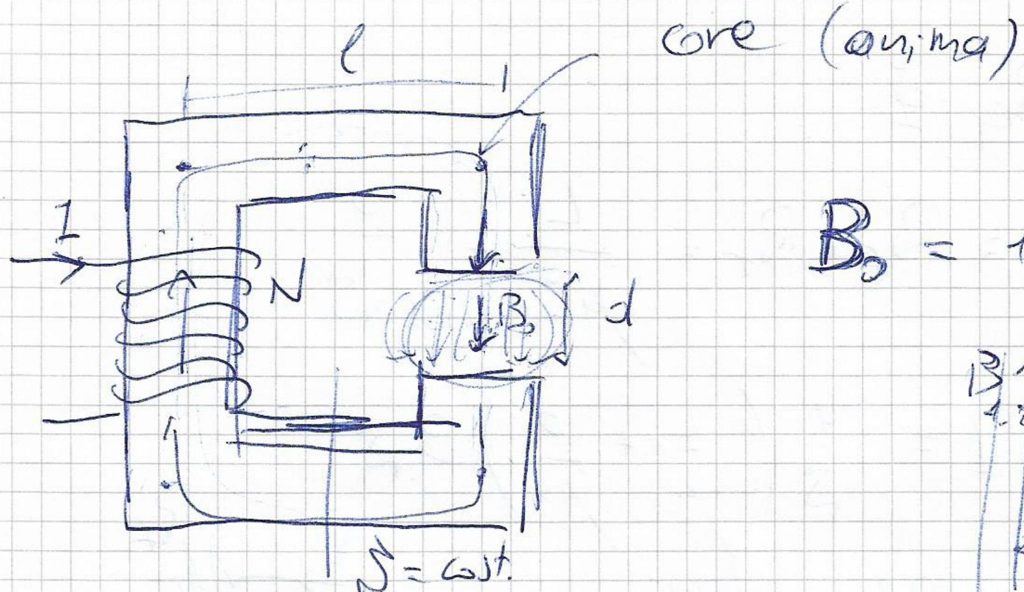
$F_{\text{forza magnetomotrice}} = \mathcal{R}\, \Phi(\bar{B})$ legge di Hopkinson



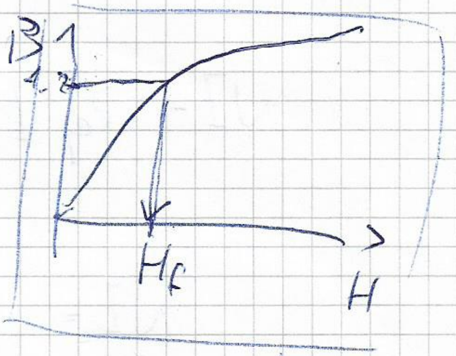
$$NI = \Phi \sum_i \mathcal{R}_i$$

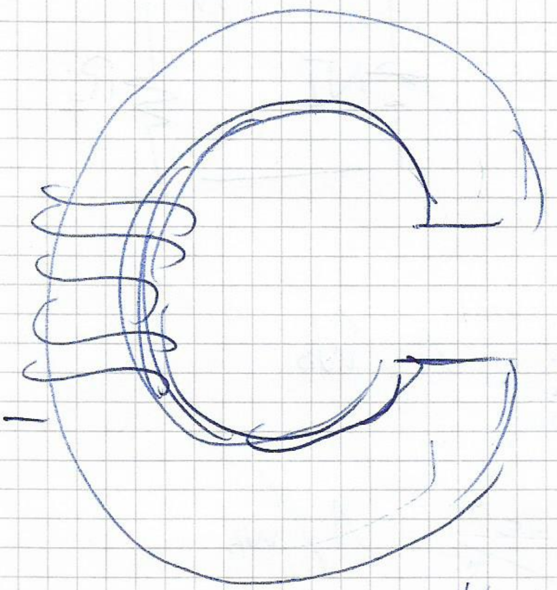
$$\mathcal{R}_i = \int \frac{dl_i}{\mu_i S_i}$$

$$\frac{1}{\mathcal{R}_{\text{eq}//}} = \sum_i \frac{1}{\mathcal{R}_i}$$



$B_0 = 1{,}2\ T$ $\quad L$





$B_f = B_0 = 1.2T$

$H_0 = \frac{B_0}{\mu_0}$

$$\oint \vec{H}\cdot d\vec{l} = H_f \cdot L + \underbrace{H_0}_{\hookrightarrow B_0/\mu_0} d = NI$$

$L = 4l - d$

$$I = \frac{H_f L}{N} + \frac{B_0 d}{\mu_0 N} = 3.17 + 795.77 = 798.94\ A$$

$H_f(@B = 1.2T) = 200\ A/m \quad (As/m)$

$d = 5\ cm \qquad N = 60$

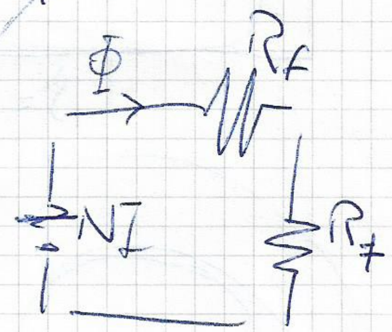
$4l = 1\ m$

$\mu = \frac{1.2}{200} = 6 \cdot 10^{-3} \qquad \mu_r = \frac{\mu}{\mu_0} = 4774{,}5 \gg 1$

$$NI = R_{eq} \cdot \Phi$$

$$R_{eq} = R_f + R_t = \frac{L}{\mu S} + \frac{d}{\mu_0 S}$$

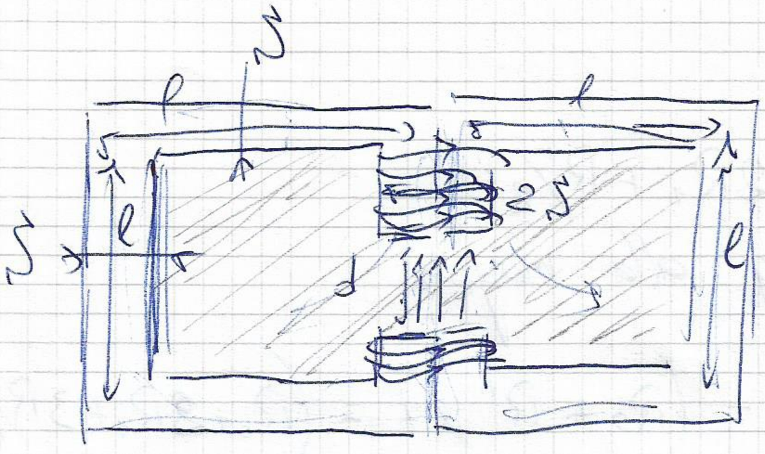
$$NI = R_{eq}\,\Phi = \frac{1}{\cancel{S}}\left(\frac{L}{\mu} + \frac{d}{\mu_0}\right) B \cdot \cancel{S}$$



$$NI = \underbrace{\frac{BL}{\mu}}_{H_f} + \underbrace{\frac{Bd}{\mu_0}}_{H_0}$$

$$R_f = \frac{L}{\mu S} = \frac{158.33}{S} \ As/Wb$$

$$R_t = \frac{d}{\mu_0 S} = \frac{3.98 \cdot 10^4}{S} \ A/Wb$$

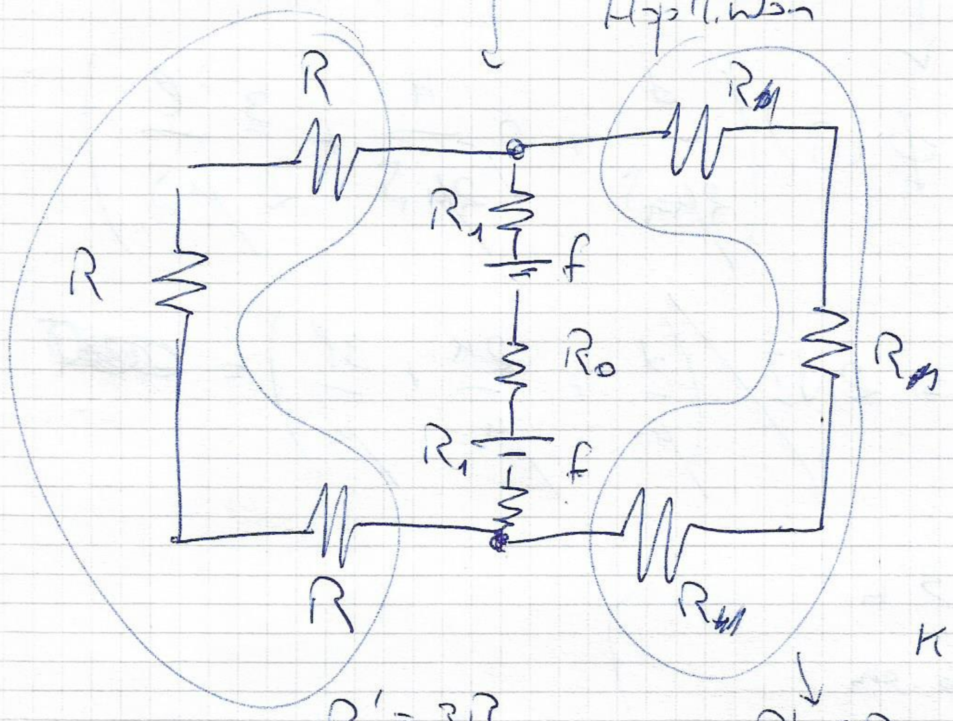


N, I

μ_r lineare

ferro- (soft)

Hopkinson



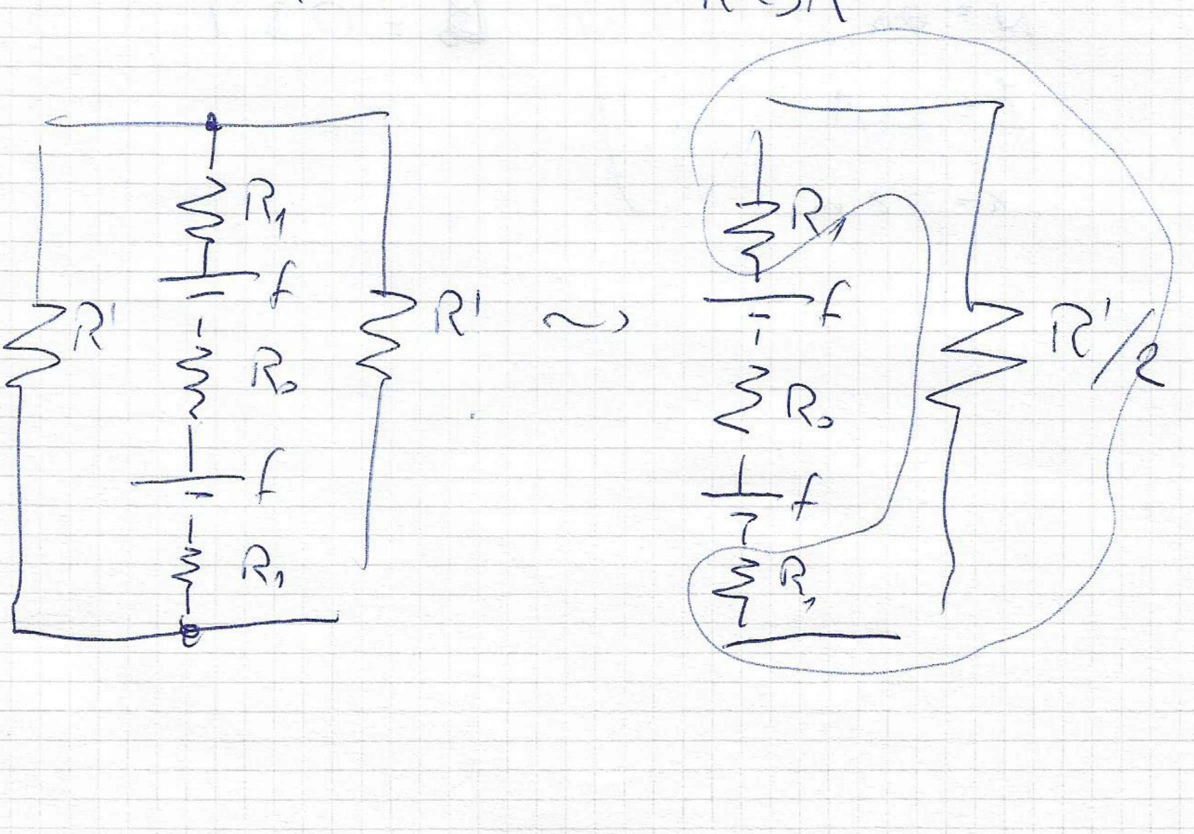
$R = l / \mu S$

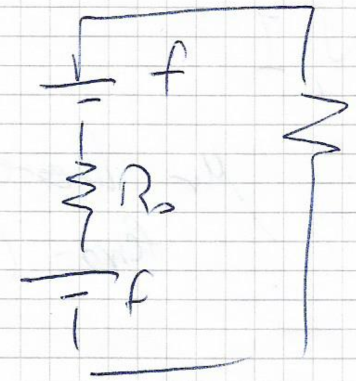
$R_0 = d / \mu_0 2S$

$R_1 = K / \mu 2S$

$K = (l-d)/2$

$R' = 3R$ $\qquad$ $R' = 3R$





$$R'' = 2R_1 + R'/_2$$

eq. alla maglia

$$2f = (R_0 + R'')\,i = (R_0 + 2R_1 + 3R/_2)\,i$$

$$2NI = \Phi(\bar{B})\left(\frac{d}{2\mu_0 S} + 2\frac{h}{2\mu S} + \frac{3}{2}\frac{\ell}{\mu S}\right)$$

$$B_0 = \frac{\Phi}{2S} = 2NI \Big/ \left(\frac{d}{\mu_0} + \frac{2h}{\mu} + \frac{3\ell}{\mu}\right) =$$

$\ell = 0.5$ m

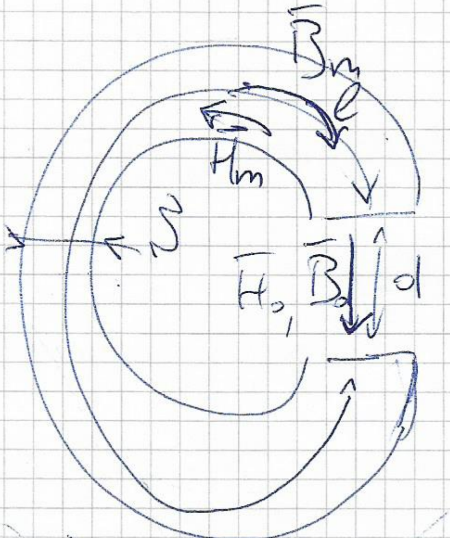
$d = 10$ cm

$N = 600$

$I = 20$ A

$h = 0.2$ m

$B_0 = 0.3$ T

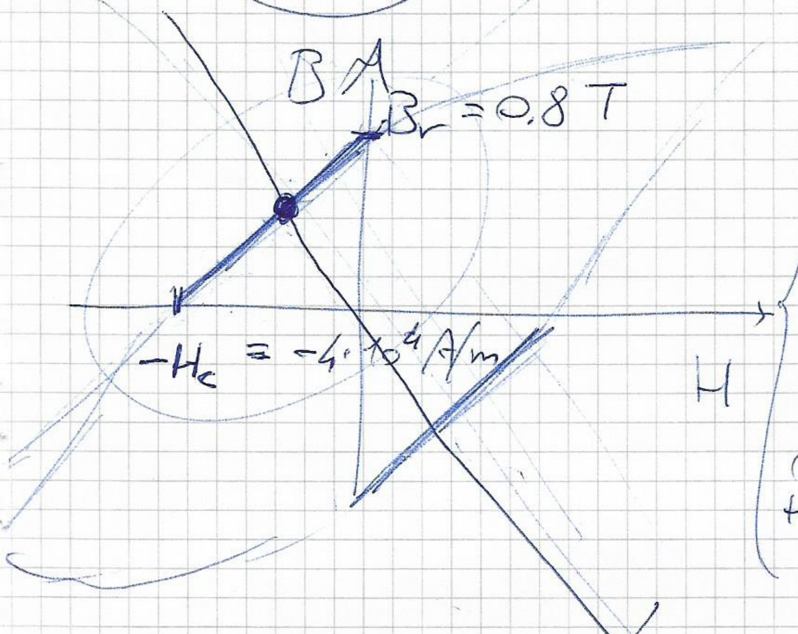


Magnete permanente

$$\oint \bar H \cdot d\bar l = H_m l + H_0 d = 0$$

$$H_m l + \frac{B_0}{\mu_0} d = \boxed{H_m l + \frac{B_m}{\mu_0} d = 0}$$

$$B_m = B_0$$



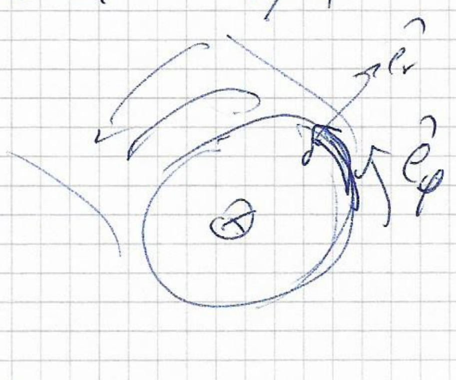
$$\begin{cases} B_m = \dfrac{B_r}{H_c} H_m + B_r \\ B_m = -\dfrac{\mu_0 l}{d} H_m \end{cases}$$

$$B_m = B_r \Big/ \left[1 + \frac{B_r}{H_c}\frac{d}{\mu_0 l}\right] = 0.35\ T$$

$l = 25$ cm

$d = 2$ cm

$$H_m = -2.24 \cdot 10^4\ A/m$$



$$B_m = \frac{\mu_0}{d} H_c l \Big/ \left[1 + \frac{\mu_0 l}{d}\frac{H_c}{B_r}\right]$$

$$\bar M = \frac{\bar B_m}{\mu_0} - \bar H = 3.0 \cdot 10^5\ A/m$$

$$\bar J_m = \bar\nabla \times \bar M = 0 \qquad \bar H_m = \bar M \times \underbrace{\hat e_n}_{\hat e_r} = M \hat e_\varphi = 3 \cdot 10^5\ A/m$$