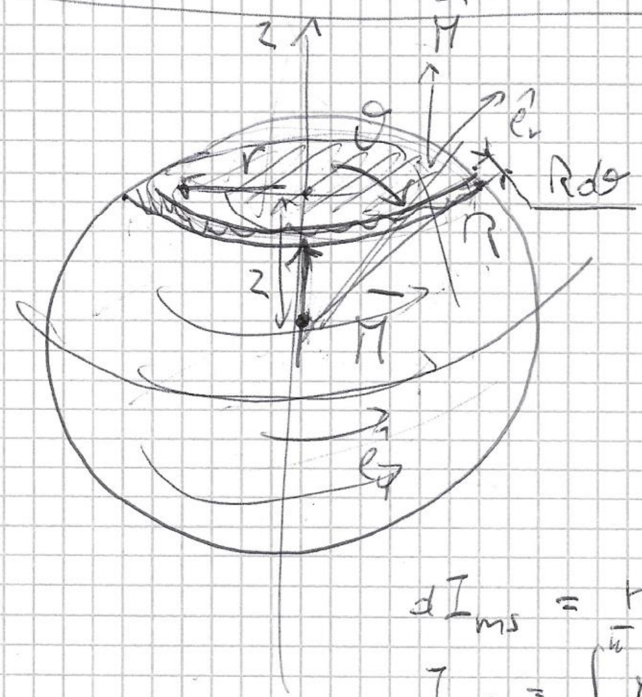
ESERC. 04/VI/2020
MAGNETISMO NELLA MATERIA
ONDE ELETTROMAGNETICHE



$\bar{M} = M\hat{e}_z$

$\bar{J}_m = \bar{\nabla} \times \bar{M} = \emptyset$

$\bar{K}_m = \bar{M} \times \hat{e}_n = \bar{M} \times \hat{e}_r =$

$= M\hat{e}_z \times \hat{e}_r = M\sin\vartheta\, \hat{e}_\varphi$

$dI_{ms} = K_m R d\vartheta = M R \sin\vartheta d\vartheta$

$I_{ms} = \int_\emptyset^\pi MR\sin\vartheta d\vartheta = MR\left[-\cos\vartheta\right]_\emptyset^\pi = 2MR$

$r = R\sin\vartheta$

z

$$B(\emptyset) = \mu_0 \frac{I}{2} \frac{r^2}{\underbrace{(r^2+z^2)^{3/2}}_{R^2}} = \mu_0 \frac{I}{2} \frac{\cancel{R^2}\sin^2\vartheta}{R^3}$$

$I \rightarrow dI_{ms}$

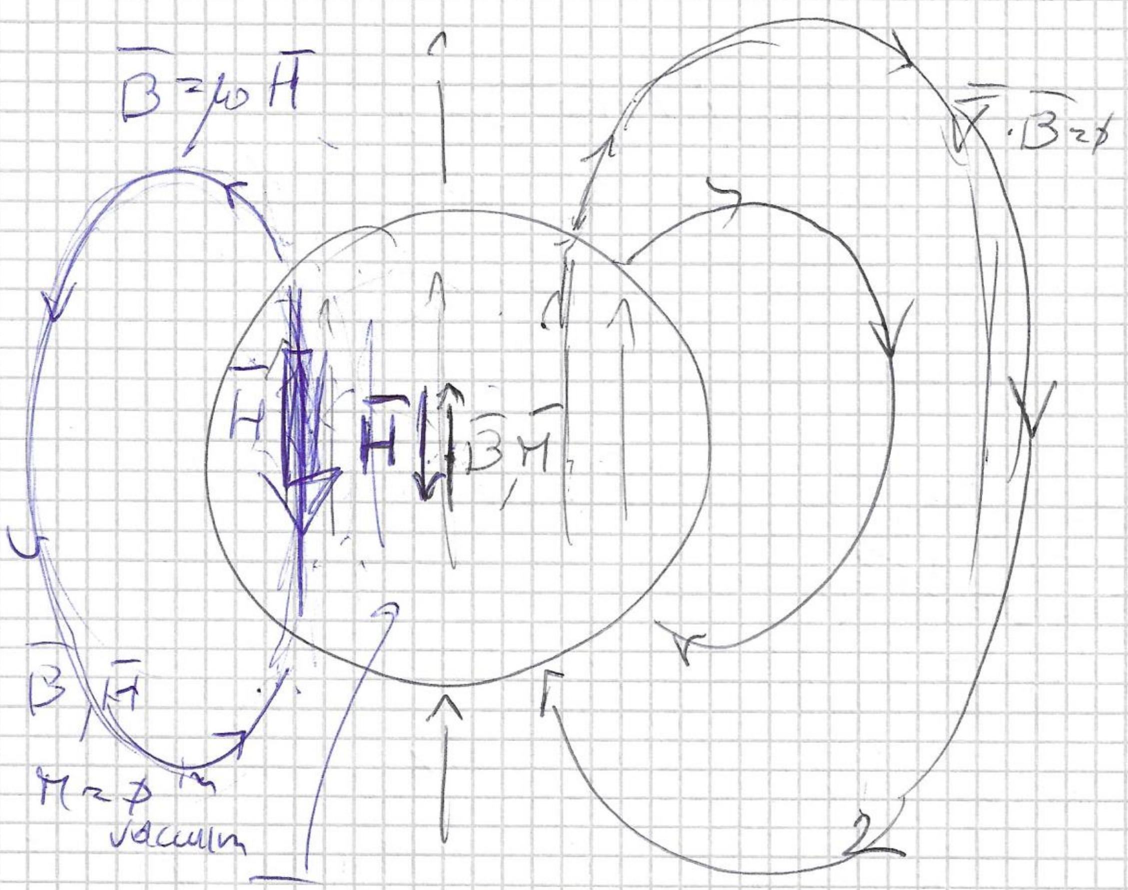
$$dB(\emptyset) = \mu_0 \frac{dI_{ms}}{2} \frac{R^2\sin^2\vartheta}{R^3} = \frac{\mu_0}{2} M\sin^3\vartheta d\vartheta$$

$$\underline{B_z(\emptyset) = \int_\emptyset^\pi \mu_0 \frac{M}{2} \sin^3\vartheta d\vartheta = \mu_0 \frac{M}{2} \frac{4}{3} = \frac{2\mu_0}{3} M}$$

$$\boxed{\bar{B}(0) = \mu_0 \frac{2}{3} \bar{M}}$$

$\bar{B} = \mu_0 (\bar{H} + \bar{M})$

$\hookrightarrow \bar{H}(0) = -\frac{1}{3}\bar{M}$



$$\bar{B} = \mu_0 (\bar{H} + \bar{M})$$

$$\oint \bar{H} \cdot d\bar{l} = 0$$

with field line as loop

$$\oint \bar{H} \cdot d\bar{l} = \underbrace{\int_{out} \bar{H} \cdot d\bar{l}}_{> 0} + \underbrace{\int_{in} \bar{H} \cdot d\bar{l}}_{< 0} = 0$$

Momento di dipolo magn. $\bar{m}$

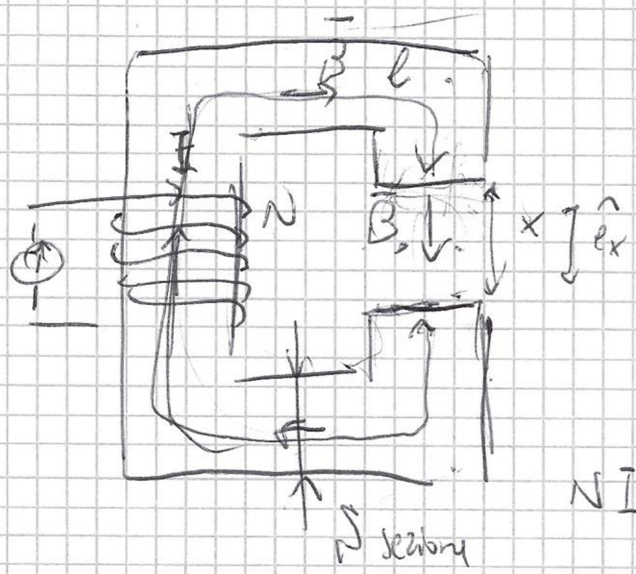
$$\bar{M} = \frac{\langle \bar{m} \rangle}{d\tau} = \frac{\langle \bar{m} \rangle}{Vol_{sph}}$$

uniforme su tutto volume

$$\bar{m} = \bar{M} \cdot Vol = \frac{4\pi}{3} R^3 \bar{M}$$

$$dm = dI_{ms} \cdot S = MR \sin\theta \, d\theta \cdot \pi r^2 = \pi R^3 M \sin^3\theta \, d\theta$$

$$m = \int dm = \pi R^3 M \int_0^{\pi} \sin^3\theta \, d\theta = \frac{4\pi}{3} R^3 M$$



$\mu_r = cost.$ lineare

Circ. magn.

$\Phi_{disperso} = \phi$

$B_0 = B$

$$NI = \oint \bar{H} \cdot d\bar{l} = Hl + H_0 x =$$

$$= \frac{B}{\mu_0 \mu_r} l + \frac{B}{\mu_0} x = \frac{1}{\mu_0} \left(\frac{l}{\mu_r} + x \right) B$$

$B_0 = B$

$$B = \frac{\mu_0 N I}{\left(\frac{l}{\mu_r} + x \right)}$$

$$U_m = U_{mf} + U_{mt} = u_{mf} V_f + u_{mt} V_t =$$

$$\left[u_m = \frac{1}{2} \frac{B^2}{\mu} \right] \qquad = \frac{1}{2} \frac{B^2}{\mu_0 \mu_r} S l + \frac{1}{2} \frac{B^2}{\mu_0} S x = \frac{1}{2\mu_0} B^2 S \left(\frac{l}{\mu_r} + x \right) \Rightarrow$$

$$\boxed{U_m = \frac{\mu_0}{2} \frac{N^2 I^2 S}{l/\mu_r + x}}$$

$$\boxed{\bar{F}_m = \bar{\nabla} U_m \Big|_{I = cost.}} \qquad F_x = \frac{\partial U_m}{\partial x} \Big|_{I = cost.} = -\frac{\mu_0}{2} \frac{N^2 I^2 S}{\left(l/\mu_r + x \right)^2} = -\frac{1}{2} \frac{B^2}{\mu_0} S$$

attrattiva

Es.: $\mu_r = 1000$
$l = 50$ cm
$x = 5$ cm
$S = 100$ cm^2
$N = 1000$
$I = 10$ A

$B = 0.248$ T
$F_x = -246.6$ N
~ 25 kg

$x = 2.5$ cm
$B = 0.493$ T
$F = -9.666$ N
(~ 98.5 kg)

$\vec{\nabla} \cdot \vec{E} = \emptyset$ $\qquad$ $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$

$\vec{\nabla} \cdot \vec{B} = \emptyset$ $\qquad$ $\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \partial \vec{E} / \partial t$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t} \right)$$

$$-\nabla^2 \vec{E} + \vec{\nabla}(\cancel{\vec{\nabla} \cdot \vec{E}}) = -\varepsilon_0 \mu_0 \partial^2 \vec{E} / \partial t^2$$

$$\begin{cases} \nabla^2 \vec{E} = \varepsilon_0 \mu_0 \dfrac{\partial^2 \vec{E}}{\partial t^2} \leftarrow \\ \nabla^2 \vec{B} = \varepsilon_0 \mu_0 \dfrac{\partial^2 \vec{B}}{\partial t^2} \leftarrow \end{cases}$$

$\vec{f}$

Eq. di d'Alembert

$$\nabla^2 \vec{f} = \frac{1}{v^2} \frac{\partial^2 \vec{f}}{\partial t^2}$$

$\vec{f}$ onda con vel. di propagazione v

$$\frac{1}{v^2} = \varepsilon_0 \mu_0 \qquad v = 1/\sqrt{\varepsilon_0 \mu_0} = c$$

$$\begin{cases} \nabla^2 \vec{E} = \dfrac{1}{c^2} \dfrac{\partial^2 \vec{E}}{\partial t^2} \\ \nabla^2 \vec{B} = \dfrac{1}{c^2} \dfrac{\partial^2 \vec{B}}{\partial t^2} \end{cases} \leftarrow \text{Maxwell}$$

+ eq. di Maxwell (accoppiamento)

Onda piana

Fronte d'onda piano

$\perp \quad \vec{c} = c\hat{k}$ $\qquad \vec{k}$ vettore d'onda

$$\vec{E} = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \qquad \vec{k} = k\hat{k}$$

$$\vec{B} = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{k} = k\hat{e}_x \qquad \vec{k}\cdot\vec{r} = kx$$

$\vec{E}(x,t) =$ cost. $\forall\, y, z$

$(\vec{B})$

$$\vec{E}_0 = E_{0y}\,\hat{e}_y$$
$$\vec{B}_0 = B_{0z}\,\hat{e}_z$$
polarizzazione lineare

$$\vec{E}(x,t) = E_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_y$$
$$\vec{B}(x,t) = B_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_z$$

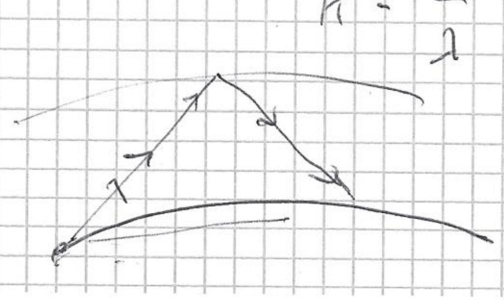
in $\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$

(1) $$\nabla^2 \vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E} = -k^2\,\vec{E}(x,t)$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2}\cdot\left[-\omega^2\,\vec{E}(x,t)\right]$$

$$\boxed{k^2 = \frac{\omega^2}{c^2}} \rightarrow k = \pm\frac{\omega}{c}$$

$k = \frac{2\pi}{\lambda}$ Relazione di dispersione (banale)



② Maxwell ?

$$\vec{\nabla}\times\vec{E} = \left[\frac{\partial E_y}{\partial x} - \frac{\partial}{\partial y}\phi\right]\hat{e}_z = ik E_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_z$$

$$\parallel$$

$$-\frac{\partial \vec{B}}{\partial t} = i\omega B_{0z}\, e^{i(kx-\omega t)}\,\hat{e}_z$$

$$\boxed{\frac{E_{0y}}{B_{0z}} = \frac{\omega}{k} = c}$$

E_{0z} , B_{0y} $\quad \frac{E_{0z}}{B_{0y}} = -c$

$$\boxed{\vec{E} = \vec{B}\times\vec{v}}$$

$$\vec{B} = \frac{\vec{v}\times\vec{E}}{v^2} = \frac{\hat{k}\times\vec{E}}{v}$$

$$\vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$$

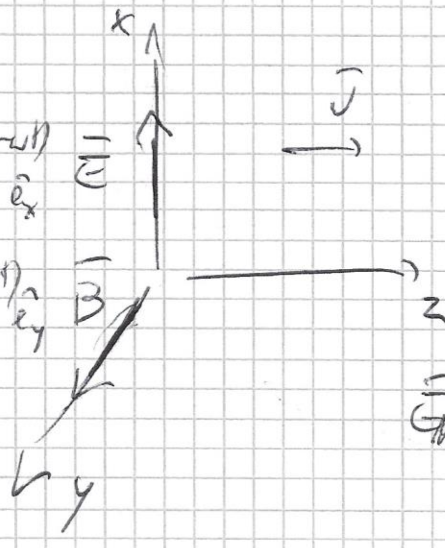
$$\begin{cases} \vec{E}(x,t) = E_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_y \\ \vec{B}(x,t) = \frac{E_{0y}}{c}\, e^{i(kx-\omega t)}\,\hat{e}_z \end{cases}$$

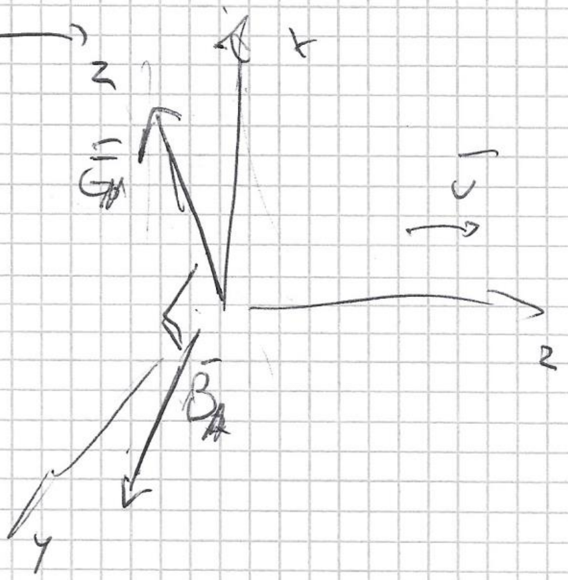
Polarizzazione

$\bar{E}(z,t) = E_{0x}\, e^{i(kz-\omega t)}\, \hat{e}_x$

$\bar{B}(z,t) = B_{0y}\, e^{i(kz-\omega t)}\, \hat{e}_y$

$\downarrow \frac{E_{0x}}{c}$





$E_{0x}\hat{e}_x + E_{0y}\hat{e}_y$

$\bar{E} = \bar{B} \times \bar{J}$

$\bar{E}_1 = E_{01} \cos(kz - \omega t + \varphi_1)\, \hat{e}_x$

$\bar{E}_2 = E_{02} \cos(kz - \omega t + \varphi_2)\, \hat{e}_y$

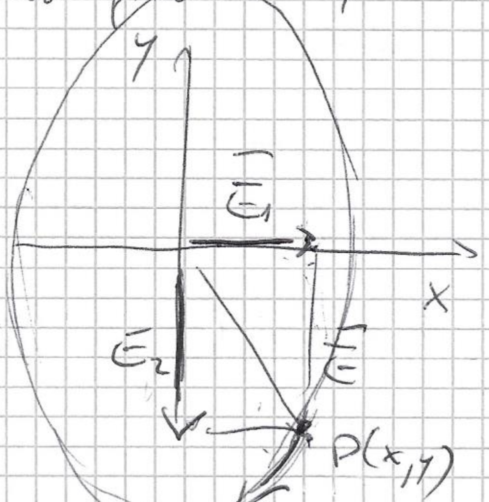
$\bar{E} = \bar{E}_1 + \bar{E}_2$

pol. lineare $\varphi_1 = \varphi_2$

Es. $\varphi_1 = 0$ $\quad \bar{E}_1 = E_{01} \cos(kz - \omega t)\, \hat{e}_x$

$\varphi_2 = -\pi/2$ $\quad \bar{E}_2 = E_{02} \sin(kz - \omega t)\, \hat{e}_y$

Nel piano $z = 0$



$$\begin{cases} x = E_{01} \cos(\omega t) \\ y = -E_{02} \sin(\omega t) \end{cases}$$

$$\frac{x^2}{E_{01}^2} + \frac{y^2}{E_{02}^2} = \cos^2(\omega t) + \sin^2(\omega t) = 1$$

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ polarizz. ellittica

$E_{01} = E_{02} \rightsquigarrow$ pol. circolare

Vettore di Poynting - q.d.m.

$$\bar{S} = \frac{\bar{E} \times \bar{B}}{\mu} = \frac{\bar{E} \times \hat{n} \times \bar{E}}{v\mu} = \frac{1}{v\mu} (\bar{E} \cdot \bar{E}) \hat{n} = \frac{1}{v\mu} E^2 \hat{n} =$$

$$\bar{B} = \frac{\bar{v} \times \bar{E}}{v^2}$$

$$\bar{E} \cdot \bar{E} = E^2$$

$$\bar{E} = E_x \hat{e}_x + E_y \hat{e}_y$$

$$= \sqrt{\frac{\varepsilon}{\mu}} E^2 \hat{n} = \frac{1}{Z} E^2 \hat{n}$$

$$v = \frac{1}{\sqrt{\varepsilon \mu}}$$

$Z \doteq \sqrt{\frac{\mu}{\varepsilon}}$ impedenza caratteristica del mezzo

$$Z_0 \doteq \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$$

Intensità (I. media) $= I = \langle S \rangle = \frac{1}{T} \int_T S(t)\,dt$

Pressione di radiazione $\bar{p} = \bar{S}/v$ forza/superficie =

$= \frac{q.d.m}{\text{superficie} \cdot \text{tempo}}$

$$(\overline{q.d.m.})_{wave} = \int \bar{p}\,dt = \frac{1}{v} \int \bar{S}\,dt$$

$$\langle \overline{q.d.m.} \rangle = \frac{1}{v} \langle \bar{S} \rangle T = \frac{1}{v} \bar{I} T$$