ESERC. 08/VI/2020

ONDE ELETTROMAGNETICHE
POTENZIALI ELETTRODINAMICI

$$\boxed{\bar{I} = \langle \bar{S} \rangle = \frac{1}{T}\int_T \bar{S}\,dt}$$

$\nu = 100\ \text{MHz}$ vuoto ($\bar{v} = \bar{c} = c\hat{e}_x$)

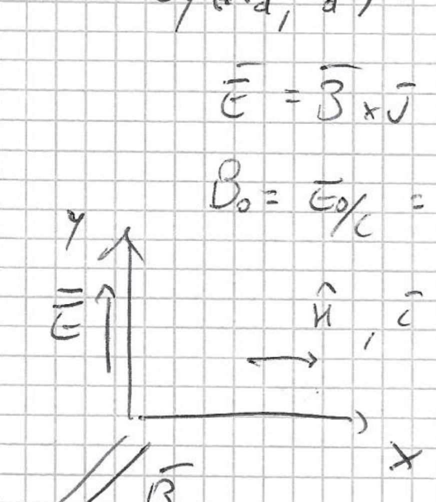
pol. lineare

$\bar{E} \parallel \hat{e}_y$ $\quad E_0 = 10\ V/m$

$E_y(x_1, t_1) = 10\ V/m$ $\qquad t_1 = 7.5\ s$; $\quad x_1 = 5\ km$

$$\bar{E} = \bar{B} \times \bar{v} = \bar{B} \times c\hat{e}_x$$

$$B_0 = E_0/c = 3.34 \cdot 10^{-8}\ T \qquad \bar{B} \parallel \hat{e}_z$$



$$\bar{E} = E_0 \cos(kx - \omega t + \varphi)\hat{e}_y$$

$$\bar{B} = \frac{E_0}{c} \cos(kx - \omega t + \varphi)\hat{e}_z$$

$$\cos(kx_1 - \omega t_1 + \varphi) = 1$$

$$kx_1 - \omega t_1 + \varphi = 2n\pi \quad \forall n \in \mathbb{Z}$$

$$\varphi = -kx_1 + \omega t_1 + 2n\pi = \omega(t_1 - x_1/c) + 2\pi n =$$

$\hookrightarrow k = \frac{\omega}{c}$

$$= 2\pi\Big[\underbrace{\nu(t_1 - x_1/c)}_{\tilde{\phi}} + n\Big]$$

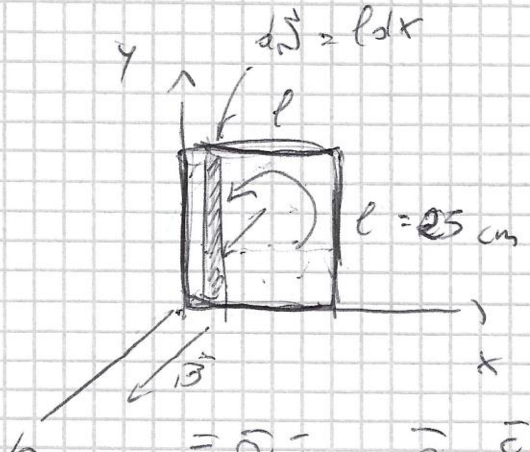
$$\tilde{\phi} = \text{floor}(\tilde{\phi}) = 0.33$$

onda pol. lineare $\quad \vec{E} \parallel \hat{e}_y \quad ; \quad \nu = 300\ MHz$

$\vec{E}_{0y} = 100\ mV/m$

vacuum $\Rightarrow \vec{c} = c\,\hat{e}_x$

$\lambda = ? \qquad R = 30\ \Omega$



$\vec{E}(x,t) = E_{0y} \cos(kx - \omega t)\,\hat{e}_y$

$\vec{E} = \vec{B} \times \vec{c} \qquad \vec{B} = \dfrac{\vec{c} \times \vec{E}}{c^2} \Rightarrow B_{0z} = E_{0y}/c \qquad \vec{B}(x,t) = \underbrace{\dfrac{E_{0y}}{c}}_{B_{0z}} \cos(kx - \omega t)\,\hat{e}_z$

$k = \dfrac{\omega}{c} \qquad \lambda = \dfrac{2\pi}{k} = 100\ cm = 1\,m$

$l = \lambda/4$

$f_{ind} = -\dfrac{d}{dt}\phi(\vec{B}(t))$

$\phi(t) = \displaystyle\int_{S_{spira}} \vec{B} \cdot d\vec{S} = \int_0^l B_z(t)\, l\,dx = \dfrac{E_{0y}\, l}{c} \int_0^l \cos(kx - \omega t)\,dx =$

$= \dfrac{E_{0y}\, l}{\underbrace{ck}_{=\omega}} \Big[\sin(kx - \omega t)\Big]_0^l = \dfrac{E_{0y}\, l}{\omega}\Big[\sin(\overbrace{kl - \omega t}^{\pi/2 - \omega t}) - \sin(-\omega t)\Big] \Rightarrow$

$kl = k\dfrac{\lambda}{4} = \dfrac{k}{4}\dfrac{2\pi}{k} = \dfrac{\pi}{2}$

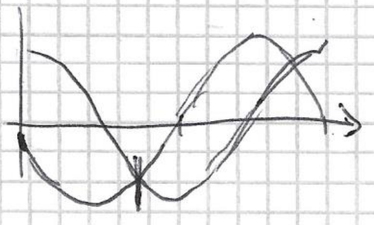
$$\boxed{\phi(t) = \dfrac{E_{0y}\, l}{\omega}\big[\cos(\omega t) + \sin(\omega t)\big]}$$

$\cancel{i_{ind}(t) = f_{ind}(t)/R}$

$I_0 = 0.83\ mA$

$f_{ind} = -\dfrac{d}{dt}\big[\phi(t)\big] = E_{0y}\, l\,\big[\sin(\omega t) - \cos(\omega t)\big]$

$i_{ind}(t) = f_{ind}/R = \dfrac{E_{0y}\, l}{R}\big[\ldots\big] = I_0\,\big[\sin(\omega t) - \cos(\omega t)\big]$



$\dfrac{di}{dt} = 0 \qquad \cos(\omega t) = -\sin(\omega t)$

$\omega t = \dfrac{3\pi}{4} + n\pi \qquad I_{max} = I_0\Big[\sin\Big(\dfrac{3\pi}{4}\Big) + \cos\Big(\dfrac{3\pi}{4}\Big)\Big]$

$I_{max} = I_0\sqrt{2} = 1.17\ mA$

Onda elm $\quad \bar{v} = c\hat{e}_x$

$\bar{B}$ noto $\quad B_0 = 10^{-8}\,T$

$$\bar{B} = B_0\left[2\cos(kx-\omega t)\hat{e}_y + 3\sin(kx-\omega t)\hat{e}_z\right]$$

polarizzazione ?

$\bar{E} = ?$

$A = 9\,m^2 \quad \hat{n}_A$ 60° rispetto $\hat{e}_x$

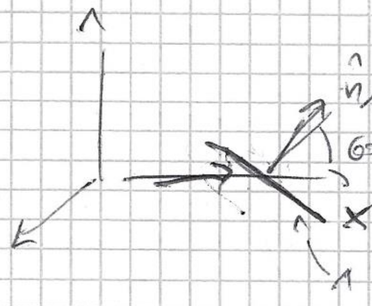
$\langle P\rangle(A)$, q.d.m. ceduta ad A → assorbente / riflettente

pol. <u>ellittica</u>

$$\bar{E} = \bar{B}\times\bar{c} = E_0\left[3\sin(kx-\omega t)\hat{e}_y - 2\cos(kx-\omega t)\hat{e}_z\right]$$

$E_0 = B_0 c = 3\,V/m$

$\langle\bar{S}\rangle \qquad \langle W\rangle = \langle\bar{S}\rangle\cdot\bar{A} = \frac{1}{2}\langle S\rangle A =$



$$= \frac{1}{2}A\left\langle\left|\frac{\bar{E}\times\bar{B}}{\mu_0}\right|\right\rangle =$$

$$= \frac{1}{2\mu_0}A\frac{\langle E^2\rangle}{c} = \frac{A}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\langle E^2\rangle = \qquad \bar{E}\cdot\bar{E}$$

$$= \frac{A}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\frac{1}{T}\int_0^T\left[9\sin^2(kx-\omega t)+4\cos^2(kx-\omega t)\right]dt =$$

$$= \frac{A}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\left[9\cdot\frac{1}{2}+4\cdot\frac{1}{2}\right] = 0.70\,W$$

$p = \langle S\rangle/c =$ q.d.m. $= p\,\Delta t$

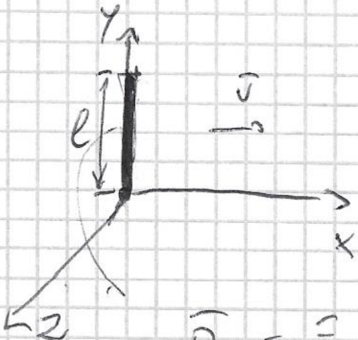
$\quad = 2.32\cdot 10^{-2}\,N/m^2$ per sup. assorbente

conservazione q.d.m. $\quad F_{ass} = 2pA = 41.76\,N$

wave $\rightarrow$ q.d.m. $\quad \phi \qquad$ wave $\leftarrow$ q.d.m. $\qquad 2\times$ q.d.m. $\rightarrow$

onda elm $\quad I = 1.19 \cdot 10^{-2}\ W/m^2$

pol. circolare ; mezzo $\varepsilon_r = K = 4$, $\mu_r = 1$



$\bar{v} = v\hat{e}_x$

$l = 20$ cm

$$\bar{E}(x,t) = E_0\left[\cos(kx-\omega t)\hat{e}_y + \sin(kx-\omega t)\hat{e}_z\right]$$

$$\bar{B} = \frac{\bar{v}\times\bar{E}}{v^2} \qquad \bar{B}(x,t) = \frac{E_0}{v}\left[-\sin(kx-\omega t)\hat{e}_y + \cos(kx-\omega t)\hat{e}_z\right]$$

$$I = \langle S \rangle \qquad \bar{S} = \frac{\bar{E}\times\bar{B}}{\mu} = \frac{\bar{E}\times\bar{v}\times\bar{E}}{\mu v^2} = \frac{\bar{E}\cdot\bar{E}}{\mu v}\hat{e}_x = \sqrt{\frac{\varepsilon}{\mu}}\,E^2\,\hat{e}_x$$

$$I = \langle S \rangle = \sqrt{\frac{\varepsilon}{\mu}}\,\frac{1}{T}\int_0^T(\bar{E}\cdot\bar{E})\,dt = \sqrt{\frac{\varepsilon}{\mu}}\,\frac{E_0^2}{T}\int_0^T\underbrace{\left[\cos^2(kx-\omega t)+\sin^2(kx-\omega t)\right]}_{1}dt$$

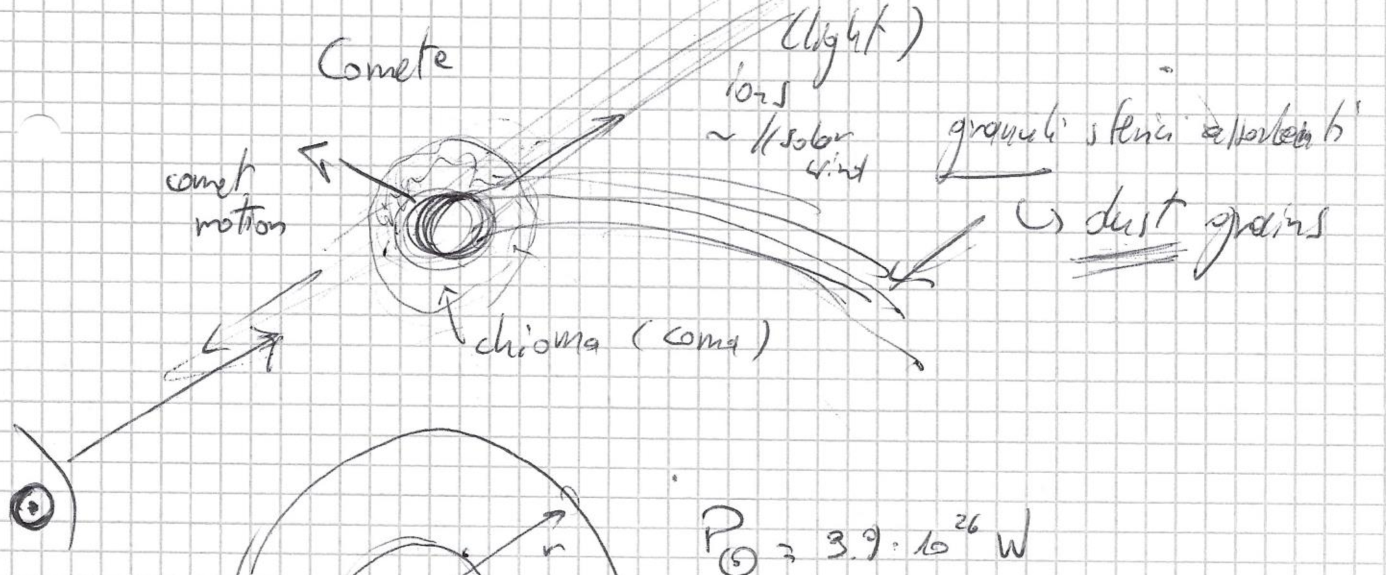
$$I = \langle S \rangle = \sqrt{\frac{\varepsilon_m}{\mu}}\,E_0^2 \qquad E_0 = \left(I\sqrt{\mu/\varepsilon}\right)^{\frac{1}{2}} = \underline{1.5\ V/m}$$

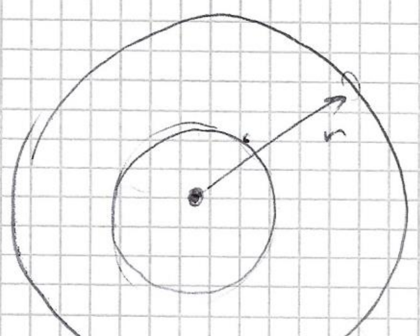
$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{2}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = \underline{1.5\cdot 10^8\ m/s} \qquad \underline{B_0 = E_0/v = 10^{-8}\ T}$$

$$f_0 = \int_0^l \bar{E}\cdot d\bar{l} = \int_0^l E_y(0,t)\,dy = E_0\int_0^l \cos(k\cancel{x}-\omega t)\,dy = \underline{E_0\,l\cos(\omega t)} =$$

$$= \underline{0.3\cos(\omega t)\ V}$$

Comete





$$P_{\odot} = 3.9 \cdot 10^{26}\ W$$

$$\bar{p} = \frac{\bar{S}}{c}$$

$$S(r) = \frac{P}{4\pi r^2} \rightsquigarrow p = \frac{P}{4\pi r^2 c}$$

$$F_{rad} = pA = p\pi a_0^2 \qquad a_0 = \text{raggio granulo}$$

$$F_{rad} = F_g$$

$$\frac{P}{4\pi r^2 c}\pi a_0^2 = G\frac{M_{\odot} m}{r^2} \qquad m = \frac{4\pi}{3} a_0^3 \rho$$

$$\text{bilancio} \quad a_0 = \frac{3P}{16\pi c \rho G M}$$

$$\left.\begin{array}{l} G = 6.67 \cdot 10^{-11}\ Nm^2/Kg^2 \\ M = 1.99 \cdot 10^{30}\ Kg \\ \rho = 3\ g/cm^3 \end{array}\right\} \quad \begin{array}{l} a_0 = 0.19\ \mu m \\ a_0 < 0.19\ \mu m \quad \text{prevale } F_{rad} \end{array}$$

Campo elm

$$\boxed{V'(\bar r,t)=\phi \quad ; \quad \bar A'(\bar r,t) = -\frac{q\,t}{4\pi\varepsilon_0 r^2}\,\hat e_r}$$

$\bar E, \bar B = ?$

sorgenti = ?

$V, \bar A$ "più familiari"? $\quad \lambda(\bar r,t) \,/\, (V,\bar A) \to (V',\bar A')$

$$\bar E(\bar r,t) = -\bar\nabla V' - \frac{\partial \bar A'}{\partial t} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat e_r$$

$$\bar B(\bar r,t) = \bar\nabla\times\bar A' = \frac{q\,t}{4\pi\varepsilon_0}\,\bar\nabla\times(\hat e_r/r^2) = \phi$$

q puntiforme in $\bar r$ = origine del sdr, ferma

$$\boxed{V(\bar r,t) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} \quad ; \quad \bar A(\bar r,t) = \phi}$$

$$V' = V - \frac{\partial\lambda}{\partial t} \quad ; \quad \bar A' = \bar A + \bar\nabla\lambda$$

① $V'(\bar r,t) = V(\bar r,t) - \partial\lambda/\partial t$

$$\phi = \frac{q}{4\pi\varepsilon_0 r} - \frac{\partial\lambda(\bar r,t)}{\partial t} \rightsquigarrow \lambda(\bar r,t) = \frac{q\,t}{4\pi\varepsilon_0 r} + f(\bar r)$$

② $\bar A'(\bar r,t) = \bar A(\bar r,t) + \bar\nabla\lambda$

$$-\frac{q\,t}{4\pi\varepsilon_0 r^2}\,\hat e_r = \phi + \bar\nabla\left(\frac{q\,t}{4\pi\varepsilon_0 r}\right) + \bar\nabla f(\bar r)$$

$$= -\frac{q\,t}{4\pi\varepsilon_0}\frac{\hat e_r}{r^2} + \bar\nabla f(\bar r)$$

$\bar\nabla f(\bar r) = \phi \qquad f$ costante in $\bar r, t \Rightarrow f = \phi$

$$\lambda(\bar r,t) = \frac{q\,t}{4\pi\varepsilon_0 r}$$