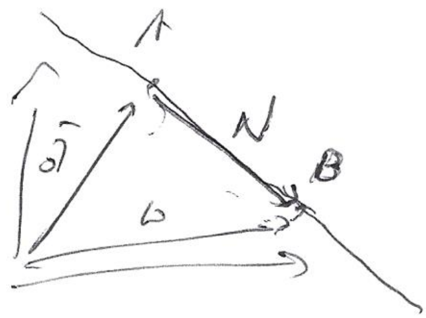
ES. #1 Calcolo vettoriale 02/X/2020

① eq. parametrica retta per A, B

vettori pos $\bar{a}, \bar{b}$

sdr cartesiano O origine



$\bar{N}$ congiungente $\bar{a}, \bar{b}$

$\bar{N} = \bar{b} - \bar{a}$

forma parametrica (t)

$$\boxed{\bar{r}(t) = \bar{P}_0 + t\bar{N}}$$

$\bar{P}_0$ $\forall$ pt $\in \bar{r}$

(anche $\bar{a}$)

$$\bar{r}(t) = \bar{a} + t(\bar{b} - \bar{a})$$

$$\begin{cases} x(t) = a_x + t(b_x - a_x) = a_x + tN_x \\ y(t) = a_y + t(b_y - a_y) = a_y + tN_y \\ z(t) = a_z + t(b_z - a_z) = a_z + tN_z \end{cases} \quad \forall t \in \mathbb{R}$$

eq. cartesiane

$$\begin{cases} t = \dfrac{x - a_x}{b_x - a_x} = \dfrac{x - a_x}{N_x} \\ t = \dfrac{y - a_y}{b_y - a_y} = \dfrac{y - a_y}{N_y} \\ t = \dfrac{z - a_z}{b_z - a_z} = \dfrac{z - a_z}{N_z} \end{cases} \rightsquigarrow \begin{cases} N_y x - N_x y - (N_y a_x - N_x a_y) = \phi \\ \quad \hookrightarrow \text{piano} \\ N_z y - N_y z - (N_z a_y - N_y a_z) = \phi \end{cases}$$

②

$\bar{A} = 3\hat{e}_x - 6\hat{e}_y + 2\hat{e}_z$

$(\hat{e}_x, \hat{e}_y, \hat{e}_z)$

→ Angoli tra $\bar{A}$ e gli assi (sdr)

→ Versore di $\bar{A}$

α, β, γ ;

$\hat{e}_x = (1; 0; 0)$

$\hat{e}_y = (0; 1; 0)$

$\hat{e}_z = (0; 0; 1)$

$\bar{A} = (3; -6; 2)$

$A = |\bar{A}| = (3^2 + (-6)^2 + 2^2)^{\frac{1}{2}} = 7$

prodotto scalare

$\underbrace{\bar{A} \cdot \hat{e}_x} = A \cos\alpha$

$3 = 7\cos\alpha \rightsquigarrow \cos\alpha = \frac{3}{7} \rightsquigarrow \alpha \simeq 64.6°$

$\bar{A} \cdot \hat{e}_y = A\cos\beta$

$-6 = 7\cos\beta \rightsquigarrow \beta = \arccos(-6/7) \simeq 149°$

$\bar{A} \cdot \hat{e}_z = A\cos\gamma$

$2 = 7\cos\gamma \rightsquigarrow \gamma = \arccos(2/7) \simeq 73.4°$

coseni direttori

$\hat{a} \parallel \bar{A}, \quad |\hat{a}| = 1$

$\hat{a} = \bar{A}/A = \frac{1}{7}\bar{A} = \left(\frac{3}{7}; -\frac{6}{7}; \frac{2}{7}\right)$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(03) $\bar{A} = 2\hat{e}_x + 2\hat{e}_y - \hat{e}_z$

$\bar{B} = 6\hat{e}_x - 3\hat{e}_y + 2\hat{e}_z$

→ Calcolare ϑ angolo $(\hat{A}, \bar{B})$ (→ svolgere)

$\bar{A} \cdot \bar{B} = AB \cos\vartheta$

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(04) $\bar{A} = 2\hat{e}_x - 6\hat{e}_y - 3\hat{e}_z$

$\bar{B} = 4\hat{e}_x + 3\hat{e}_y - \hat{e}_z$

2 vettori non paralleli ⇝ definiscono un piano

→ det. versore ⊥ piano

Cerchiamo $\bar{C} \perp \bar{A}, \perp \bar{B}$; poi normalizziamo

$$\begin{cases} \bar{A} \cdot \bar{C} = \phi \\ \bar{B} \cdot \bar{C} = \phi \end{cases} \Rightarrow \begin{cases} 2c_x - 6c_y - 3c_z = \phi \\ 4c_x + 3c_y - c_z = \phi \end{cases}$$

$\bar{C} = c_x \hat{e}_x + c_y \hat{e}_y + c_z \hat{e}_z$

(1) $c_z = 4c_x + 3c_y$ ⇝ $2c_x - 6c_y - 12c_x - 9c_y = \phi$

$c_y = -\frac{2}{3} c_x$

$c_z = 4c_x + 3\left(-\frac{2}{3} c_x\right) = 2c_x$

$c_z = 2c_x$

$\bar{C} = \cancel{c_x}\left(1 ; -\frac{2}{3} ; 2\right)$ $c_x \in \mathbb{R}$

$\bar{C}\left(1 ; -\frac{2}{3} ; 2\right)$ $\boxed{|C| = \left(1 + \frac{4}{9} + 4\right)^{1/2}} = \pm \frac{7}{3}$

$\hat{c} = \pm\left(\frac{3}{7} ; -\frac{2}{7} ; \frac{6}{7}\right)$

0.4 - bis prodotto vettoriale

$$\overline{C} = \overline{A} \times \overline{B} \quad \perp \overline{A}, \overline{B}$$

$$\overline{C} = \overline{A} \times \overline{B} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} =$$

$$= (6+9)\,\hat{e}_x + (-12+2)\,\hat{e}_y + (6+24)\,\hat{e}_z =$$

$$= 15\,\hat{e}_x - 10\,\hat{e}_y + 30\,\hat{e}_z$$

$$C = |\overline{C}| = 35$$

$$\hat{c} = \frac{\overline{C}}{C} = \left(\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}\right)$$

$$\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \ldots$$

$$\bar{C} = \bar{A} \times \bar{B}$$

$$C = AB\sin\theta \quad \text{con } \theta \text{ angolo tra } \bar{A} \text{ e } \bar{B}$$

$$\bar{C} \perp \bar{A}, \bar{B}$$

0.5 Piano $\perp \bar{A} = 2\hat{e}_x + 3\hat{e}_y + 6\hat{e}_z$

passante per l'estremità di $\bar{B} = \hat{e}_x + 5\hat{e}_y + 3\hat{e}_z$

$= Q(1; 5; 3)$

$P(x, y, z)$ estremità di $\bar{r}$

$\overline{PQ} = \bar{B} - \bar{r} \in$ piano

$$\bar{A} \cdot (\bar{B} - \bar{r}) = 0$$

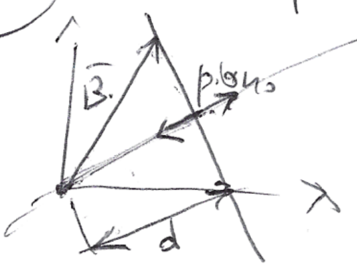
$$\bar{B} - \bar{r} = (1-x;\ 5-y;\ 3-z)$$

$$\bar{A} \cdot \bar{r} = \bar{A} \cdot \bar{B}$$

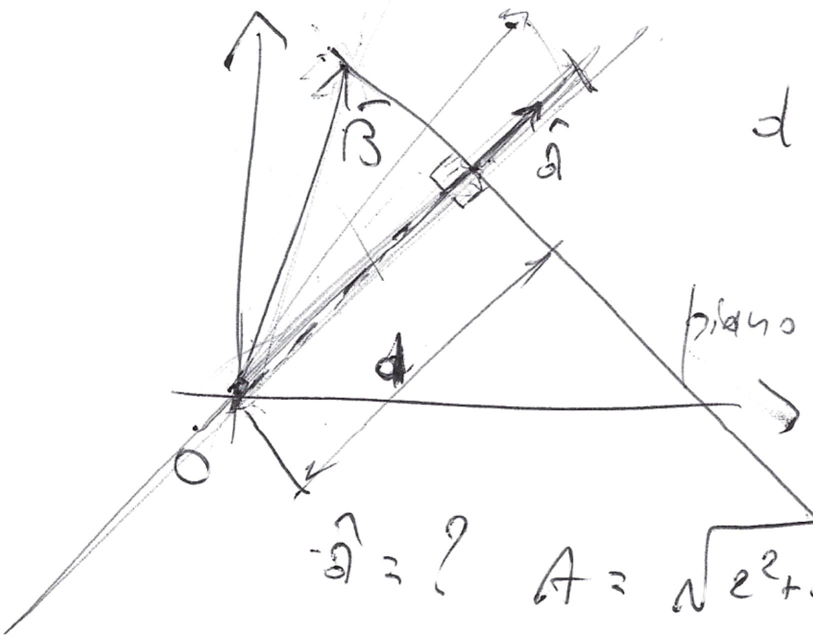
$$2x + 3y + 6z = 2 \cdot 1 + 3 \cdot 5 + 6 \cdot 3 = 35$$

$\boxed{2x + 3y + 6z - 35 = 0}$ eq. del piano

0.6 Dato il piano, $d(\text{piano}, O) = ?$



$$d = \bar{B} \cdot \hat{a} = \bar{B} \cdot \frac{\bar{A}}{A}$$



$d = \bar{B} \cdot \hat{a}$

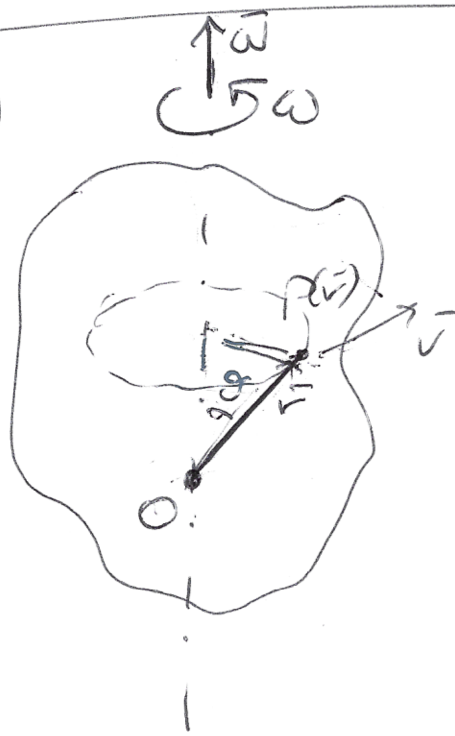
$\hat{a} = \frac{\bar{A}}{A}$

$\hat{a} = ?$ $A = \sqrt{2^2+3^2+6^2} = 7$

$\hat{a} = \left(2/7 \, ; \, 3/7 \, ; \, 6/7 \right)$

$d = \bar{B} \cdot \hat{a} = 1 \cdot \frac{2}{7} + 5 \cdot \frac{3}{7} + 3 \cdot \frac{6}{7} = \ldots = 5$

Q.8



$\bar{\omega}$

$\bar{v} = \bar{\omega} \times \bar{r}$

$R =$ raggio di rot. $= r \sin\vartheta$

$v = \omega R = \omega r \sin\vartheta = |\bar{\omega} \times \bar{r}|$

$\bar{v} \perp \bar{r}$

$\bar{v} \perp \bar{\omega}$

$\bar{\omega}, \bar{r}, \bar{v}$

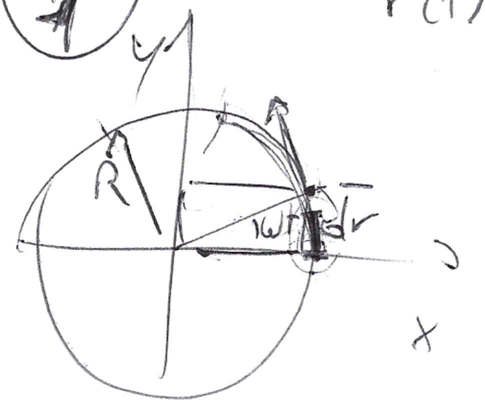
$\bar{v}, \bar{\omega}, \bar{r}$

$\bar{r}, \bar{v}, \bar{\omega}$

$\bar{v} = \bar{\omega} \times \bar{r}$

09

C



$$\bar r(t) = R\cos(\omega t)\hat e_x + R\sin(\omega t)\hat e_y \quad \forall t \in \mathbb{R}$$

a) eq. vettore tg $\bar T$

b) calc. arco $t \in [\phi, t^*]$

$\bar r(\phi)$, $\bar r(t^*)$

c) riscrivere $\bar r$ parametrizzata in funz. di s = lunghezza arco

a) $$\bar T = \frac{d\bar r}{dt} = -R\omega\sin(\omega t)\hat e_x + R\omega\cos(\omega t)\hat e_y$$

$$\hat t = \frac{\bar T}{T} \qquad T = |\bar T| = \left[R^2\omega^2\sin^2(\omega t) + R^2\omega^2\cos^2(\omega t)\right]^{\frac{1}{2}} = R\omega$$

$$\hat t = -\sin(\omega t)\hat e_x + \cos(\omega t)\hat e_y$$

b) $$d\bar r = dx\,\hat e_x + dy\,\hat e_y = -R\omega\sin(\omega t)dt\,\hat e_x + R\omega\cos(\omega t)dt\,\hat e_y$$

$$(ds)^2 = (d\bar r)^2 = d\bar r \cdot d\bar r = R^2\omega^2\sin^2(\omega t)dt^2 + R^2\omega^2\cos^2(\omega t)dt^2 = R^2\omega^2 dt^2$$

$$ds = R\omega\,dt$$

$$\Delta s = s(t^*) - s(\phi) = \int_\phi^{t^*} ds = \int_\phi^{t^*} R\omega\,dt = R\omega t$$

c) $\Delta s = R\omega t = s(t) - \cancel{s(\phi)}$ ϕ

$s = R\omega t \qquad t \to \frac{s}{R\omega}$

$$\bar r(s) = R\cos\left(\frac{s}{R}\right)\hat e_x + R\sin\left(\frac{s}{R}\right)\hat e_y$$

$$\bar T = \frac{d\bar r}{ds} = -\sin\left(\frac{s}{R}\right)\hat e_x + \cos\left(\frac{s}{R}\right)\hat e_y$$