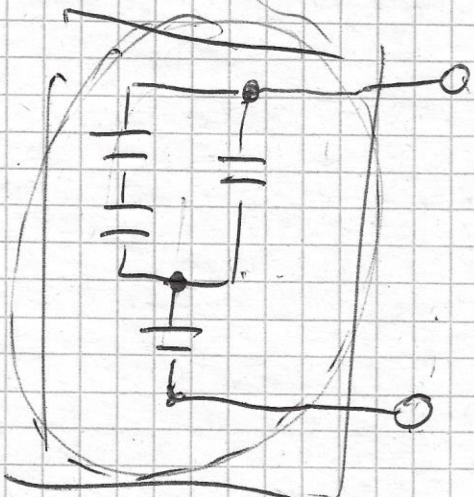
ES. # 10 25/XI/2020

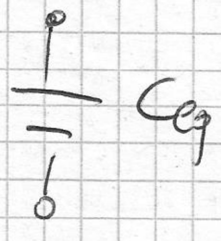
Condensatori; forze su conduttori

1

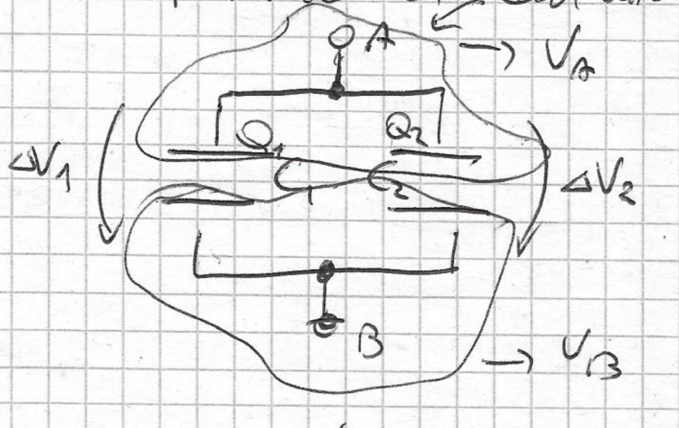
Rete — insieme di C collegati

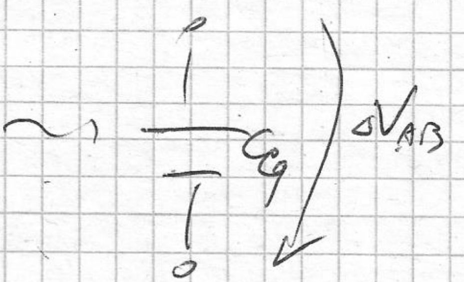


bipolo



Parallelo di condensatori





$$\Delta V_{AB} = \Delta V_1 = \Delta V_2$$

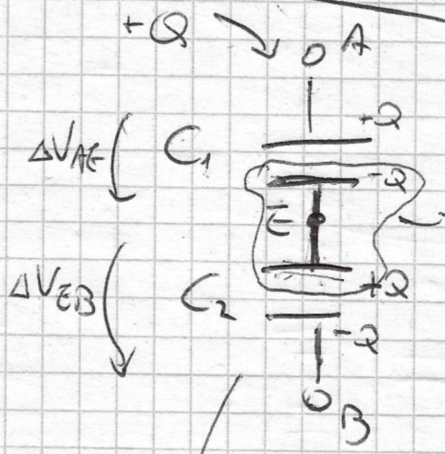
$$Q_1 = C_1 \Delta V_1 = C_1 \Delta V_{AB}$$

$$Q_2 = C_2 \Delta V_2 = C_2 \Delta V_{AB}$$

$$Q_{tot} = Q_1 + Q_2 = (C_1 + C_2)\Delta V_{AB}$$

$$C_{eq} = \frac{Q_{tot}}{\Delta V_{AB}} = C_1 + C_2 \qquad C_{eq} = \sum_{i=1}^{N} C_i$$

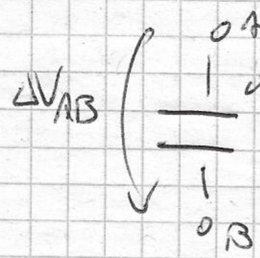
Serie di condensatori



$$\Delta V_{AE} = V_A - V_E = \frac{Q}{C_1}$$

$$\Delta V_{EB} = V_E - V_B = \frac{Q}{C_2}$$

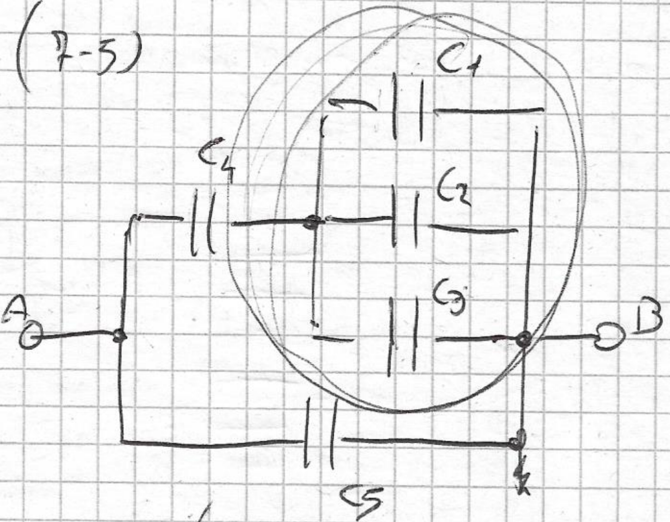
$$\Delta V_{AB} = \Delta V_{AE} + \Delta V_{EB} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\,\frac{C_1 + C_2}{C_1 C_2}$$



$$C_{eq} = \frac{Q}{\Delta V_{AB}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \rightsquigarrow \frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

(7-5)



$C_1 = 1\,pF$

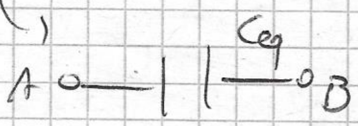
$C_2 = 2\,pF$

$C_3 = 3\,pF$

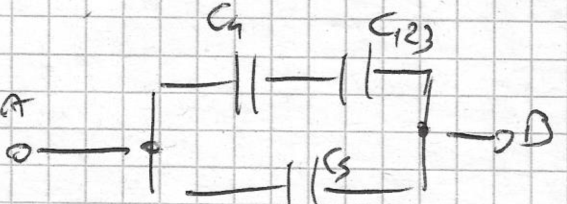
$C_4 = 4\,pF$

$C_5 = 5\,pF$

$\Delta V_{AB} = 100\,V$



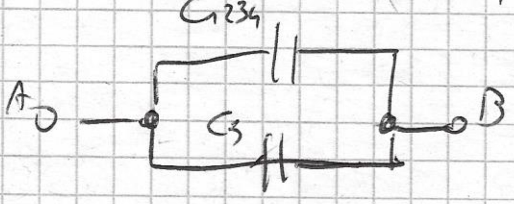
① $C_1 \parallel C_2 \parallel C_3 \rightarrow C_{123} = C_1 + C_2 + C_3 = 6\,pF$



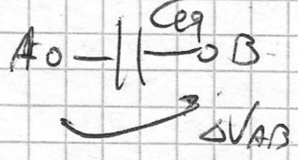
② C_4 serie C_{123} $\rightsquigarrow$ $C_{1234} = \dfrac{1}{\dfrac{1}{C_{123}} + \dfrac{1}{C_4}} = \dfrac{C_{123}\,C_4}{C_{123} + C_4}$

$\left(\dfrac{1}{C_{1234}} = \dfrac{1}{C_{123}} + \dfrac{1}{C_4}\right)$

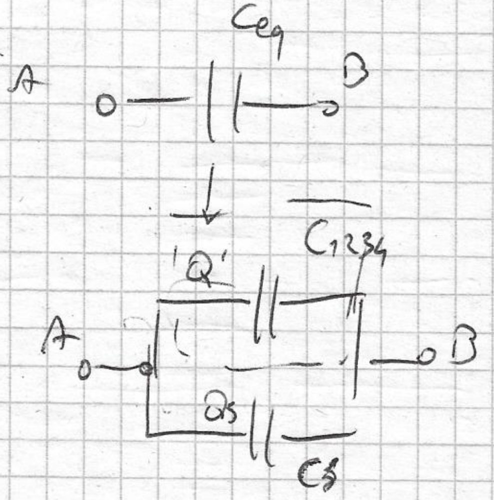
$C_{1234} = 2.4\,pF$



③ $C_{1234} \parallel C_5$ $\quad C_{eq} = C_{1234} + C_5 = 7.4\,pF$

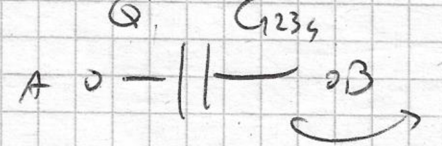


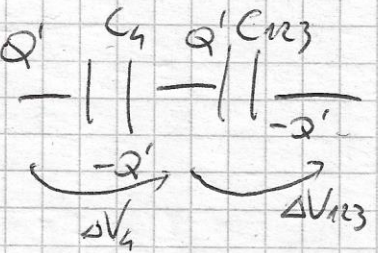
$Q_{tot} = C_{eq}\,\Delta V_{AB} = 0.74\,nC$



$$Q_5 = C_5 \Delta V_5 = C_5 \Delta V_{AB} = 0.5\ nC$$

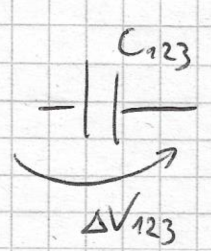
$$Q' = C_{1234} \Delta V_{AB} = Q_{Tot} - Q_5 = 0.24\ nC$$

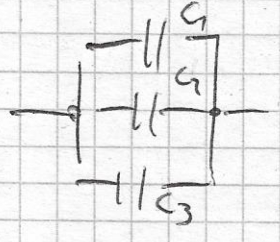




$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{Q'}{C_4} = 60\ V$$

$$\Delta V_{123} = \Delta V_{AB} - \Delta V_4 = 40\ V$$





$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_{123}$$

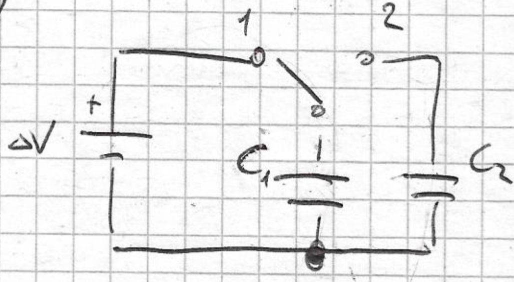
$$Q_1 = C_1 \Delta V_1 = 40\ pC$$

$$Q_2 = C_2 \Delta V_2 = 80\ pC$$

$$Q_3 = C_3 \Delta V_3 = 120\ pC$$

$$U_{Tot} = \frac{1}{2} C_{eq} (\Delta V_{AB})^2 = 37\ nJ$$

(7-6)



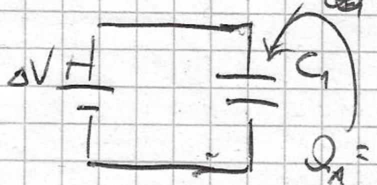
C_i scarichi

$C_1 = 0.1\,\mu F$

$C_2 = 0.2\,\mu F$

$\Delta V = 12\,V$

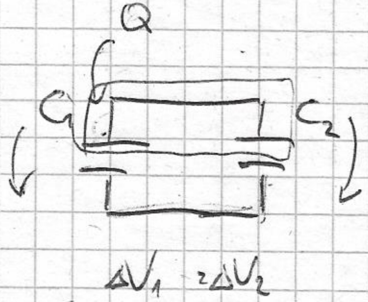
(1)



$Q_1 = C_1 \Delta V = 1.2\,\mu C \quad (Q_2 = 0)$

$W = W_1 = \frac{1}{2} C_1 (\Delta V)^2 = 7.2\,\mu J$

(2)



ridistribuzione di Q

$Q = Q_1 + Q_2 \qquad \Delta V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$

$Q = Q_1 + Q_2 = C_1 \Delta V' + C_2 \Delta V' = (C_1 + C_2)\Delta V' =$

$= C_{eq} \Delta V'$

$\Delta V' = Q/(C_1 + C_2) = 4\,V$

$Q_1 = C_1 \Delta V' = 0.4\,\mu C$

$Q_2 = C_2 \Delta V' = 0.8\,\mu C$

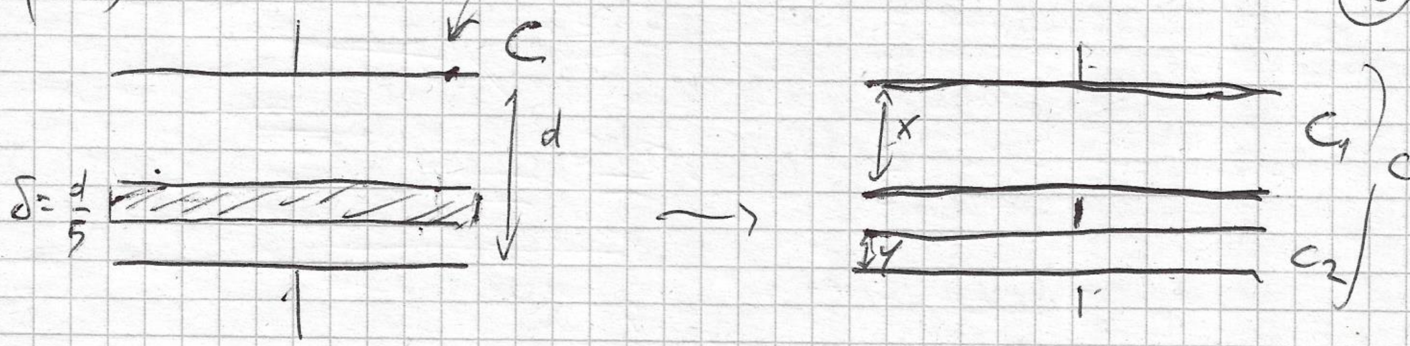
W: $C_{eq} = C_1 + C_2 = 0.3\,\mu F$

$W = \frac{1}{2} C_{eq} (\Delta V')^2 = \frac{1}{2} \frac{Q^2}{C_{eq}} = 2.4\,\mu J$

$W_1 + W_2 = \frac{1}{2} C_1 (\Delta V')^2 + \frac{1}{2} C_2 (\Delta V')^2$

(7-7)

Q carica costante



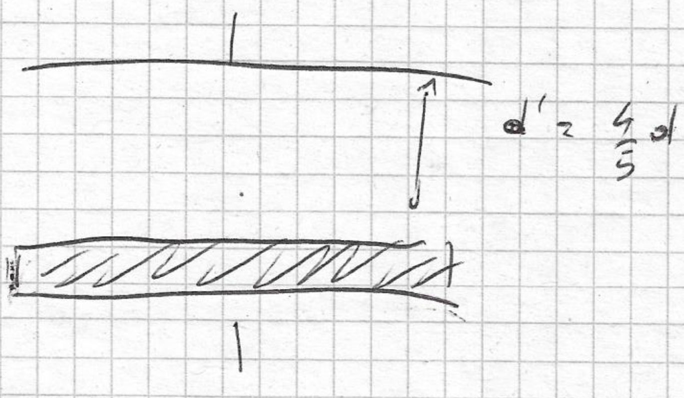
$$d = x + y + \delta$$

$$x + y = \frac{4}{5} d$$

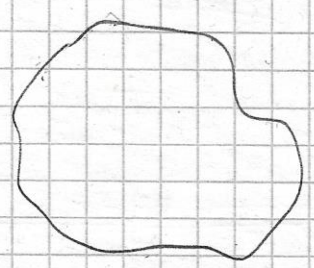
$$C' = 1\Big/\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 1\Big/\left(\frac{x}{\varepsilon_0 S} + \frac{y}{\varepsilon_0 S}\right) = \frac{\varepsilon_0 S}{x+y} = \frac{5}{4}\underbrace{\frac{\varepsilon_0 S}{d}}_{C}$$

$$C' = \frac{5}{4} C$$

$$\frac{W'}{W} = \frac{1}{2}\frac{Q^2}{C'}\Big/\frac{1}{2}\frac{Q^2}{C} = \frac{C}{C'} = \frac{x+y}{d} = \frac{4}{5}$$



Pressione elettrostatica = densità di en. elettr.

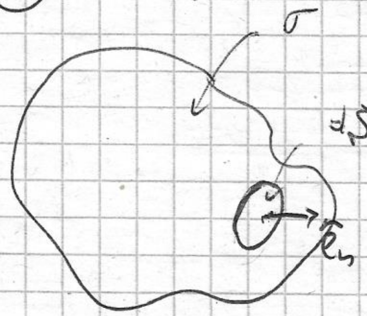


p = forza normale alla sup.

(A) metodo diretto

(B) " dei lavori virtuali

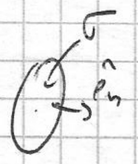
(A) Diretto



$$S = (dS) + (S - dS)$$

$$\bar{E}_0 = \bar{E}_0^{dS} + \bar{E}_0^{S-dS}$$

$$\bar{E}_0 \begin{cases} \sigma/\varepsilon_0 \, \hat{e}_n & \text{fuori in prossimità della sup.} \\ \emptyset & \text{dentro} \end{cases}$$



$$\bar{E}_0^{dS} \begin{cases} \sigma/2\varepsilon_0 \, \hat{e}_n & \text{fuori} \\ -\sigma/2\varepsilon_0 \, \hat{e}_n & \text{dentro} \end{cases}$$

$$\bar{E}_0^{S-dS} = \frac{\sigma}{2\varepsilon_0} \hat{e}_n$$

$$\bar{F} = q^{dS} \cdot \bar{E}_0^{S-dS} = \sigma \, dS \frac{\sigma}{2\varepsilon_0} \hat{e}_n = \frac{\sigma^2}{2\varepsilon_0} dS \, \hat{e}_n$$

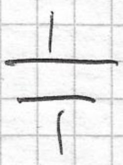
forza normale alla sup.

$$p = \frac{F}{dS} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\bar{E}_0 = \frac{\sigma}{\varepsilon_0} \hat{e}_n$$

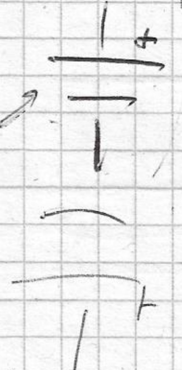
$$p(\bar{r}) = \frac{\varepsilon_0 E_0^2(\bar{r})}{2} = u(\bar{r}) \text{ densità}$$

pressione



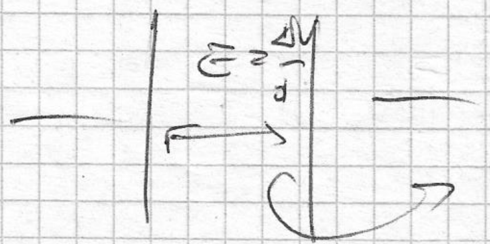
elettrolitic

$\geq 1 \mu F$



BNC
Bayonet Neil Concelman

SHV



E_{max} breakdown field

rigidità dielettrica

Ⓑ Lavori virtuali

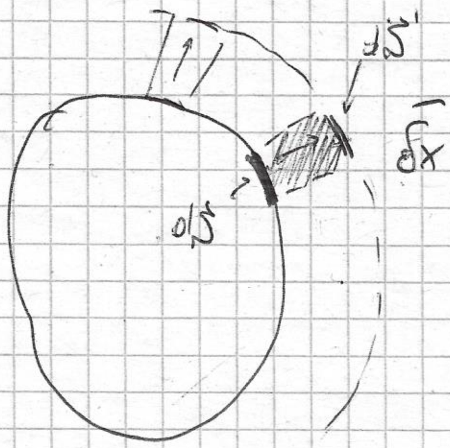
9

Spostamenti virtuali coerenti

Sist. equilibrio $\rightarrow$ trasformazione (rot. / trasl. / deform.)

! per stati di equ. (quasi statica)

$\nexists$ G_{n}



dilatazione

Forza esterna $\delta \bar{F}_{ext}$

$$\delta L_{ext} = \delta \bar{F}_{ext} \cdot \bar{\delta x} = \delta F_{ext,x} \delta x$$

$$\delta U = \delta L_{ext}$$

Forza elettrost. $\delta \bar{F} = -\delta \bar{F}_{ext}$

$$\delta F_x = -\delta F_{ext,x} = -\frac{\delta U}{\delta x} \qquad \delta U = U_f - U_i$$

$$U_i = u\, d\bar{S}\, \delta x \qquad u = \varepsilon_0 E_0^2/2$$

$$U_f = \phi \cdot d\bar{S}\, \delta x$$

$$\delta U = -u\, d\bar{S}\, \delta x$$

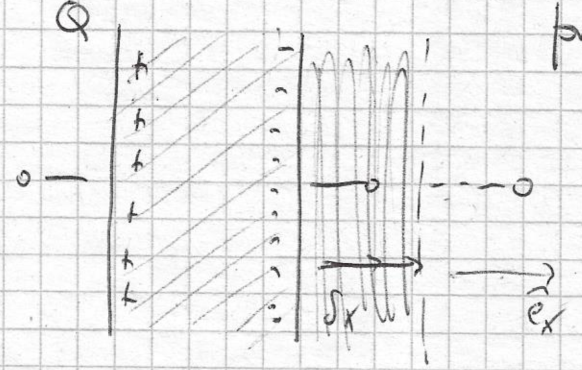
$$\boxed{\delta F_x = -\frac{\delta U}{\delta x}} = u\, d\bar{S} \qquad \underline{p} = \frac{\delta F_x}{d\bar{S}} = \underline{u}$$

$\bar{F}_{ext}$ (meccanica)

$$F_{ext,x} = -\frac{\partial U}{\partial x}$$

$$\boxed{\bar{F}_{ext} = -\bar{\nabla} U}$$

(8-2)



processo a carica costante

$$\overline{E}_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S}$$

$$u = \varepsilon_0 \overline{E}_0^2 / 2 = \frac{Q^2}{2\varepsilon_0 S^2} \qquad u_i = u_f$$

$$\delta U = u_f \tau_f - u_i \tau_i = u(\tau_f - \tau_i) = u S \delta x$$

$$F_x = -\frac{\delta U}{\delta x} = -uS = -\frac{Q^2}{2\varepsilon_0 S^2} S = -\frac{\varepsilon_0 E_0^2 S}{2} = -\frac{\sigma^2 S}{2\varepsilon_0}$$

attrattiva

$$C = \frac{\varepsilon_0 S}{d} = \frac{\varepsilon_0 S}{d(\text{variabile})}$$

d_0 = dist. iniz.

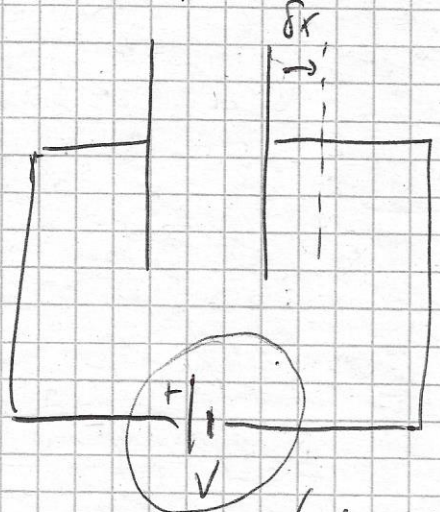
$d_0 + x$

$$C = \frac{\varepsilon_0 S}{(d_0 + x)}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{\varepsilon_0 S} (d_0 + x)$$

$$F_x = -\frac{dU}{dx} = -\frac{Q^2}{2\varepsilon_0 S} = -\frac{\sigma^2 S}{2\varepsilon_0}$$

(8-3) processo a tensione costante



C varia!

ma $V =$ cost.

$\Rightarrow Q$ varia anch'essa!

U_c en. immagazzinata nel condensatore

$$dU_c = d\left(\frac{1}{2}CV^2\right) = \frac{V^2}{2}dC = \frac{V^2}{2}d\left(\frac{\varepsilon_0 S}{x}\right) = -\frac{V^2}{2}\frac{\varepsilon_0 S}{x^2}dx$$

$S\,dx = d\tau$

$$\Rightarrow dU_c = -\frac{\varepsilon_0 V^2}{2x^2}d\tau = -\frac{\varepsilon_0 E_0^2}{2}d\tau = -\frac{\sigma^2}{2\varepsilon_0}d\tau$$

$E_0 = V/d$

$$F_x = -\frac{dU_c}{dx} = +\frac{\sigma^2}{2\varepsilon_0}S$$

$$\bar{F} = -\bar{\nabla}U_{tot}$$

$$dQ = V dC$$

$$dW_g = V dQ = V^2 dC = -dU_g$$

$$dU_{tot} = dU_c + dU_g = \frac{V^2}{2}dC - V^2 dC = -\frac{V^2}{2}dC = -dU_c$$

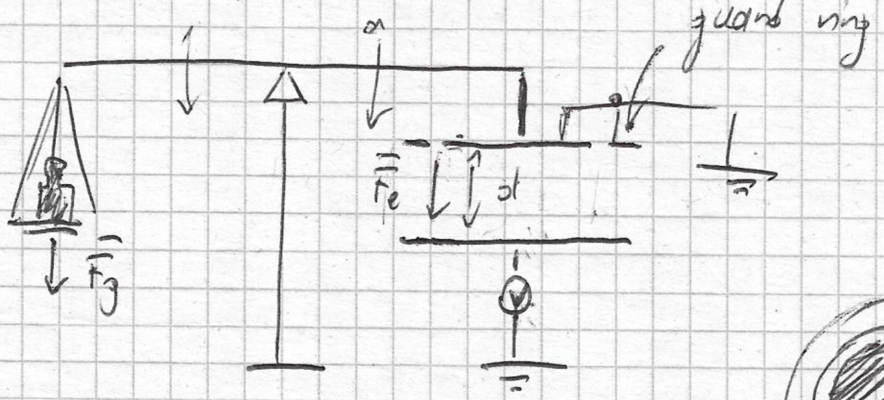
$$F_{x\,tot} = -\frac{dU_{tot}}{dx} = +\frac{dU_c}{dx} = -\frac{\sigma^2 S}{2\varepsilon_0} = -\frac{\varepsilon_0 E_0^2 S}{2}$$

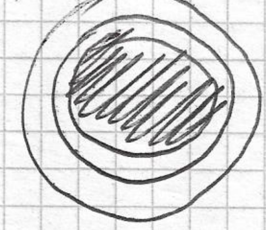
$\boxed{\bar{F} = -\bar{\nabla}U_{tot}}$ sempre

tensione costante $\rightarrow$ $\boxed{\bar{F} = +\bar{\nabla}U_c}$

(8-4) Elettrometro di Thomson (Lord Kelvin)

Misura assoluta





$$\bar{F}_g = \bar{F}_e \qquad E_0 = V/d$$

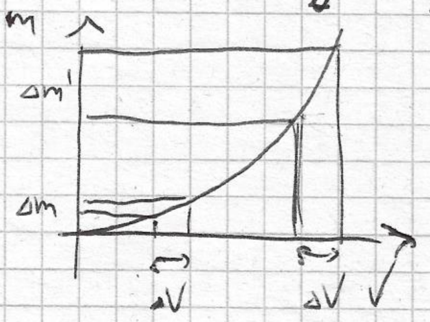
$$mg = \frac{\varepsilon_0 E_0^2 S}{2} = \frac{\varepsilon_0 V^2 S}{2d^2}$$

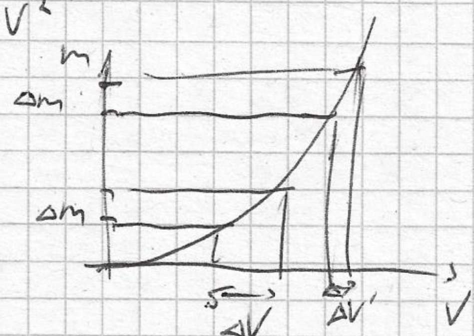
$$m = \frac{\varepsilon_0 S}{2d^2 g} V^2 \longrightarrow V = \left(\frac{2g\, d^2 m}{\varepsilon_0 S}\right)^{\frac{1}{2}}$$

$m \propto V^2$ scala quadratica

$$\frac{dV}{V^2} \times \frac{dm}{dV} = \frac{\varepsilon_0 S}{d^2 g} \qquad \frac{dm}{V^2} = \frac{\varepsilon_0 S}{d^2 g}\frac{V dV}{V^2}$$

$$\frac{\Delta V}{V} = \frac{g d^2}{\varepsilon_0 S}\frac{\Delta m}{V^2}$$





Es.: $S = 400\ cm^2$; $d = 5\ mm$ → $m_1 = 20\ mg$ → $m_2 = 800\ mg$

$V_1 = 166.4V$ / $V_2 = 1052.6V$

$\Delta m = 1\ mg$

$$\frac{\Delta V}{V} = \frac{g d^2}{\varepsilon_0 S}\frac{\Delta m}{V^2} \begin{cases} (1)\ 0.025 \\ (2)\ 6.25 \cdot 10^{-4} \end{cases}$$