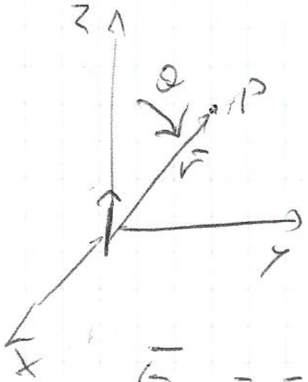
ES #10 Dipoli elettrostatici

02/XII/2020

$\bar p$ in O $\quad \bar p = (\phi, \phi, p)$

$$V_0(\bar r) = \frac{1}{4\pi\varepsilon_0}\frac{\bar p\cdot\bar r}{r^3}\left(=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\right) = -\frac{1}{4\pi\varepsilon_0}\,\bar p\cdot\bar\nabla\left(\frac{1}{r}\right)$$



$\bar p(\bar r')$

$$V_0(\bar r) = \frac{1}{4\pi\varepsilon_0}\frac{\bar p(\bar r')\cdot(\bar r-\bar r')}{|\bar r-\bar r'|^3}$$

$$E_{0r} = -\frac{\partial V_0}{\partial r} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_{0\theta} = -\frac{1}{r}\frac{\partial V_0}{\partial\theta} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$E_{0\varphi} = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}V_0 = \phi$$

$z = r\cos\theta$; $r = (x^2+y^2+z^2)^{\frac{1}{2}}$

$$V_0(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{pz}{(x^2+y^2+z^2)^{3/2}}$$

$$E_{0x} = \frac{1}{4\pi\varepsilon_0}\frac{3pxz}{(x^2+y^2+z^2)^{5/2}}$$

$$E_{0y} = \frac{1}{4\pi\varepsilon_0}\frac{3pyz}{(x^2+y^2+z^2)^{5/2}}$$

$$E_{0z} = \frac{1}{4\pi\varepsilon_0}\frac{p(2z^2-x^2-y^2)}{(x^2+y^2+z^2)^{5/2}}$$

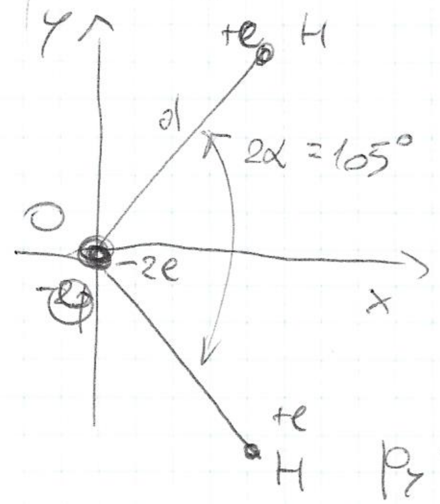
Forze $\quad U = -\bar E_0\cdot\bar p \quad \bar F = -\bar\nabla U \Rightarrow$

$$\bar F = \bar\nabla(\bar p\cdot\bar E_0) = \underline{(\bar p\cdot\bar\nabla)\bar E_0} \qquad \bar M = \bar p\times\bar E_0$$

(2)

$$\bar{p} = \int_{\tau_{distr}} \rho(\bar{r}')\bar{r}' d\tau'$$

H_2O



$d = 0.98\ \text{Å} = 0.98 \cdot 10^{-10}\ m$

$$\bar{p} = \sum_i q_i \bar{r}_i$$

$q = e = 1.6 \cdot 10^{-19}\ C$

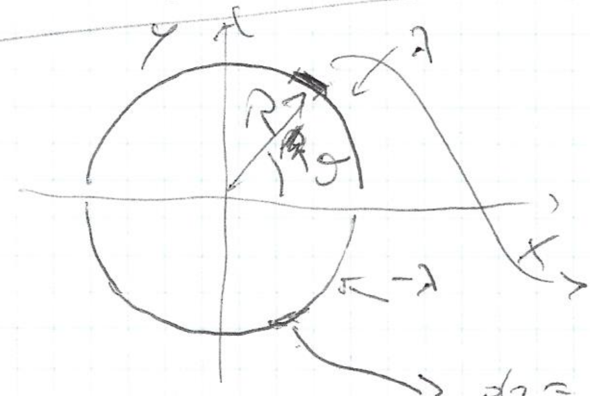
$$\sum q_i = \phi$$

$$p_y = -2q \cdot \phi + qd\sin\alpha + qd\sin(-\alpha) = \phi$$

$$p_x = -2q \cdot \phi + qd\cos\alpha + qd\cos(-\alpha) =$$

$$= 2qd\cos\alpha = 1.91 \cdot 10^{-29}\ C \cdot m$$

$$p_{H_2O}^{exp} = 6.2 \cdot 10^{-30}\ C \cdot m$$



$$\bar{p} = \int_\ell \lambda(\bar{r})\bar{r}\, d\ell$$

$$dq = \lambda d\ell = \lambda R d\theta$$

$$dq = -\lambda d\ell = -\lambda R d\theta$$

$$dp_y^{UP} = y\,dq = R\sin\theta \cdot \lambda R d\theta = \lambda R^2 \sin\theta\, d\theta$$

$$p_y^{UP} = \int_\phi^\pi \lambda R^2 \sin\theta\, d\theta = \lambda R^2 [-\cos\theta]_\phi^\pi = 2\lambda R$$

$$dp_y^{DOWN} = y\,dq = -\lambda R^2 \sin\theta\, d\theta$$

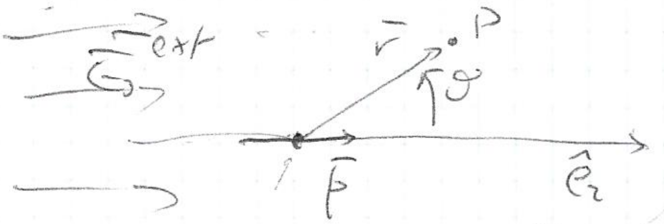
$$p_y^{DOWN} = \int_{\pi}^{2\pi} -\lambda R^2 \sin\theta \, d\theta = -\lambda R^2 \left[-\cos\theta\right]_{\pi}^{2\pi} = $$

$$= 2\lambda R^2$$

$$p_y = p_y^{UP} + p_y^{DOWN} = 4\lambda R^2 = \frac{4QR}{\pi}$$

$$\bar{p} = \frac{4QR}{\pi}\hat{e}_y \qquad Q \doteq \lambda \pi R$$

$\bar{p} \parallel \bar{E}_0^{ext}$: dim. $\exists \; R \;/\; V_A(R) = \text{cost.}$



$$z = r\cos\theta$$

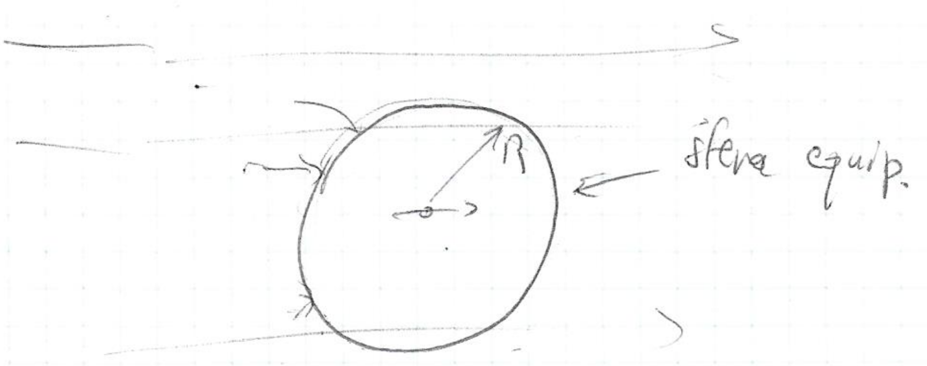
$$V(P) = \underbrace{U_0 - \bar{E}_0^{ext} z}_{\text{pot. campo ext}} + \frac{p\cos\theta}{4\pi\varepsilon r^2} = U_0 - \bar{E}_0^{ext} r\cos\theta + \frac{p\cos\theta}{4\pi\varepsilon r^2}$$

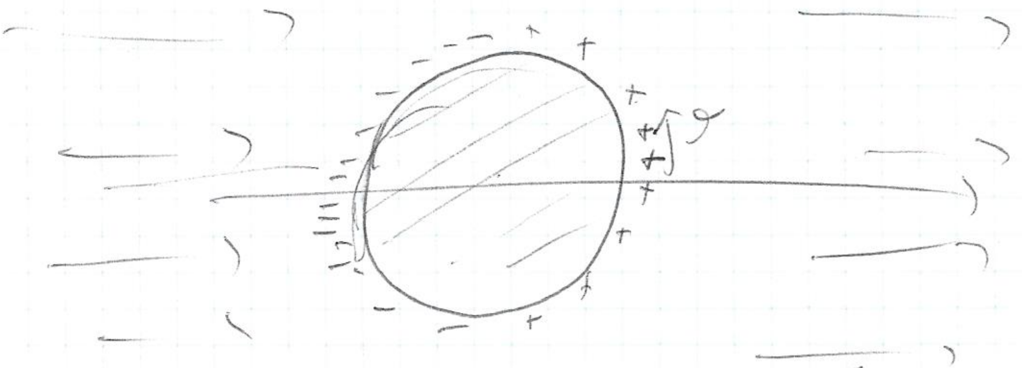
$$\cancel{V(r=R,\theta) = U_0 \left[\bar{E}_0^{ext} R\right.}$$

$$V(r=R,\theta) = U_0 - \underbrace{\left[\bar{E}_0^{ext} R - \frac{p}{4\pi\varepsilon R^2}\right]}_{=0}\cos\theta = \text{cost.}$$

$$R = \left(\frac{p}{4\pi\varepsilon \bar{E}_0^{ext}}\right)^{1/3} \qquad V(R,\theta) = U_0$$

$$V(P) - \underbrace{V(P^*)}_{U_0} = -\int_{P}^{P^*} \bar{E}_0^{ext} \cdot d\bar{l} = -\bar{E}_0^{ext} z$$





$$E_{0r}(R) = -\frac{\partial V}{\partial r}\Big|_{r=R} = -\frac{\partial}{\partial r}\left[V_0 - E_0^{ext}\underbrace{z}_{r\cos\vartheta} + \frac{p\cos\vartheta}{4\pi\varepsilon_0 r^2}\right]_{r=R} =$$

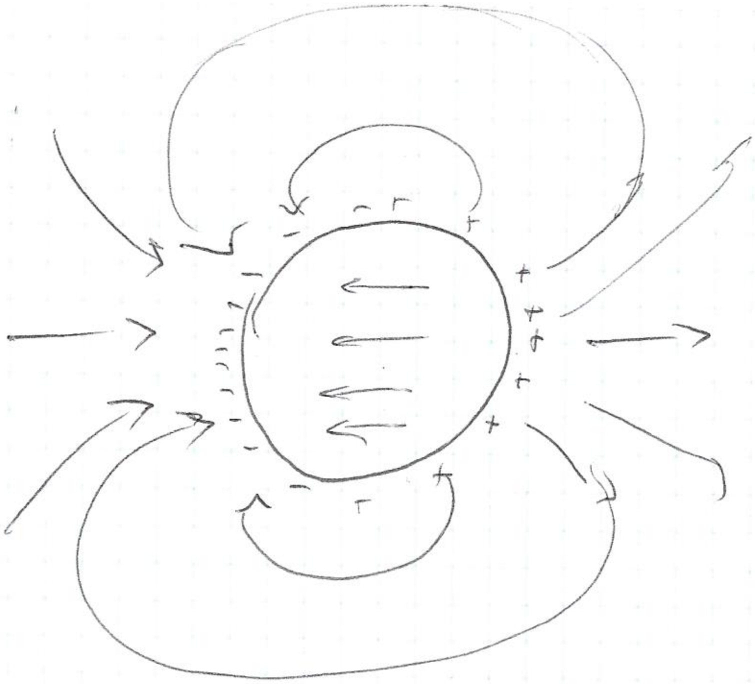
$$= E_0^{ext}\cos\vartheta + \frac{p\cos\vartheta}{2\pi\varepsilon_0 R^3}$$

$$R = \left(\frac{p}{4\pi\varepsilon_0 E_0^{ext}}\right)^{\frac{1}{3}} \longrightarrow \frac{p}{4\pi\varepsilon_0 R^3} = E_0^{ext}$$

$$E_{0r}(R) = E_0^{ext}\cos\vartheta + 2E_0^{ext}\cos\vartheta =$$

$$= 3E_0^{ext}\cos\vartheta$$

$$\sigma_{ind}(R,\vartheta) = \varepsilon_0 E_n = 3\varepsilon_0 E_0^{ext}\cos\vartheta$$



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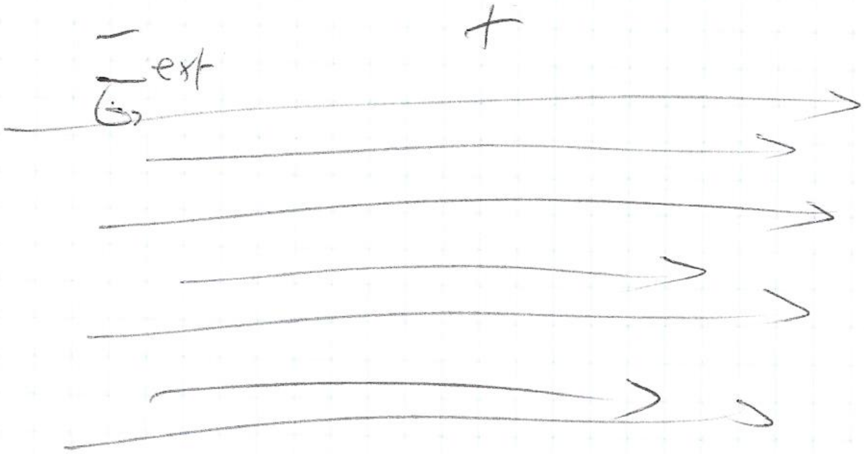
$\underline{E}_{ext}$

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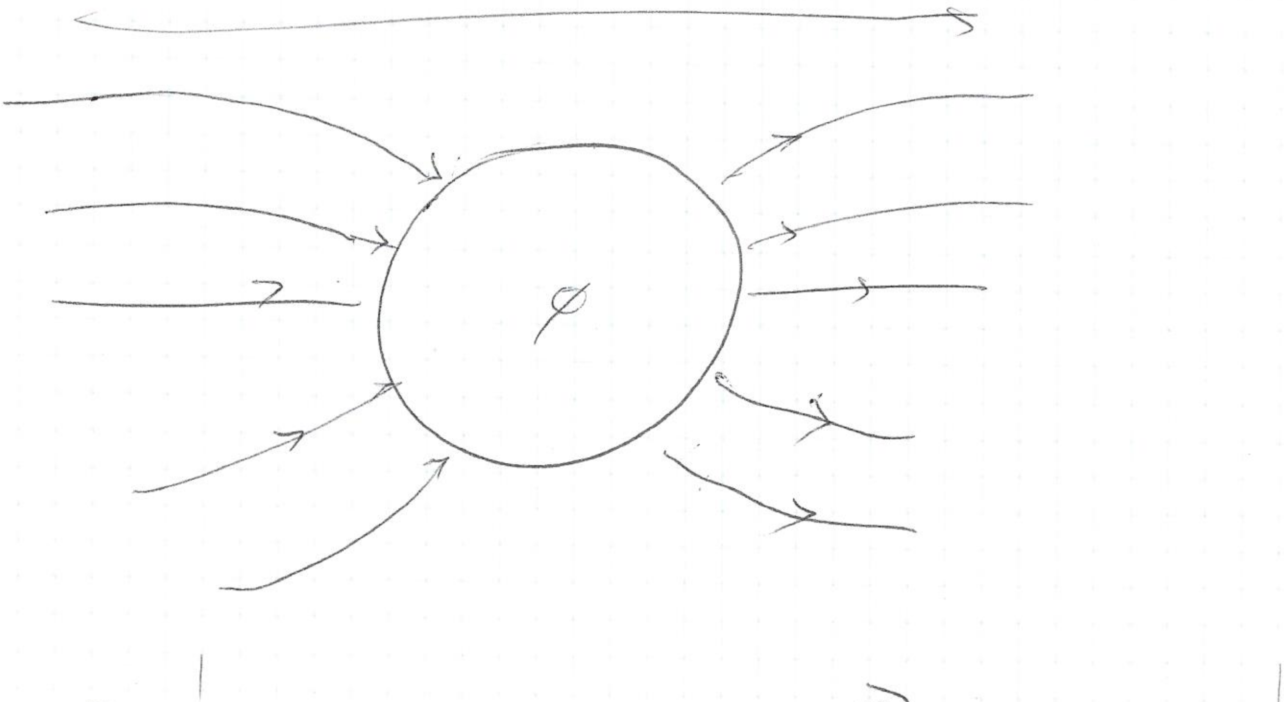
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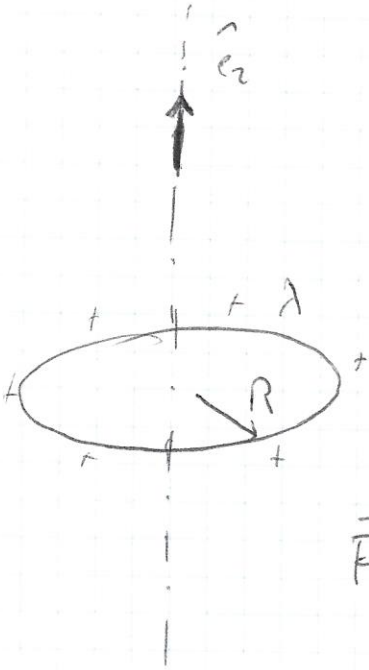

$$\bar{E}_0(\phi,\phi,z) = \frac{\lambda R z}{2\varepsilon_0 (R^2+z^2)^{3/2}}\,\hat{e}_z$$

$$\bar{F} = \bar{\nabla}(\bar{p}\cdot\bar{E}_0) = \underline{(\bar{p}\cdot\bar{\nabla})\bar{E}_0}$$

$$\bar{p} = p\hat{e}_z$$

$$\bar{F} = p_z \frac{\partial}{\partial z}\bar{E}_0 = F_z\,\hat{e}_z$$

$$F_z = \frac{p\lambda R}{2\varepsilon_0}\left[\frac{1}{(R^2+z^2)^{3/2}} - \frac{3z^2}{(R^2+z^2)^{5/2}}\right] =$$

$$= \frac{p\lambda R}{2\varepsilon_0}\,\frac{R^2-2z^2}{(R^2+z^2)^{5/2}}$$

$$R^2 - 2z_0^2 = \phi$$

$$|z_0| = R\sqrt{2}$$

$$F_z > \phi \qquad |z| < R\sqrt{2}$$

$$F_z < \phi \qquad |z| > R\sqrt{2}$$

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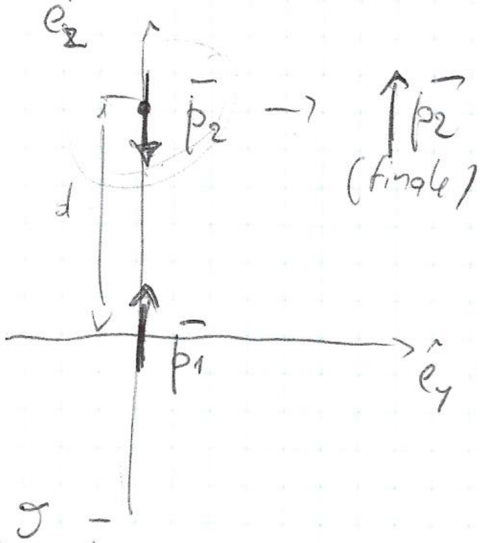
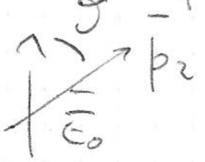


$$U_0^i = -\bar{p}_2 \cdot \bar{E}_0$$

$$E_{0x}(\phi,\phi,d) = \frac{1}{4\pi\varepsilon_0}\frac{3p_1 x z}{(x^2+y^2+z^2)^{5/2}} = \phi$$

$$E_y = \phi$$



$$E_{0z} = \frac{1}{4\pi\varepsilon_0}\frac{p_1(2z^2-x^2-y^2)}{(x^2+y^2+z^2)^{5/2}}\Bigg|_{\phi,\phi,d} = \frac{1}{2\pi\varepsilon_0}\frac{p_1}{d^3}$$

$$U_0^i = -\bar{p}_2 \cdot \bar{E}_0 = -p_2 E_{0z}\cos\vartheta =$$

$$= p_2 E_{0z} = \frac{p_1 p_2}{2\pi\varepsilon_0 d^3} > \phi$$

$U_0(\vartheta)$

$U_0^f = U_{0\,min}$ per $\bar{p}_2 \parallel \bar{E}_0 = \bar{p}_2 \parallel \bar{p}_1$

$$U_0^f = -p_1 p_2 / 2\pi\varepsilon_0 d^3 \qquad \Delta U_0 = -p_1 p_2 / \pi\varepsilon_0 d^3 < \phi$$

$$\bar{F} = \bar{\nabla}(\bar{p}_2 \cdot \bar{E}_0) = (\bar{p}_2 \cdot \bar{\nabla})\bar{E}_0$$

$$= \bar{\nabla}(p_2 E_{0z})$$

$$\frac{\partial}{\partial x}\left[\frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{5/2}}\right]_{(\phi,\phi,d)} = \frac{-2x(x^2+y^2+z^2)^{5/2} - \frac{5}{2}(2z^2-x^2-y^2)(\dots)^{3/2}\cdot 2x}{(\dots)^5}$$

$$= \phi$$

$$\frac{\partial}{\partial y}\Big[\quad\Big]_{(\phi,\phi,d)} = \phi$$

$$\frac{\partial}{\partial z}\left[\cdots\right] = \frac{4z(x^2+y^2+z^2)^{5/2} - \frac{5}{2}(2z^2-y^2-z^2)\cdot(x^2+y^2+z^2)^{3/2}\cdot 2z}{(x^2+y^2+z^2)^5} =$$

$$= \frac{4z(x^2+y^2+z^2) - 5z(2z^2-x^2-y^2)}{(x^2+y^2+z^2)^{7/2}} =$$

$$= \frac{z(9x^2+9y^2-6z^2)}{(x^2+y^2+z^2)^{7/2}} = -\frac{6d^3}{d^7} = -\frac{6}{d^{\cancel{7}4}}$$

$$F_z = \frac{p_1p_2}{4\pi\varepsilon_0}\cdot\left(-\frac{6}{d^4}\right) = -\frac{3}{2\pi\varepsilon_0}\frac{p_1p_2}{d^4}$$

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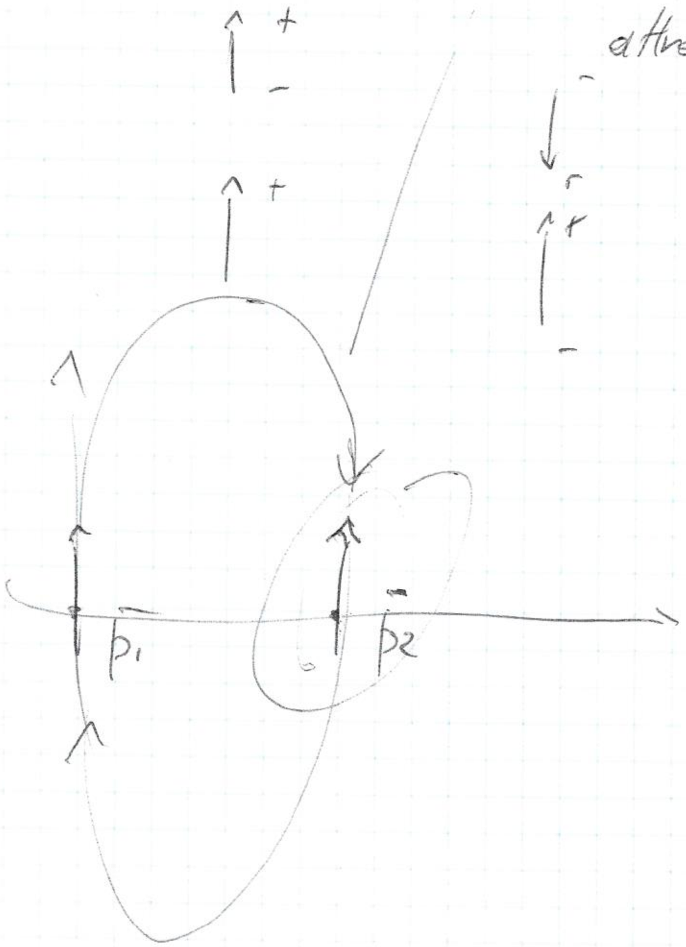
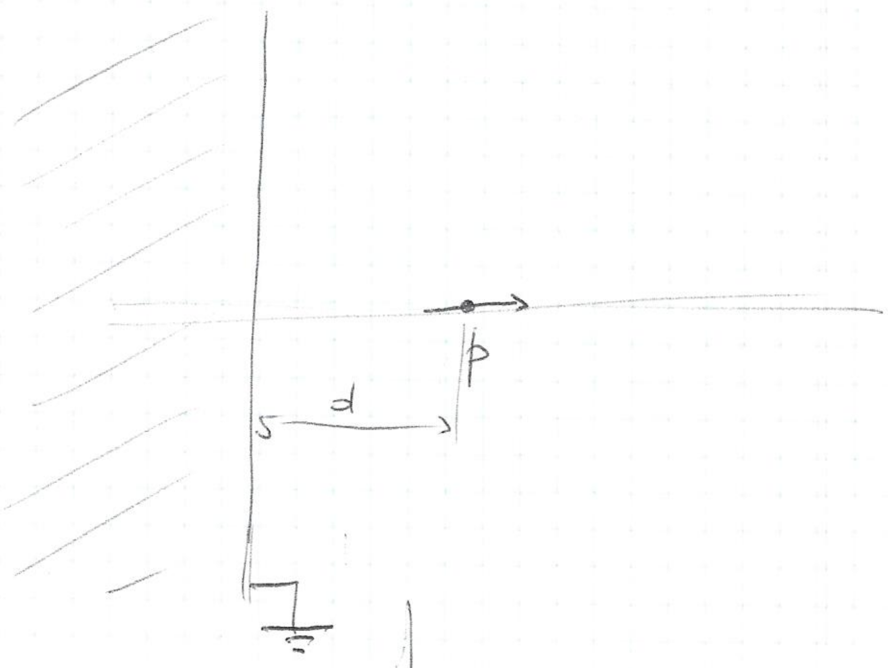
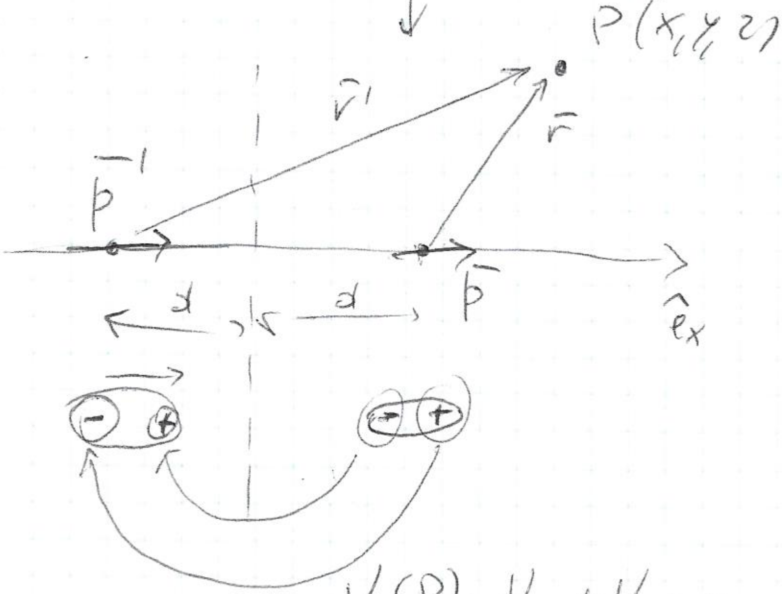






$\bar{p} = (p, 0, 0)$

$\bar{r}(x-d, y, z)$

$\bar{p}'(p, 0, 0)$

$\bar{r}'(x+d, y, z)$

$$V_0(P) = V_{0p} + V_{0p'} =$$

$$= \frac{1}{4\pi\varepsilon}\left[\frac{\bar{p}\cdot\bar{r}}{r^3} + \frac{\bar{p}'\cdot\bar{r}'}{r'^3}\right] =$$

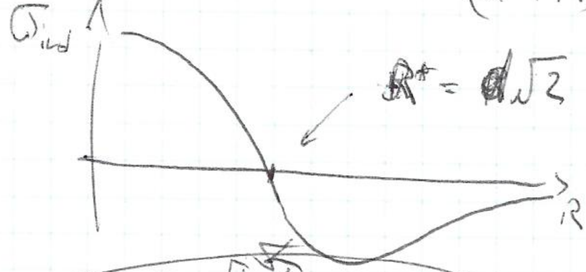
$$= \frac{p}{4\pi\varepsilon_0}\left[\frac{x-d}{((x-d)^2+y^2+z^2)^{3/2}} + \frac{\cancel{\phi} x+d}{((x+d)^2+y^2+z^2)^{3/2}}\right]$$

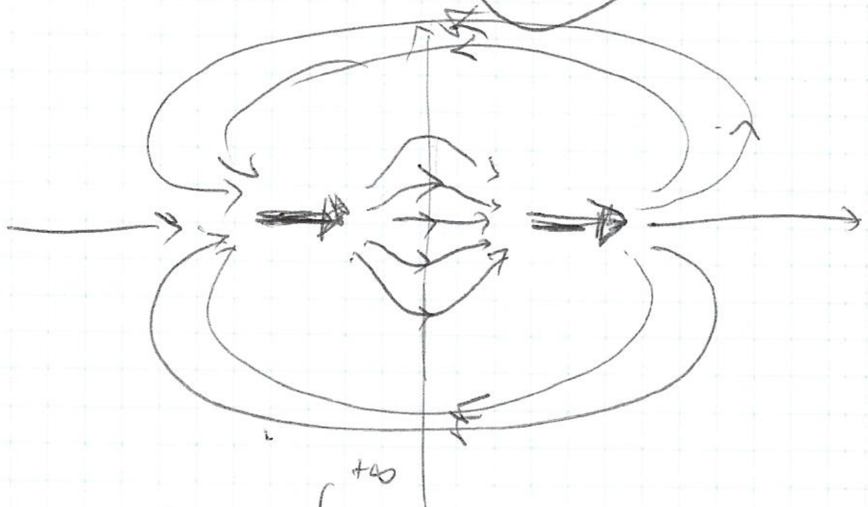
$$E_{0x} = -\frac{\partial V_0}{\partial x} = \frac{p}{4\pi\varepsilon_0}\left[\frac{2(x-d)^2-y^2-z^2}{((x-d)^2+y^2+z^2)^{5/2}} + \frac{2(x+d)^2\dots}{\dots}\right]$$

$$E_{0x}(0, y, z) = \frac{p}{2\pi\varepsilon_0}\,\frac{2d^2-y^2-z^2}{(d^2+y^2+z^2)^{5/2}}$$

$$R^2 = y^2 + z^2$$

$$\sigma_{ind}(\phi, R) = \frac{p}{2\pi} \cdot \frac{2d^2 - R^2}{(d^2+R^2)^{5/2}}$$





$$Q_{ind} = \int_{0}^{+\infty} \sigma_{ind}(R)\, 2\pi R\, dR =$$

$$= p \int_{0}^{+\infty} \frac{2d^2 - R^2}{(d^2+R^2)^{5/2}} R\, dR =$$

$$= p \left(\underbrace{-\frac{2d^2}{3}\left[\frac{1}{(d^2+R^2)^{3/2}}\right]_{0}^{+\infty}}_{\hookrightarrow\, 2/3d} - \underbrace{\int_{0}^{+\infty} \frac{R^3}{(d^2+R^2)^{5/2}}\, dR}_{2/3d} \right) = 0$$

$$f'(R) = R/(d^2+R^2)^{5/2} \quad \longrightarrow f(R) = -\frac{1}{3}\frac{1}{(d^2+R^2)^{3/2}}$$

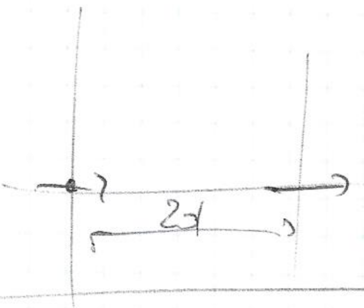
$$g(R) = R^2 \quad \longrightarrow g'(R) = 2R$$

$$\bar{F} = \bar{\nabla}(\bar{p}\cdot\bar{E}_0) =$$

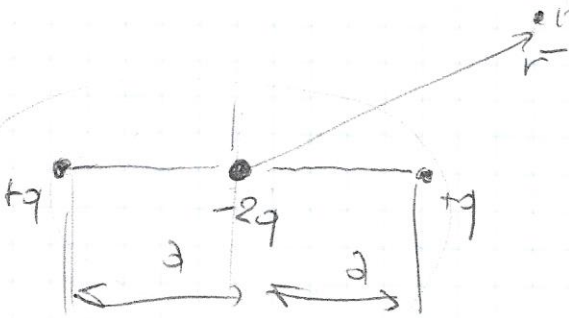
$$= \bar{\nabla}(E_{0x}\cdot p + E_{0y}\cdot 0 + E_{0z}\cdot 0) =$$

$$= \underbrace{p\frac{\partial E_{0x}}{\partial x}}_{F_x}\hat{e}_x + \cancel{p\frac{\partial E_{0x}}{\partial y}\hat{e}_y} + \cancel{p\frac{\partial E_{0x}}{\partial z}\hat{e}_z}$$

$$F_x = p\frac{\partial E_{0x}}{\partial x}\bigg|_{(2d,0,0)} = p\frac{\partial}{\partial x}\left[\frac{p}{4\pi\varepsilon}\,\frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}\right]_{(2d,0,0)} =$$



$$= -\frac{3p^2}{32\pi\varepsilon_0 d^4}$$



$r \gg a$

$$\sum q_i = 0$$

$$\bar{p} = \sum q_i \bar{r}_i = 0$$

$$V_0(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\bar{r}')}{|\bar{r}-\bar{r}'|}\,d\tau' = \frac{1}{4\pi\varepsilon}\int\frac{\rho(x',y',z')}{[(x-x')^2+(y-y')^2+(z-z')^2]^{1/2}}\,d\tau'$$

$$\bar{r}' = 0,0,0$$

$$f(\bar{r}') = \frac{1}{|\bar{r}-\bar{r}'|} \simeq f(\bar{r}'=0) + \;\longrightarrow\; \frac{1}{r}$$

$$+\frac{\partial f}{\partial x'}\bigg|_{r'=0}\cdot x' + \frac{\partial f}{\partial y'}\bigg|_{0}\cdot y' + \frac{\partial f}{\partial z'}\bigg|_{0}\cdot z' +$$

$$+\frac{\partial^2 f}{\partial x'\partial y'}x'y' + \frac{\partial^2 f}{\partial x'\partial z'}x'z' + \frac{\partial^2 f}{\partial y'\partial z'}y'z' + \frac{1}{2}\frac{\partial^2 f}{\partial x'^2}x'^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y'^2}y'^2 + \frac{1}{2}\frac{\partial^2 f}{\partial z'^2}z'^2 +$$

$\frac{1}{|\bar{r}-\bar{r}'|}$ $[\ldots] = [(x-x')^2+(y-y'^2)+(z-z')^2]$ 12

A $\frac{\partial f}{\partial x'} = \frac{\partial}{\partial x'}\left[[\ldots]^{-\frac{1}{2}}\right] = -\frac{1}{2}\frac{2(x-x')(-1)}{[\ldots]^{3/2}} =$

$= \left.\frac{x-x'}{[\ldots]^{3/2}}\right|_{\bar{r}'=\emptyset} = \frac{x}{r^3}$

$\left.\frac{\partial f}{\partial y'}\right|_{\bar{r}'=\emptyset} = \frac{y}{r^3}$; $\left.\frac{\partial f}{\partial z'}\right|_{\bar{r}'=\emptyset} = \frac{z}{r^3}$

B $\left.\frac{\partial^2 f}{\partial x'^2}\right|_{\bar{r}'=\emptyset} = \frac{\partial}{\partial x'}\left.\left[\frac{x-x'}{[\ldots]^{3/2}}\right]\right|_{\bar{r}'=\emptyset} = \frac{-1\cdot[\ldots]^{3/2}-\frac{3}{2}(x-x')[\ldots]^{\frac{1}{2}}\cdot 2(x-x')\cdot(-1)}{[\ldots]^3} =$

$= \left.\frac{-(x-x')^2-(y-y')^2-(z-z')^2+3(x-x')^2}{[\ldots]^{5/2}}\right|_{\bar{r}'=\emptyset} =$

$$\boxed{\frac{\partial^2 f}{\partial x'^2} = \frac{3x^2}{r^5} - \frac{1}{r^3}}$$

$$\frac{\partial^2 f}{\partial y'^2} = \frac{3y^2}{r^5} - \frac{1}{r^3}$$

$$\frac{\partial^2 f}{\partial z'^2} = \frac{3z^2}{r^5} - \frac{1}{r^3}$$

(c) $\dfrac{\partial^2 f}{\partial x'\partial y'} = \dfrac{\partial}{\partial x'}\left[\dfrac{y-y'}{[\dots]^{3/2}}\right] =$ (13)

$$= \left.\frac{3(x-x')(y-y')}{[\dots]^{5/2}}\right|_{\bar{r}'=0} = \frac{3xy}{r^5}$$

$$\frac{\partial^2 f}{\partial x'\partial z'} = \frac{3xz}{r^5} \; ; \; \frac{\partial^2 f}{\partial y'\partial z'} = \frac{3yz}{r^5}$$

$$f(x',y',z')\Big|_{\bar{r}'=0} = \frac{1}{|\bar{r}-\bar{r}'|} \simeq \frac{1}{r} +$$

$$+ \underbrace{\frac{xx'}{r^3} + \frac{yy'}{r^3} + \frac{zz'}{r^3}}_{\frac{\bar{r}\cdot\bar{r}'}{r^3}} +$$

$$* \quad + \frac{1}{2}\left(\frac{3x^2}{r^5} - \frac{1}{r^3}\right)x'^2 + \frac{1}{2}\left(\frac{3y^2}{r^5} - \frac{1}{r^3}\right)y'^2 + \frac{1}{2}\left(\frac{3z^2}{r^5} - \frac{1}{r^3}\right)z'^2$$

$$+ \frac{3xx'yy'}{r^5} + \frac{3xx'zz'}{r^5} + \frac{3yy'zz'}{r^5} =$$

$$* \Rightarrow -\frac{1}{2}\frac{r'^2}{r^3}$$

$$\equiv \Rightarrow \frac{3}{2}\frac{(\bar{r}\cdot\bar{r}')^2}{r^5}$$

$$U_0(\bar{r}) = \int \rho(\bar{r}') \boxed{\frac{1}{|\bar{r}-\bar{r}'|}} d\tau' =$$

$$= \frac{1}{r}\underbrace{\int \rho(\bar{r}')d\tau'}_{\to q} +$$

...