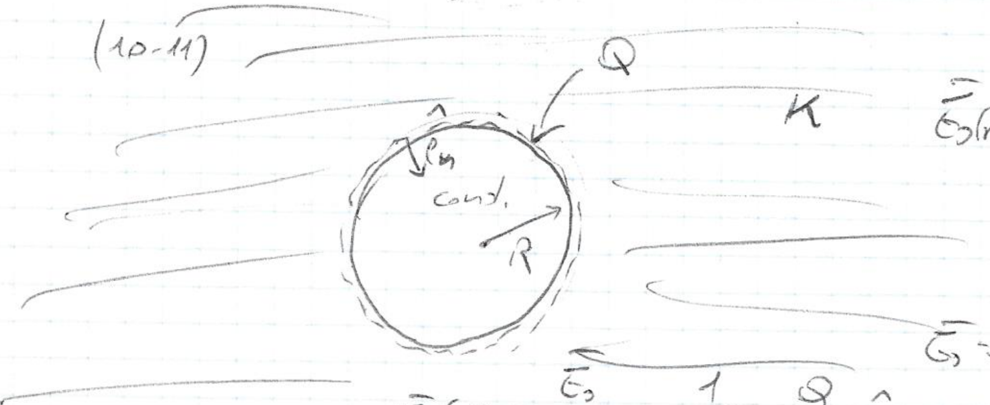
Es. #14 Dielettrici circuiti in corrente continua

18/XII/2020

1

(10-11)



K $\qquad \vec{E}_0(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{e}_r \qquad r \geq R$

$\vec{E}_0 =$ nullo per $r < R$

$$\vec{E}(r) = \frac{\vec{E}_0}{K} = \frac{1}{4\pi\varepsilon_0 K}\frac{Q}{r^2}\hat{e}_r$$

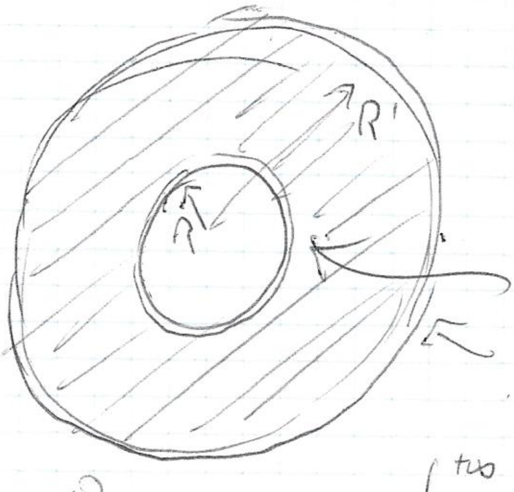
$$\vec{P} = (K-1)\varepsilon_0\vec{E}_{in} = \frac{1}{4\pi}\frac{K-1}{K}\frac{Q}{r^2}\hat{e}_r$$

$$\rho_p = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{d}{dr}\left(\cancel{r^2 P_r}\right) = 0$$

$$\sigma_p(R) = \vec{P}\cdot(-\hat{e}_r) = -\frac{K-1}{4\pi K}\frac{Q}{R^2} = -\frac{K-1}{K}\sigma$$

$\sigma = Q/4\pi R^2$

$$\sigma' = \sigma + \sigma_p = \sigma - \frac{K-1}{K}\sigma = \frac{\sigma}{K}$$

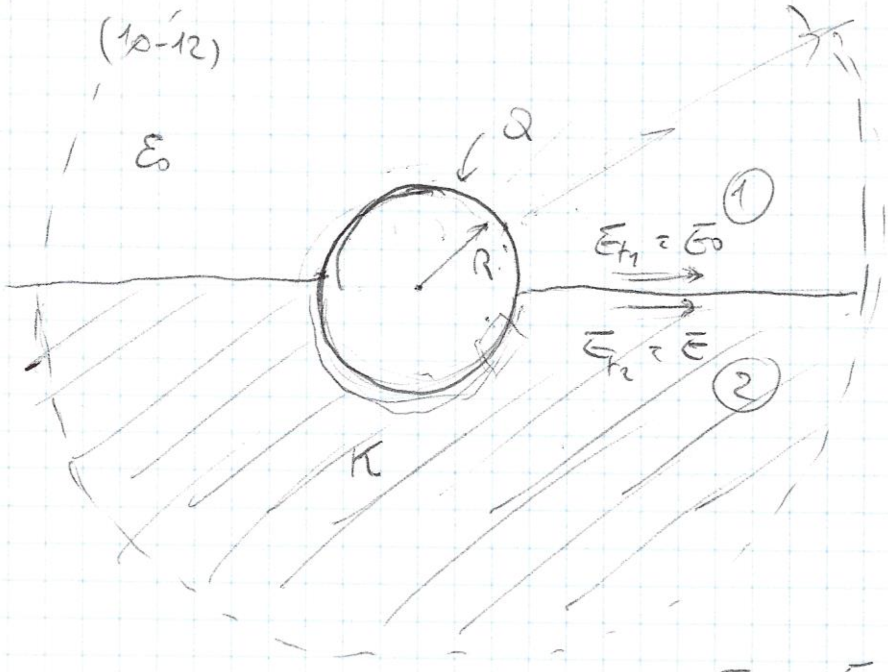


$Q_p(R) = -\frac{K-1}{K}Q$

$Q_p(R') =$

$$C = \frac{Q}{\Delta V} \qquad \Delta V = \int_R^{+\infty} E_r\,dr = \frac{Q}{4\pi\varepsilon_0 K}\int_R^{+\infty}\frac{1}{r^2}\,dr = \frac{Q}{4\pi\varepsilon_0 K R}$$

$$C = 4\pi\varepsilon_0 K R$$



R – ~~...~~

R' → +∞ equipot.

$V \sim \frac{1}{r}$

$\bar{E} \sim \frac{1}{r^2}\hat{e}_r$

$$E_0 = E_{t1} = E_{t2} = E$$

$$\bar{D} = D\hat{e}_n$$

(1) $D_0 = \sigma_0 = \varepsilon_0 E_0$ $\qquad E_0 = \sigma_0/\varepsilon_0$

(2) $D = \sigma = \varepsilon_0 K E$ $\qquad E = \sigma/\varepsilon_0 K$ $\qquad \Rightarrow \sigma = K\sigma_0$

$$Q = \sigma 2\pi R^2 + \sigma_0 2\pi R^2 = 2\pi R^2 \sigma_0 (1+K) \Rightarrow \sigma_0 = \frac{Q}{2\pi\varepsilon(1+K)}$$

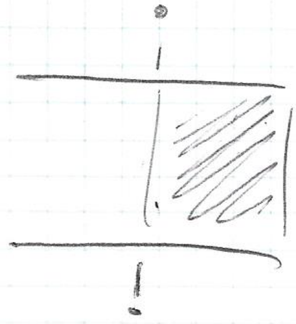
$$E(R) = E_0 = \frac{\sigma_0}{\varepsilon} = \frac{Q}{2\pi\varepsilon R^2 (1+K)}$$

$$Q' = \sigma_0 4\pi R^2 = \frac{2}{1+K}Q \qquad E(r) = \frac{1}{4\pi\varepsilon}\frac{Q'}{r^2} = \frac{1}{2\pi\varepsilon(1+K)}\frac{Q}{r^2}$$

$$\Delta V = \int_R^{+\infty} E(r)\,dr = \frac{Q'}{4\pi\varepsilon_0}\int_R^{+\infty}\frac{1}{r^2}\,dr = \frac{Q}{2\pi\varepsilon(1+K)R}$$

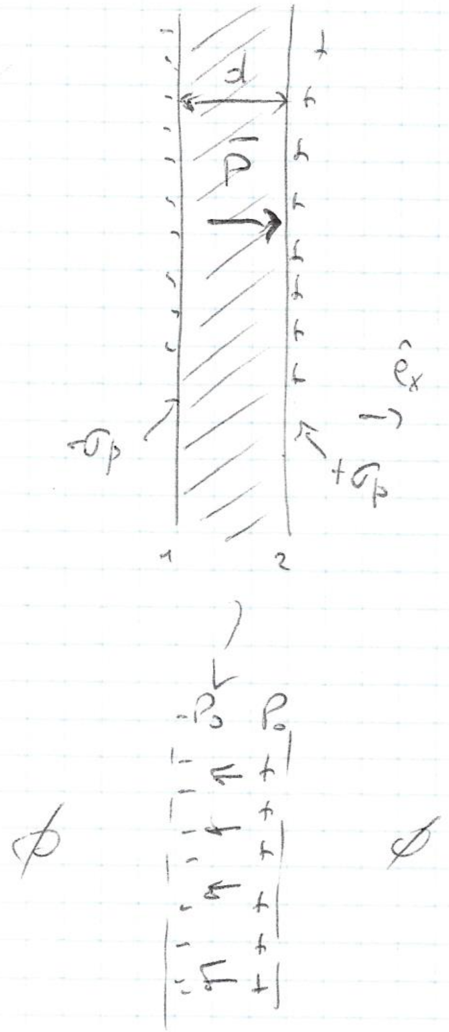
$$C = Q/\Delta V = 2\pi\varepsilon(1+K)R$$

vuoto ($K=1$) $\longrightarrow$ $C = 4\pi\varepsilon R$



(10-13) $\bar{P}$

vacuum



(polarizzazione permanente)
ferroelettrici

vacuum

$\bar{P} = P_0 \hat{e}_x$

$\rho_p = -\bar{\nabla} \cdot \bar{P} = 0$

$\sigma_p^{12} = \bar{P}(1,2) \cdot \underbrace{\hat{e}_n}_{-\hat{e}_x \,;\, \hat{e}_x} = \mp P_0$

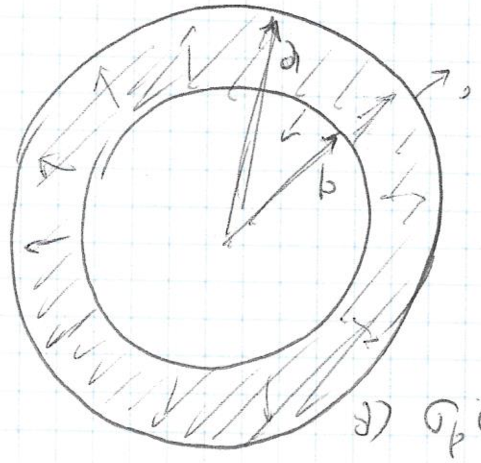
$\bar{E} = -\frac{\sigma_p}{\varepsilon} \hat{e}_x = -\frac{P_0}{\varepsilon_0} \hat{e}_x$

nel dielettrico

$\bar{E} = 0$ fuori

Esame 12/VII/2019 ④

es. #1



$a = 10\,cm$, $b = 5\,cm$

$\bar{P}(\bar{r}) = P_0\, \hat{e}_r$ $P_0 = 8.85 \cdot 10^{-9}\ C/m^2$

a) $\sigma_p(b) = \bar{P}(b) \cdot (-\hat{e}_r) = -P_0$

$\sigma_p(a) = \bar{P}(a) \cdot \hat{e}_r = P_0$

$\rho_p(r) = -\bar{\nabla} \cdot \bar{P} = -\frac{1}{r^2} \frac{d}{dr}(r^2 P_0) = -\frac{2P_0}{r}$

b) $Q_{ps}(b) = \sigma_p(b) \cdot 4\pi b^2 = -P_0 4\pi b^2 = -2.78 \cdot 10^{-10}\,C$

$Q_{ps}(a) = P_0 4\pi a^2 = 1.11 \cdot 10^{-9}\,C$

$\cancel{Q_{ps}}$ $Q_{pv} = -\int_b^a \rho_p(r) 4\pi r^2 dr = -8\pi P_0 \int_b^a r\,dr = 4\pi P_0 (b^2 - a^2) =$

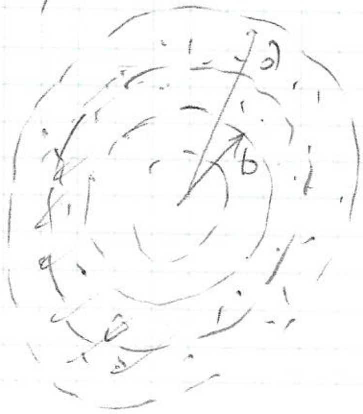
$= -8.34 \cdot 10^{-10}\,C$

$Q_{p\,tot} = \phi$

c) $\bar{E} = ?$ $\bar{D} \leftarrow \rho$ free (no) $\rightarrow \bar{D} = \phi$

$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$ $\bar{E} = -\bar{P}/\varepsilon_0$ $\begin{cases} -P_0/\varepsilon_0\, \hat{e}_r & b < r < a \\ \phi & r < b,\ r > a \end{cases}$

alternatively (equivalent charge system in vacuum)

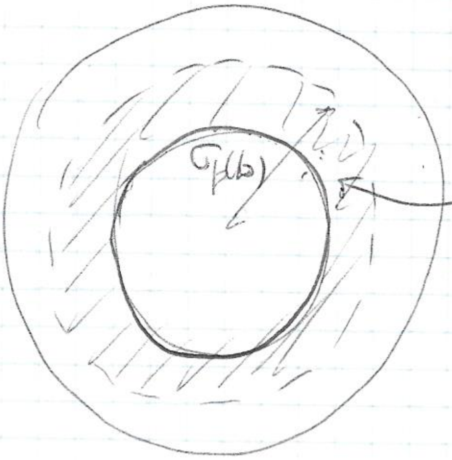


Gauss' law $r < b$

$r < b$ $Q_{INT} = \phi \Rightarrow \bar{E} = \phi$

$a < r < b$

Gauss : $a < r < b$



$$E_r(r)\,4\pi r^2 = \frac{1}{\varepsilon_0}\Big[-4\pi P_0 b^2 + 4\pi P_0 (b^2 - r^2)\Big] = -\frac{4\pi}{\varepsilon_0} P_0 r^2$$

$$\Rightarrow E_r(r) = -\frac{P_0}{\varepsilon_0} = 5 \cdot 10^3 \ V/m$$

Gauss : $r > a$

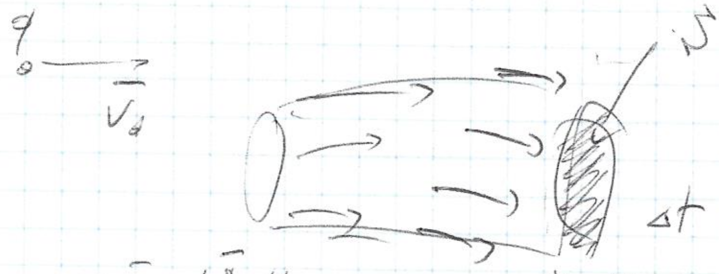
$$Q_{INT} = Q_{pb} + Q_{pa} + Q_{pv} = \phi$$

$$E_r(r)\,4\pi r^2 = \phi/\varepsilon_0 \quad \Rightarrow \quad E_r = \phi$$

$$\Delta E = E_r(b^+) - E_r(b^-) = \sigma_p(b)/\varepsilon_0$$

d) $$V_a - V_b = -\int_{-b}^{a} E_r\,dr = \frac{P_0}{\varepsilon_0}(a-b) = 50 V$$

Circuiti in corrente stazionaria



$$dq = nq\,\bar v_d \cdot d\bar S\, dt = nq\, v_d\, dS_n\, dt$$

n = part./vol

$$\bar J \doteq nq\,\bar v_d \qquad [J] = [A/m^2]$$

$$dq = \bar J \cdot d\bar S\, dt \qquad [A] = \left[\frac{C}{s}\right]$$

$$dI = \frac{dq}{dt} = \bar J \cdot d\bar S$$

$$I = \int_{S_{tubo}} \bar J \cdot d\bar S$$

$\bar E$ motore della corrente

$\bar J = f(\bar E)$ mezzi lineari, omo-, iso- = OHMICI

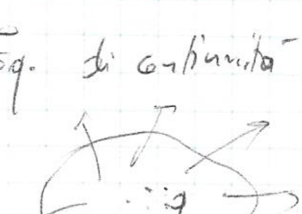
$\bar J = \sigma \bar E$ $\qquad \sigma$ conducibilità elettrica

$\bar J = \sum_i n_i q_i \bar v_i$ $\qquad \rho = \frac{1}{\sigma}$ resistività

$$[\sigma] = [E/J] = \left[\frac{V}{m}\cdot\frac{m^2}{A}\right] = \left[\frac{V}{A}\cdot m\right] = [\Omega\cdot m]$$

$[\Omega] = [V/A]$ OHM

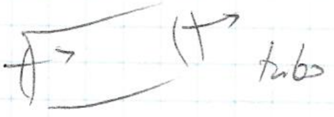
Eq. di continuità



$$\boxed{\frac{\partial\rho}{\partial t} + \bar\nabla\cdot\bar J = 0} \longleftarrow -\frac{dQ}{dt} = \oint_S \bar J \cdot d\bar S$$

Corr. Stazionaria

$\rho(t) = \text{cost.}$ $\qquad \vec{\nabla} \cdot \vec{J} = \phi$

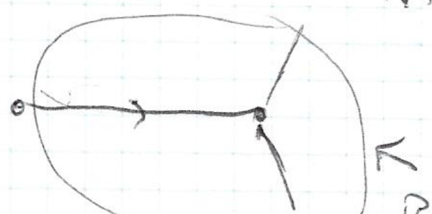


staz.: $\displaystyle\sum_i \iint \vec{J}_i \cdot d\vec{S}_i = \boxed{\sum I_i = \phi}$

I LEGGE DI KIRCHHOFF.

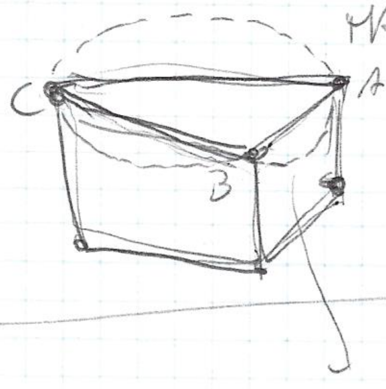
$\sum I$ entranti/uscenti da un nodo $= \phi$

RAMO : connessione tra 2 nodi, in cui circola UNA corrente determinata



RETE : insieme di rami con un nodo in comune

MAGLIA : " " " che forma una linea chiusa



$$\oint \vec{E}_s \cdot d\vec{l} = \phi$$

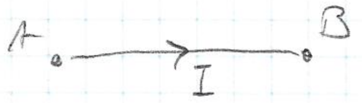
$$V_A - V_B + V_B - V_C + V_C - V_A = \phi$$

$\sum V_i = \phi$ su maglia

II LEGGE DI KIRCHHOFF

Resistenza elettrica

OHMICI



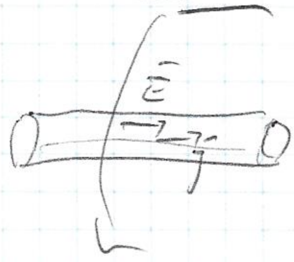
$$\Delta V = V_A - V_B = R\, I$$

↪ resistenza

$$[R] = \left[\frac{V}{A}\right] = [\Omega] \quad \text{Ohm}$$

$$\underline{\Delta V = RI}$$

<u>prima legge di Ohm</u>



$$\boxed{R = \rho \frac{l}{S} = \frac{1}{\sigma}\frac{l}{S}}$$

seconda legge di Ohm

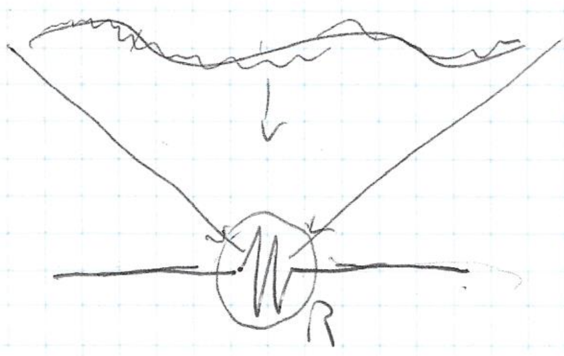
$$El = RjS$$

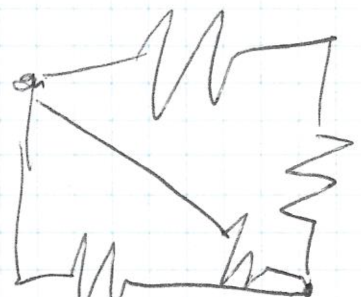
$$E = \frac{RS}{l}\, j \qquad E = \rho j \rightsquigarrow \underline{j = \sigma E}$$

(I Ohm forma locale)

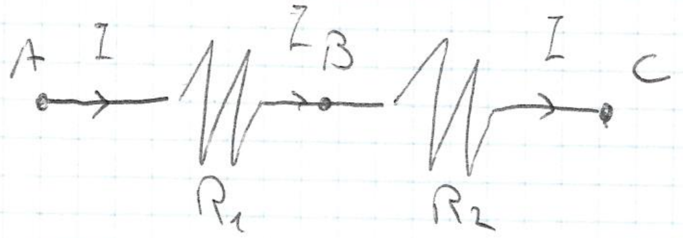
Rappresentazione a <u>parametri concentrati</u>

$$R = \rho \frac{l}{S}$$





Serie



$$V_A - V_B = R_1 I$$

$$V_B - V_C = R_2 I$$

$$V_A - V_C = (R_1 + R_2) I = R_{eq} I$$

$$R_{eq\,serie} = \sum_i R_i$$

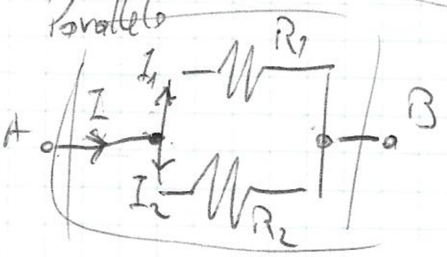
$$I = \frac{\Delta V_{AC}}{R_{eq}} = \frac{\Delta V_{AC}}{R_1 + R_2}$$

$$\Delta V_{AB} = V_A - V_B = R_1 I = \frac{R_1}{R_1 + R_2} \cdot I$$

$$\Delta V_{BC} = R_2 I = \frac{R_2}{R_1 + R_2} \cdot I$$

partitore resistivo di tensione

Parallelo



$$I = I_1 + I_2$$

$$I_1 = \Delta V_{AB} / R_1 \quad ; \quad I_2 = \Delta V_{AB} / R_2$$

$$I = I_1 + I_2 = \Delta V_{AB} \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = \Delta V_{B} / R_{eq}$$

$$\frac{1}{R_{eq\,//}} = \sum_i \frac{1}{R_i}$$

partitore res. di corrente

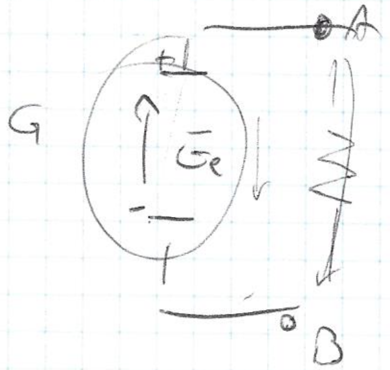
$$I_1 = \frac{\Delta V_{AB}}{R_1} = \frac{R_{eq} I}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{\Delta V_{AB}}{R_2} = \frac{R_1}{R_1 + R_2} I$$

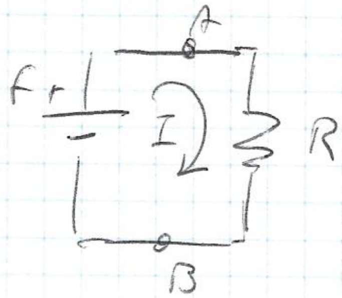
Forza elettromotrice ~ generatori

Non conservativo $\bar{E}_e$ campo elettromotore

$$f_e = \int_B^A \bar{E}_e \cdot d\bar{l} = V_A - V_B$$

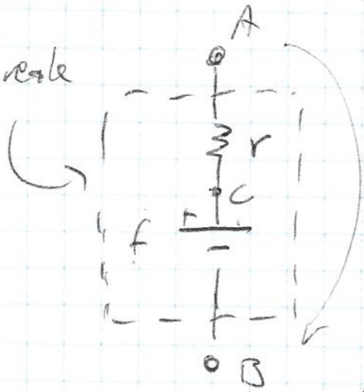


ΔV_{AB} (su carico) a circuito aperto

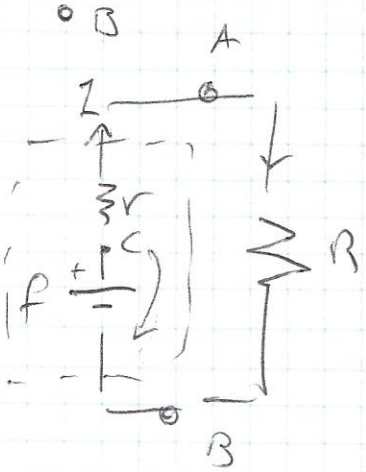


$f = \Delta V_{AB} = RI$

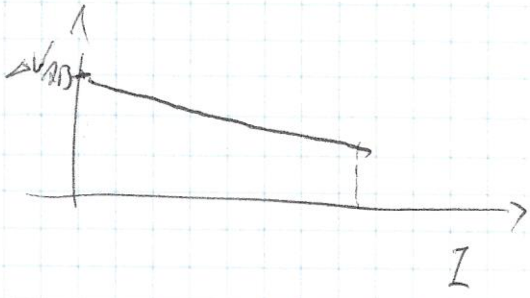
Gen. reale



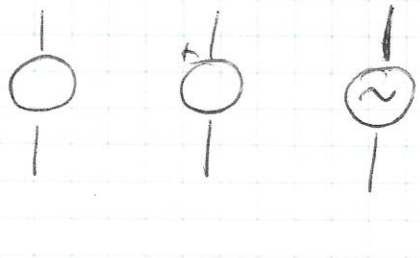
a circ. aperto $V_C = V_A$ (no I)



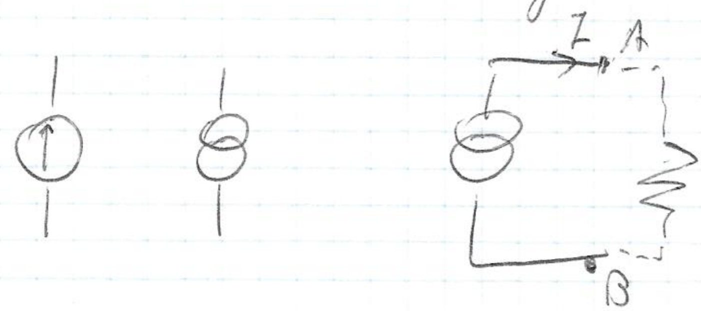
$V_A - V_B = f - rI = RI$



batteria (pila)

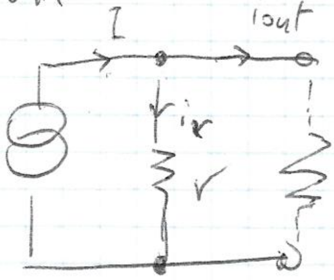


Generatore di corrente : eroga corr. fissa



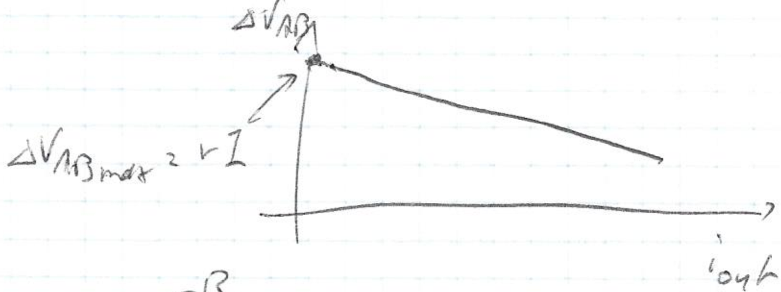
$\Delta V_{AB} = RI$

G reale di corrente



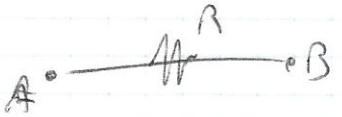
$i_{out} = I - i_r = I - \Delta V_{AB}/r$

$\Delta V_{AB} = r(I - i_{out})$



$$\Delta V_{AB} = \frac{rR}{r+R} I$$

Lavoro - effetti dissipativi



$dq \qquad d\mathcal{L} = dq\,(V_A - V_B) =$

$dq = I\,dt$

$\Rightarrow d\mathcal{L} = (V_A - V_B) I\,dt = \Delta V I\,dt$

$$W = \frac{d\mathcal{L}}{dt} = I\Delta V = RI^2 = \frac{(\Delta V)^2}{R}$$ potenza ohmica dissipata su R (effetto Joule) → calore

$d\mathcal{L} = n\,d\tau\,q\,\bar{E}\cdot d\bar{\ell} \Rightarrow d\bar{\ell} = \bar{v}_d\,dt$

$= n\,d\tau\,q\,\bar{E}\cdot\bar{v}_d\,dt$ → $\bar{J}$

$$\frac{1}{d\tau}\frac{d\mathcal{L}}{dt} = \frac{W}{d\tau} = w = qn\bar{v}_d\cdot\bar{E} = \bar{E}\cdot\bar{J}$$ densità di potenza

$\bar{E} = \rho \bar{J}$

$$\int_A^B \bar{E}_s \cdot d\bar{l} + \int_A^B \bar{E}_e \cdot d\bar{l} = \int_A^B \rho \bar{J} \cdot d\bar{l}$$

$$\underbrace{\int_A^B \bar{E}_s \cdot d\bar{l}}_{V_A - V_B} + \sum_{i,\ A\to B} f_i = \int_A^B \rho \frac{I}{S}\, dl = R_{AB}\, I$$

$\bar{J}$ uniforme $I = J/S$

lungo ramo $A \to B$
su cui $\exists\ f_i,\ R_k$

$$\boxed{V_A - V_B + \sum_{i,\ A\to B} f_i = I \sum_k R_k}$$ legge di Ohm generalizzata

$\downarrow$

su maglia

$V_A - V_A = 0$

$$\hookrightarrow \boxed{\sum_i f_i = \sum_j R_j I_j}$$ II legge di Kirchhoff (completa)