ES #16 Circuiti in corrente continua - transitori lenti 1/11/2021

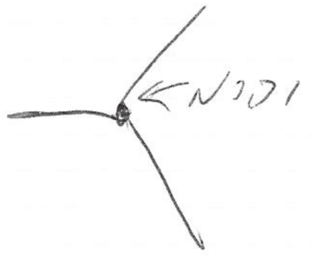
Ohmici

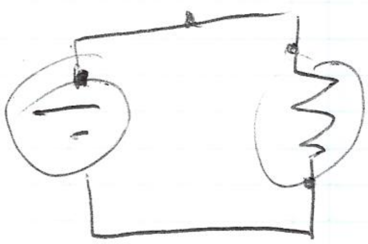
$\Delta V = RI$ ..

↳ $\vec{E} = \rho \vec{J}$

$R = \rho \frac{l}{S}$

parametri concentrati





AI NODI $\sum_i I_i = 0$

I K.

ALLE MAGLIE

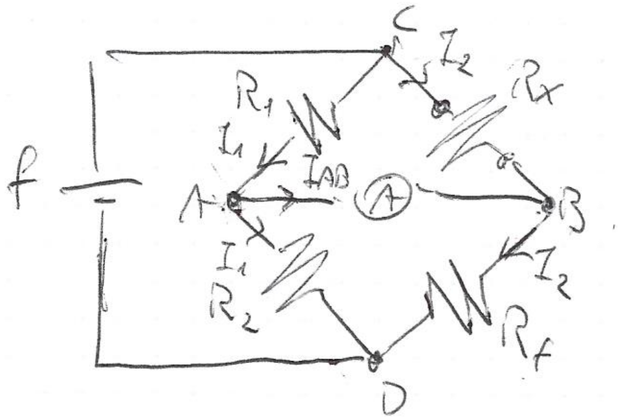
$\sum_i f_i = \sum_j R_j I_j$

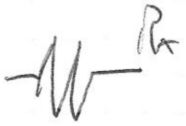
II K.

$P_G = f I$ →

(11-1)

Ponte di Wheatstone





$I_{AB} = \phi$

$V_A = V_B$

$V_C - V_A = V_C - V_B$

$R_1 I_1 = R_x I_2$

$V_A - V_D = V_B - V_D$

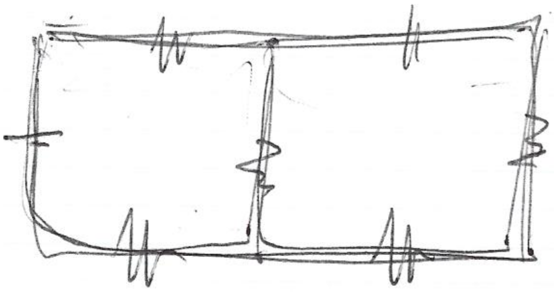
$R_2 I_1 = R_f I_2$

$$\frac{R_1}{R_2} = \frac{R_x}{R_f}$$

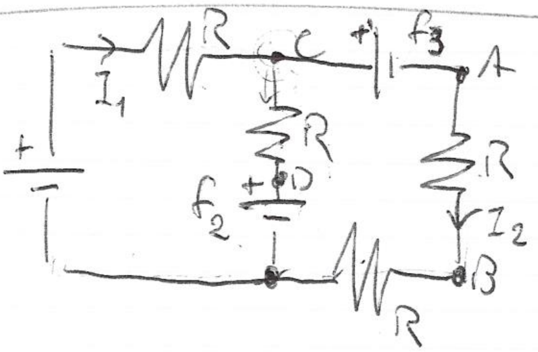
$R_x = R_f \cdot \frac{R_1}{R_2}$ quando $I_{AB} = \phi$

Metodo delle correnti di maglia

\# maglie indipendenti = \# rami − \# nodi + 1



(11-2)



ΔV_{AB}

ΔV_{CD}

$f_1 = 12\,V$

$f_2 = 9\,V$

$f_3 = 5\,V$

$R = 1\,k\Omega$

$$\begin{cases} f_1 - f_2 = RI_1 + R(I_1 - I_2) \\ f_2 - f_3 = R(I_2 - I_1) + RI_2 + RI_2 \end{cases}$$

$$\begin{cases} f_1 - f_2 = 2RI_1 - RI_2 & [1] \\ f_2 - f_3 = -RI_1 + 3RI_2 & [2] \end{cases}$$

$[1] + 2\cdot[2] \qquad f_1 + f_2 - 2f_3 = 5RI_2$

$$I_2 = (f_1 + f_2 - 2f_3)/5R = 2.2\ mA$$

$3\cdot[1] + [2]$

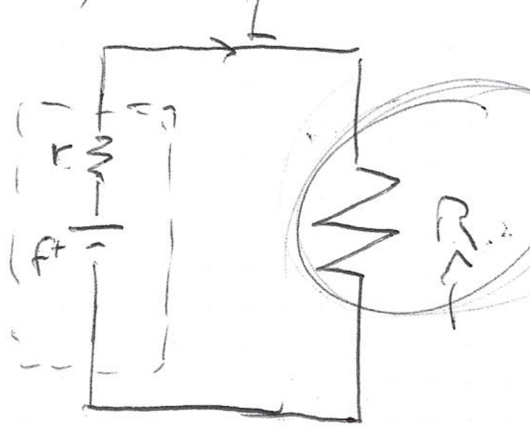
$$I_1 = (3f_1 - 2f_2 - f_3)/5R = 2.6\ mA$$

$$\Delta V_{AB} = RI_2 = 2.2\ V$$

$$\Delta V_{CD} = R(I_1 - I_2) = 0.4\ V$$

(11-3)

4



$$f = rI + RI = (r+R)I$$

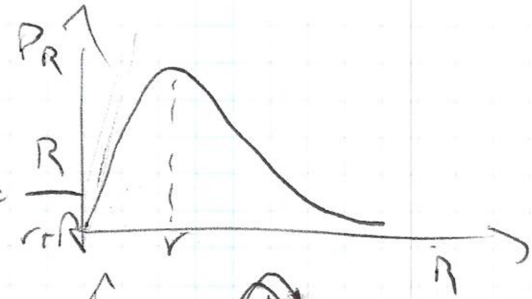
$$I = \frac{f}{r+R}$$

$$P_R = RI^2 = R\left(\frac{f}{r+R}\right)^2 \geq \phi$$

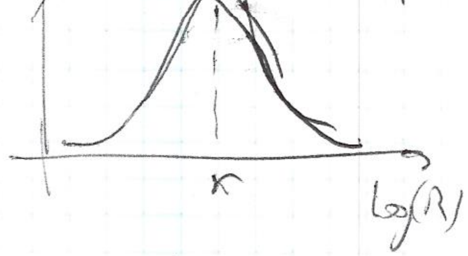
$$\frac{dP}{dR} = f^2 \cdot \frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} = \dots =$$

$$= \frac{r^2 - R^2}{(r+R)^4} = \frac{r-R}{(r+R)^3} = \phi \qquad \boxed{R = r}$$

$$P_R = R\left(\frac{f}{r+R}\right)^2 \Bigg|_{R=r} = \frac{f^2}{4r}$$



$$\eta = \frac{P_R}{P_T} = \frac{P_R}{P_r + P_R} = \frac{RI^2}{(r+R)I^2} = \frac{R}{r+R}$$

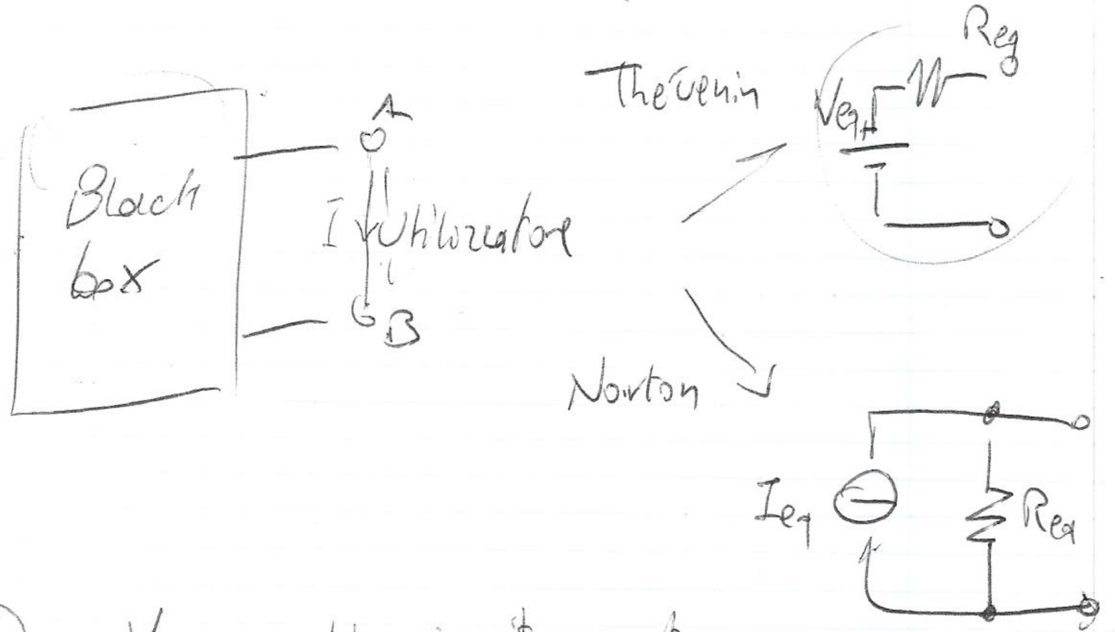


$R \to \infty$

$\eta = \frac{1}{2} \qquad R = r$

50%

Teoremi di Thévenin e di Norton



1. V_{AB} di circuito aperto → V_{eq}

2. I_{AB} di cortocircuito

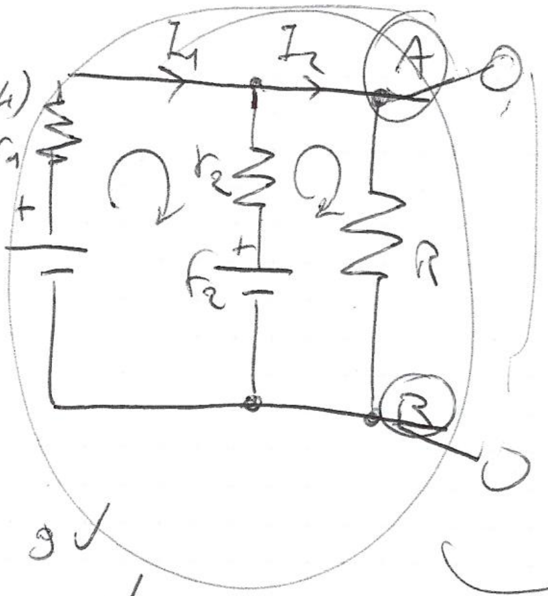
   ↳ equiv. R_{eq} a generatori disattivati

   g. tensione → c.c.

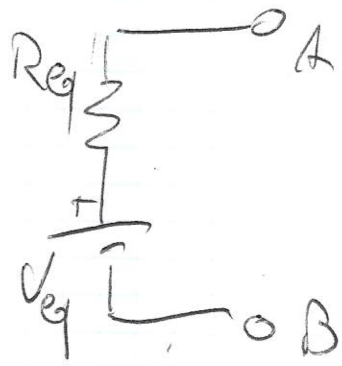
   g. corrente → c.a.

$$I_{eq} = \frac{V_{eq}}{R_{eq}}$$

(11-4)



6



$f_1 = 9\,V$

$f_2 = 12\,V$

$r_1 = 5\,\Omega$

$r_2 = 3\,\Omega$

$R = 150\,\Omega$

$$\begin{cases} f_1 - f_2 = (r_1 + r_2) I_1 - r_2 I_2 \\ f_2 = -r_2 I_1 + (r_2 + R) I_2 \end{cases}$$

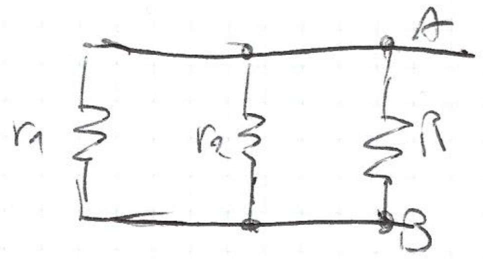
$$f_1 = r_1 I_1 + R I_2$$

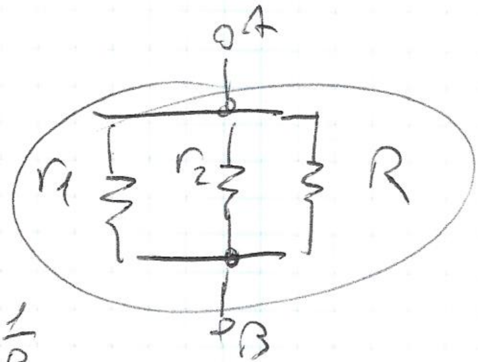
$$I_1 = \frac{f_1 - R I_2}{r_1}$$

$$f_2 = -r_2 \frac{f_1 - R I_2}{r_1} + (r_2 + R) I_2$$

$$I_2 = \frac{r_2 f_1 + r_1 f_2}{(r_1 r_2 + r_1 R + r_2 R)} = 71.6\,mA$$

$$V_{eq} = \Delta V_{AB\,ca} = R I_2 = 10.74\,V$$



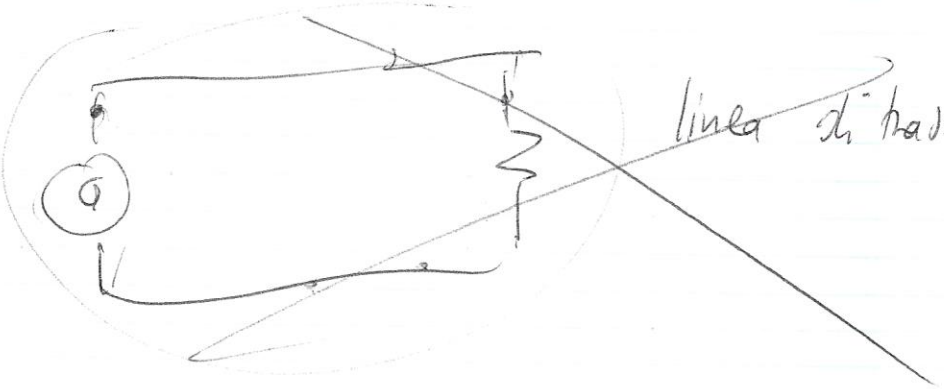


$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}$$

$$I_{eq} = \frac{V_{eq}}{R_{eq}} = 5.81\,A$$

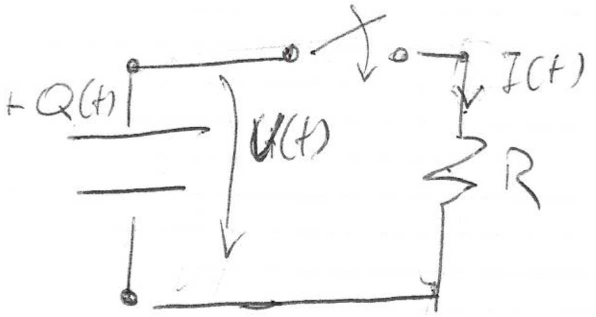
$$R_{eq} = \frac{r_1 r_2 R}{r_1 r_2 + r_1 R + r_2 R} = 1.85\,\Omega$$

Transitori <u>lenti</u>



linea di trasmissione

Scanica di un condensatore (12-1)



Q_0

$t=0 \quad Q(0)=Q_0$

$$V(t) = R\,I(t)$$

$$\frac{Q(t)}{C} = -R\,\frac{dQ(t)}{dt}$$

$$i = \frac{dq}{dt} = \cancel{\frac{dQ}{dt}}$$

$$I(t) = -\frac{dQ(t)}{dt}$$

$$\begin{cases} V_0 = Q_0/C \\ I_0 = \dfrac{V_0}{R} \end{cases} \qquad \begin{cases} V_\infty = \dfrac{Q_\infty}{C} = 0 \\ I_\infty = 0 \end{cases}$$

$$\frac{1}{RC}\int dt = -\int \frac{dQ(t')}{Q(t')}$$

$$\frac{1}{RC}t = -\log Q(t) + A' = -\log(Q(t)/A)$$

$A' = \log A$

$$Q(t) = A e^{-t/RC}$$

$$Q(t=0) = Q_0 = A \Rightarrow \boxed{Q(t) = Q_0 e^{-t/RC}} \quad \tau = RC$$

$$I(t) = -\frac{dQ}{dt} = \frac{Q_0}{RC}\, e^{-t/RC} = \frac{V_0}{R}\, e^{-t/RC}$$

$$\rightarrow I_0 = \frac{V_0}{R}$$

$$U_J = \int_0^{+\infty} R\, I^2(t)\, dt = \frac{Q_0^2}{RC^2} \int_0^{+\infty} e^{-2t/RC} =$$

$$= -\frac{Q_0^2}{RC^2}\, \frac{RC}{2} \underbrace{\left[e^{-2t/RC} \right]_0^{+\infty}}_{-1} = \frac{1}{2}\frac{Q_0^2}{C} = U_C(t=0)$$

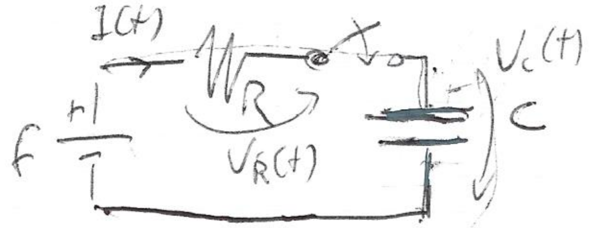
$$U_J(t^*) = \frac{1}{2C}\left[Q_0^2 - Q^2(t^*) \right]$$

$$U_C(t^*) = \frac{1}{2}\frac{Q^2(t^*)}{C}$$

(12-2)

Carica di C

9



$t = \phi$ chiusura

$$f = V_c(t) + V_R(t) =$$

$$= \frac{Q(t)}{C} + R I(t)$$

$$\boxed{f = \frac{Q(t)}{C} + R \frac{dQ(t)}{dt}}$$

$$\frac{dt}{RC} = \frac{dQ(t)}{fC - Q(t)}$$

$y \doteq fC - Q$

$\frac{dy}{dQ} = -1 \qquad dy = -dQ$

$$\frac{dt}{RC} = -\frac{dy}{y} \rightsquigarrow y(t) = Ae^{-t/RC}$$

$\tau = RC$

$$\begin{cases} Q_0 = \phi \\ V_{c0} = \phi \\ I_0 = f/R \end{cases} \qquad \begin{cases} Q_\infty = fC \\ V_{c\infty} = f \\ I_\infty = \phi \end{cases}$$

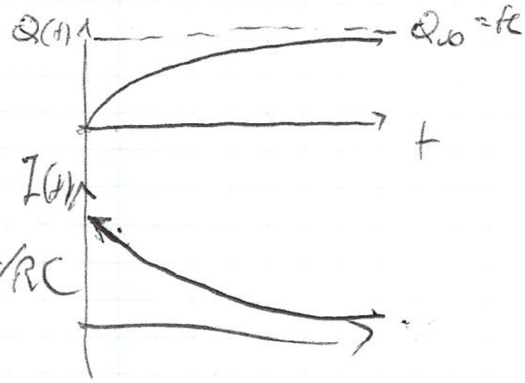
$$fC - Q(t) = Ae^{-t/RC} \rightarrow Q(t) = fC - Ae^{-t/RC}$$

$$Q(t=\phi) = \phi = fC - A \implies A = fC$$

$$Q(t) = fC\left(1 - e^{-t/RC}\right)$$

$$V(t) = \frac{Q(t)}{C} = f\left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{dQ}{dt} = \frac{f}{R} e^{-t/RC} = I_0 e^{-t/RC}$$



$$U_g = \int_\phi^{+\infty} W_g(t)\,dt = \int_\phi^{+\infty} f \cdot I(t)\,dt = \frac{f^2}{R}\int_\phi^{+\infty} e^{-t/RC}\,dt =$$

(10)

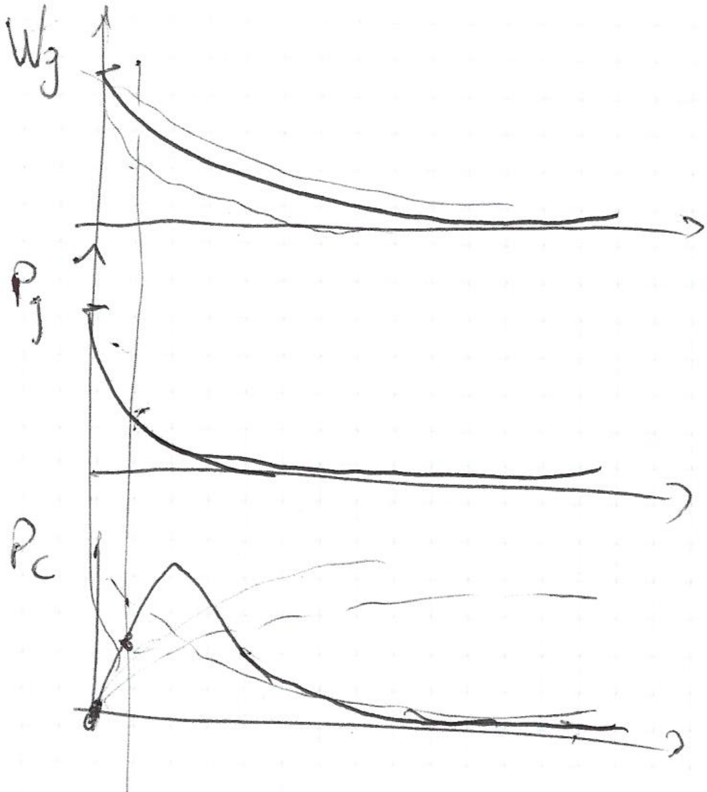
$$= -f^2 C \left[e^{-t/RC}\right]_\phi^{+\infty} = Cf^2$$

$$U_c = \frac{1}{2} C V_\infty^2 = \frac{1}{2} C f^2$$

$$U_J = \int_\phi^{+\infty} R I^2(t)\,dt = \frac{f^2}{R}\int_\phi^{+\infty} e^{-2t/RC}\,dt = \frac{1}{2} C f^2$$

$$I \times \quad f = \frac{Q(t)}{C} + R I(t) \quad \times I$$

$$f I = \frac{Q(t)\, I(t)}{C} + R I^2(t)$$



$$W_g = f I = f I_0 e^{-t/RC}$$

$$P_J = R I^2 = R I_0^2 e^{-2t/RC}$$

$$P_C = \frac{1}{2} Q(t) \cdot I(t)$$

$$W_g = P_J + P_C$$