AVVISO: da sabato 3 marzo 2021 ore 9,00 riprende il ricevimento studenti su Zoom, aula lab3:

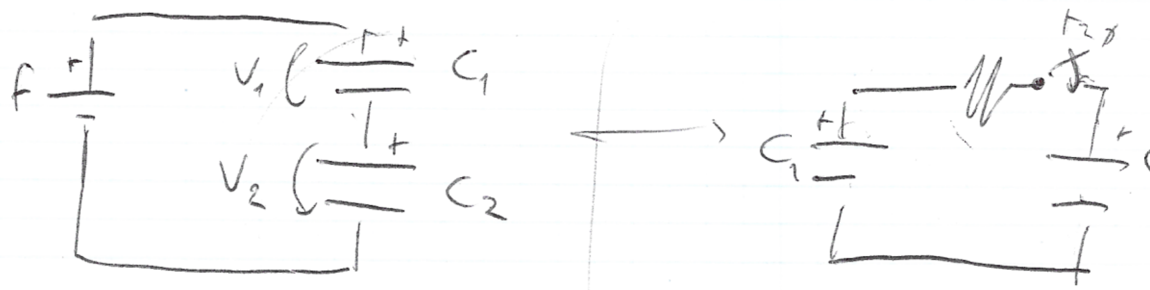
https://zoom.us/my/aula.lab3

ES #17 Transitori RC; campo $\vec{B}$ e forza di Lorentz

11/11/2021

(1)

(12-3)



(1) Q_0 carica finale fase (1)

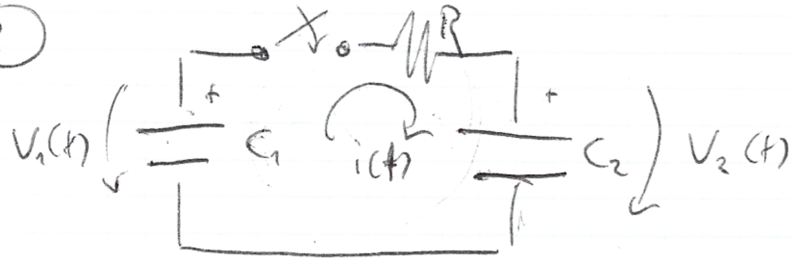
$$Q_0 = V_1 C_1 = V_2 C_2$$

$$f = V_1 + V_2 = Q_0 \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{1/C_s} = Q_0 \frac{C_1 + C_2}{C_1 C_2}$$

$$Q_0 = f C_1 C_2 / (C_1 + C_2) = f C_s$$

Energia $U_s = \frac{1}{2} \frac{Q_0^2}{C_s}$

(2)



$$0 = -V_1(t) + V_2(t) + R\,i(t)$$

$$\frac{Q_1(t)}{C_1} - \frac{Q_2(t)}{C_2} = R\,i(t)$$

derivo

$$i(t) = -\frac{dQ_1(t)}{dt} = \frac{dQ_2(t)}{dt}$$

$$\frac{1}{C_1}\frac{dQ_1}{dt} - \frac{1}{C_2}\frac{dQ_2}{dt} = R\frac{di}{dt}$$

$$-\frac{i(t)}{C_1} - \frac{i(t)}{C_2} = R\,\frac{di(t)}{dt}$$

$$-\frac{1}{RC_s}\, i(t) = \frac{di(t)}{dt}$$

$$-\frac{1}{RC_s}\int dt = \int \frac{di}{i} \rightarrow \quad i(t) = A\, e^{-t/RC_s} \qquad \tau = RC_s$$

$$t=0 \quad R\, i(0) = \left(\frac{1}{C_1} - \frac{1}{C_2}\right) Q_0 = V_{10} - V_{20} = RA$$

$$\boxed{A = (V_{10}-V_{20})/R} = I_0$$

$$i(t) = I_0\, e^{-t/RC_s} = \frac{V_{10}-V_{20}}{R}\, e^{-t/RC_s}$$

$$i(t) = \frac{dQ_2}{dt} = \quad \Rightarrow \quad Q_2(t) = \int i(t')\,dt' = -\frac{V_{10}-V_{20}}{\cancel{R}}\,\cancel{R}C_s\, e^{-t/RC_s} + K$$

$$Q_2(0) = Q_0 = -C_s(V_{10}-V_{20}) + K \Rightarrow K = Q_0 + C_s(V_{10}-V_{20})$$

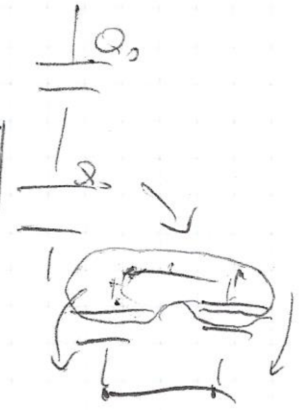
$$\boxed{Q_2(t) = Q_0 + \underbrace{C_s(V_{10}-V_{20})}_{>0 \text{ se } C_1 < C_2}\left(1 - e^{-t/RC_s}\right)}$$

$$Q_{2\infty}(t\to+\infty) = Q_0 + C_s(V_{10}-V_{20}) = 2Q_0\,\frac{C_2}{C_1+C_2}$$

$$V_{10} = Q_0/C_1 \qquad V_{20} = Q_0/C_2$$

$$\boxed{Q_1(t) + Q_2(t) = Q_{10} + Q_{20} = 2Q_0}$$

$$\boxed{Q_1(t) = 2Q_0 - Q_2(t) = Q_0 - C_s(V_{10}-V_{20})\left(1 - e^{-t/RC_s}\right)}$$



$$V_1(t) = \frac{Q_1(t)}{C_1} \qquad V_{1\infty} = \frac{Q_{1\infty}}{C_1} = \frac{2Q_0}{C_1+C_2}$$

$$V_2(t) = \frac{Q_2(t)}{C_2} \qquad V_{2\infty} = \frac{Q_{2\infty}}{C_2} = \frac{2Q_0}{C_1+C_2}$$

$$U_f = \frac{1}{2}\frac{(2Q_0)^2}{C_{//}} = \longrightarrow = 2Q_0^2/(C_1+C_2)$$

$\bar{B}$ – Forza di Lorentz

(13-1)

$$\bar{F}_L = q\bar{v} \times \bar{B} \qquad \bar{B}' = \bar{B} + \delta\bar{B}$$

$$\delta\bar{B} \parallel \bar{v}$$

$q \to \bar{v}_1$

$\searrow \bar{v}_2$

$\bar{v}_1 \perp \bar{v}_2$

$$\begin{cases} \bar{F}_1 = q\bar{v}_1 \times \bar{B} & \times\, \bar{v}_1 / q v_1^2 \\ \bar{F}_2 = q\bar{v}_2 \times \bar{B} & \times\, \bar{v}_2 / q v_2^2 \end{cases}$$

$$(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$$

$$(\bar{v}_1 \times \bar{B}) \times \bar{v}_1 = v_1^2 \bar{B} - (\bar{B} \cdot \bar{v}_1)\bar{v}_1$$

$$\Rightarrow \begin{cases} \dfrac{\bar{F}_1 \times \bar{v}_1}{q v_1^2} = \bar{B} - \dfrac{\bar{B} \cdot \bar{v}_1}{v_1^2}\bar{v}_1 \\[2ex] \dfrac{\bar{F}_2 \times \bar{v}_2}{q v_2^2} = \bar{B} - \dfrac{\bar{B} \cdot \bar{v}_2}{v_2^2}\bar{v}_2 \quad \leftarrow \quad \cdot\bar{v}_1 \end{cases}$$

$$\bar{B} \cdot \bar{v}_1 = \frac{\bar{F}_2 \times \bar{v}_2}{q v_2^2} \cdot \bar{v}_1 + \frac{\bar{B} \cdot \bar{v}_2}{v_2^2} \bar{v}_2 \cdot \bar{v}_1 \to \emptyset$$

$$\bar{B} = \frac{\bar{F}_1 \times \bar{v}_1}{q v_1^2} + \left[\frac{(\bar{F}_2 \times \bar{v}_2) \cdot \bar{v}_1}{q v_2^2} \right] \frac{\bar{v}_1}{v_1^2}$$

$$\bar{B} = \frac{1}{q v_1^2} \left[\bar{F}_1 \times \bar{v}_1 + \frac{(\bar{F}_2 \times \bar{v}_2) \cdot \bar{v}_1}{v_2^2} \bar{v}_1 \right]$$

Spettrometro di Dempster (1918)

$$\bar{F}_L = q\bar{v} \times \bar{B}$$

$\bar{v}\,dt$

$$d\mathcal{L} = \bar{F}_L \cdot d\bar{x} = q(\bar{v} \times \bar{B}) \cdot \bar{v}\,dt = 0$$

↳ $\bar{B}$ non lavora → modifica la traiettoria di q (orientazione)

MOTO DI CICLOTRONE

R_c, v, v = cost.

$$a_c = \frac{v^2}{R_c} = \frac{F_L}{m} = \frac{qvB}{m}$$

R_c = raggio di ciclotrone

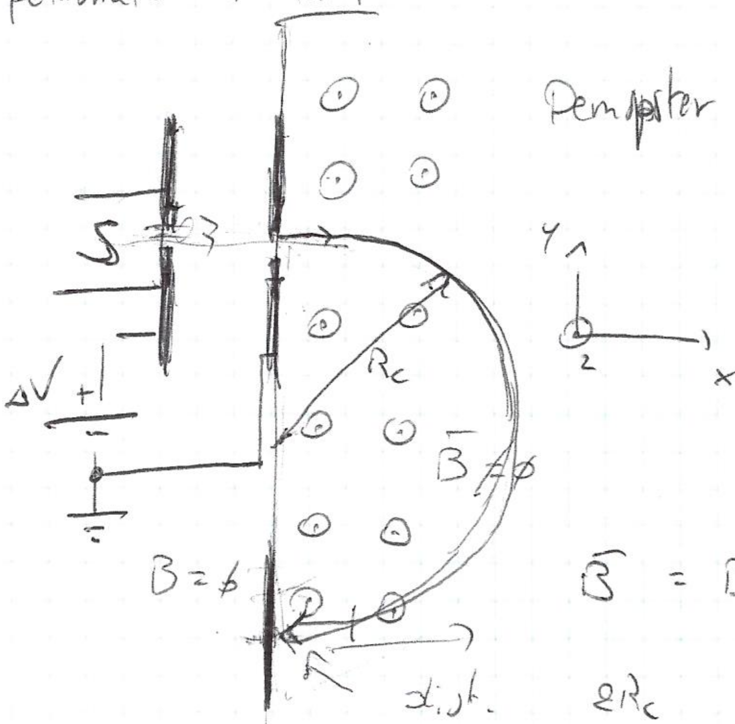
$$R_c = \frac{mv}{qB}$$

$$\boxed{\omega_c = \frac{qB}{m}}$$ freq. di ciclotrone [rad/s]

$$\left(R_c = v/\omega_c \right)$$

$$\nu_c = \frac{\omega_c}{2\pi} \quad [Hz]$$

spettrometro di massa



Dempster: part. isoenergetiche

$^{12}C^+$, $^{13}C^+$, $^{14}C^+$

$\Delta V = 1\,kV$

$q = +e$

$E_K = q\Delta V = 1\,keV$

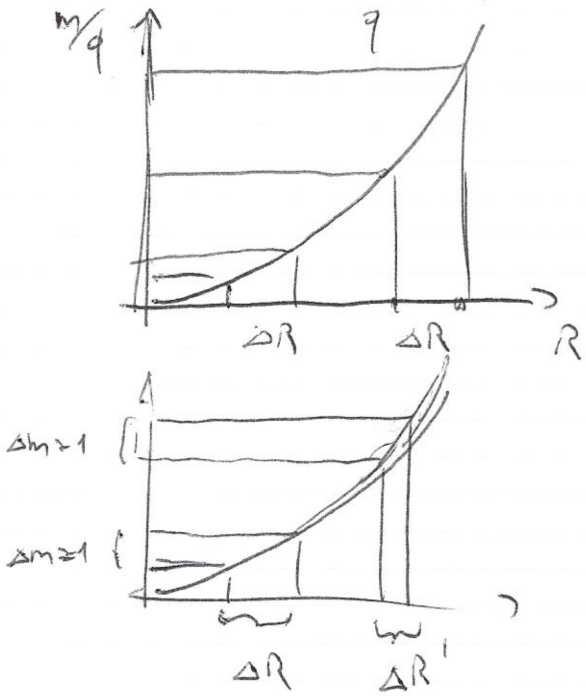
$$\bar{B} = B\hat{e}_z$$

$$R = \frac{mv}{qB} = \frac{m}{qB}\left(\frac{2E_k}{m}\right)^{\frac{1}{2}} = \left(\frac{m}{q}\right)^{\frac{1}{2}}\left(\frac{2\Delta V}{B^2}\right)^{\frac{1}{2}}$$ (5)

$E_k = q\Delta V$

Scala quadratica

$\frac{m}{q} \propto R^2$



$m_2 - m_1 = \Delta m \propto R_2^2 - R_1^2$

$\Rightarrow$

$$\frac{\Delta m}{m} = \frac{\Delta(R^2)}{R^2}$$

$^{12}C = 6p^+ + 6n =$

12 nucleoni

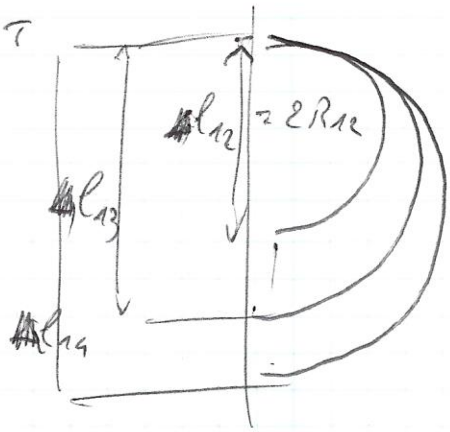
$\bar{m}$ = massa nucleone

$A = \#p + \#n$

$m = A\bar{m} = A \cdot 1.67 \cdot 10^{-27}\ Kg$

$$R = \left(\frac{m}{q}\right)^{\frac{1}{2}} (2\Delta V)^{\frac{1}{2}} / B = \sqrt{A} \cdot 4.57\ cm$$

$B = 0.1\ T$



$R_{C_{12}} = 15.83\ cm$

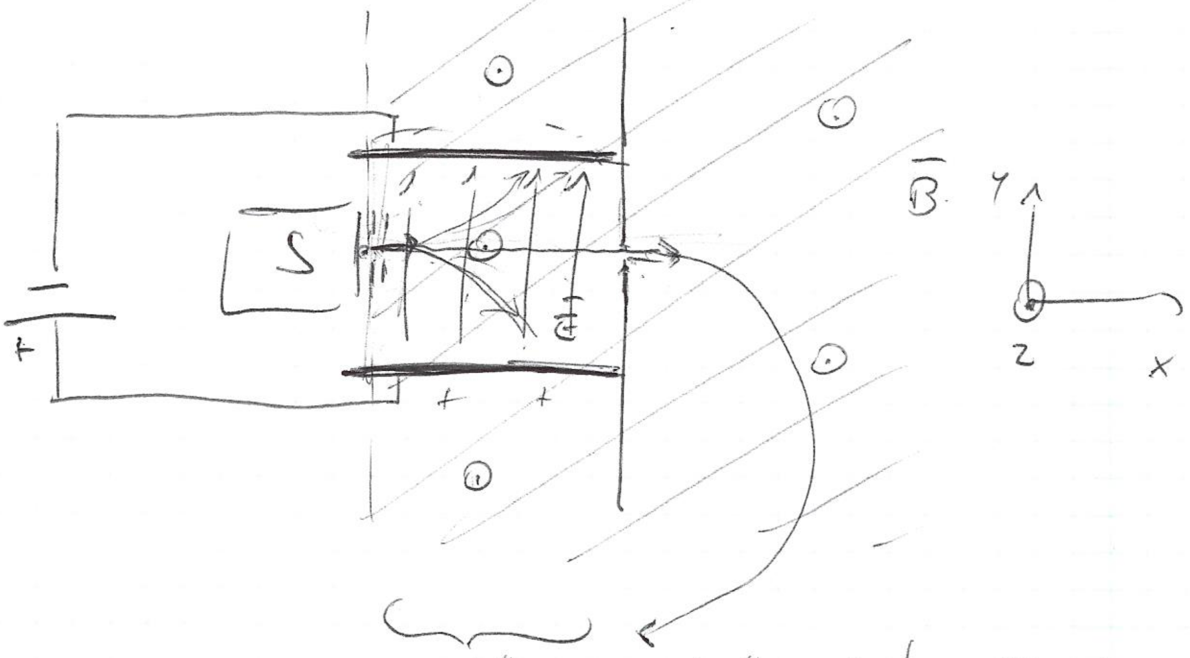
$R_{C_{13}} = 16.48\ cm$

$R_{C_{14}} = 17.10\ cm$

$l_{13} - l_{12} = 1.3\ cm$

$l_{14} - l_{13} = 1.24\ cm$

Spettrometro di massa di Bainbridge (1933)



selettore di velocità (Wien filter)

$$\bar{F}_L = q(\bar{E} + \bar{v}\times\bar{B}) \qquad \bar{E} = E_y \hat{e}_y$$

$$\hat{e}_x \times \hat{e}_z = -\hat{e}_y \qquad \bar{B} = B_z \hat{e}_z$$

$$\bar{v} = v_x \hat{e}_x$$

$$F_y = \emptyset = qE_y - q v_x B_z$$

$$E_y = v_x B_z \qquad \boxed{v_x = \frac{E}{B}}$$

$B = 0.1\ T$

${}^{12}C^+ \quad {}^{13}C^+ \quad {}^{14}C^+$

$v = 10^5\ m/s \qquad v_x = \frac{E}{B} \rightarrow E = v_x B_z = 10^4\ V/m$

$$a_c = \frac{v^2}{R_c} = \frac{F_L}{m} = \frac{qvB}{m}$$

$$R_c = \frac{mv}{qB} = \frac{m}{q} \cdot \frac{v}{B}$$

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$m = \bar{m} A$

$R_c = A \frac{\bar{m} v}{qB} = 1.04 \cdot A \text{ cm}$

$R_{C12} = 12.48 \text{ cm}$

$R_{C13} = 13.52 \text{ cm}$

$R_{C14} = 14.56 \text{ cm}$

$2R_{C13} - 2R_{C12}$

$l_{13} - l_{12} = 2.08 \text{ cm}$

$l_{14} - l_{13} = 2.08 \text{ cm}$

$E_n = 624 / 676 / 728 \text{ eV}$
(12) (13) (14)