ES. #18 Forza di Lorentz; calcolo di $\bar{B}$ tramite legge di Biot-Savart

15/11/2021

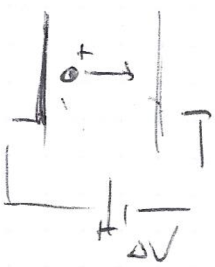
(1)

(13-4) Ciclotrone

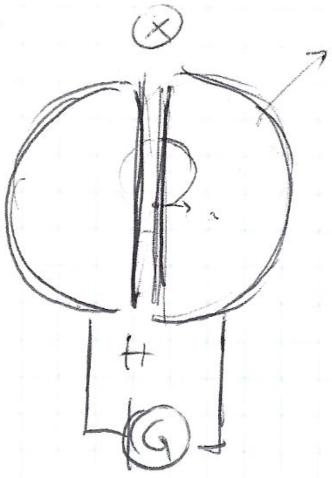
$\bar{B}_0$ $\qquad$ $\bar{F}_L = q\bar{v}\times\bar{B}$

moto di ciclotrone $\qquad \omega_c = \frac{qB}{m}$

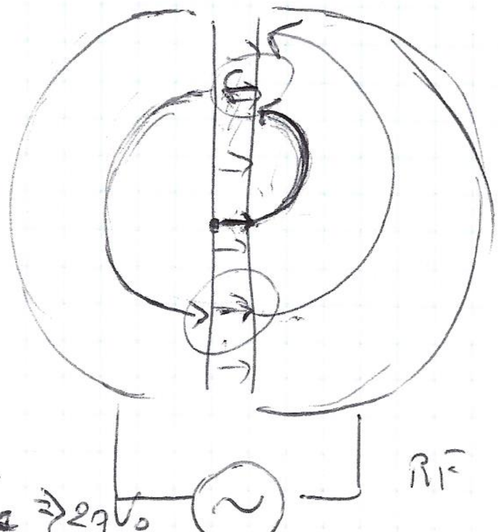
Acceleratore di particelle (cariche)

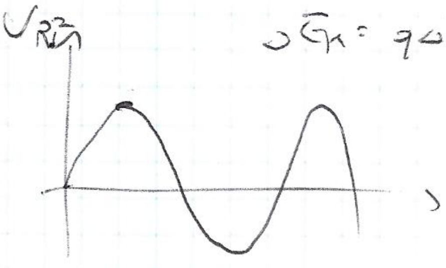


$E_k = q\Delta V$



dees "D"





$\Delta E_k = q\Delta V = qV_0$

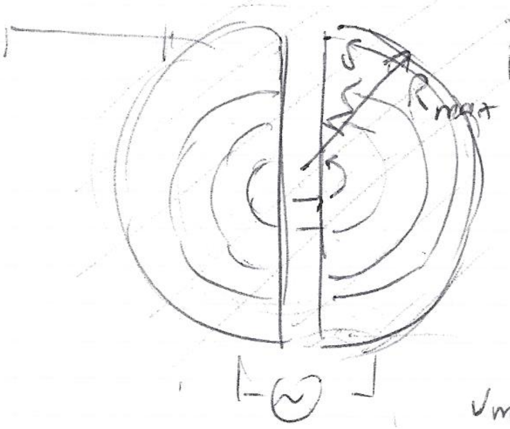
$V_{RF}(t) = V_0 f(\omega_{RF} t)$

Cond. ideale

$\omega_{RF} = \omega_c$

$T_{RF} = T_c \Rightarrow 2qV_0$

RF (radio frequency)



$\vec{B}$ uniforme

R_{max}

$$R_{max} = \frac{m v_{max}}{qB}$$

$$v_{max} = \frac{qB \cdot R_{max}}{m}$$

$$E_{Kmax} = \frac{1}{2} m v_{max}^2 = \frac{q^2 B^2 R_{max}^2}{2m} = 2 n q V_0$$

n giri (rivoluzioni)

$$n = \frac{q B^2 R_{max}^2}{4 m V_0}$$

$$n T_c = n \frac{2\pi}{\omega_c} = \frac{\pi q B R_{max}^2}{2 V_0}$$

$v \ll c$ se non vale $m = \gamma m_0 = \frac{1}{\sqrt{1-\beta^2}} m_0$

$\hookrightarrow \beta \ll 1$ $\beta = v/c$

$$\omega_c = \frac{qB}{\gamma m_0}$$

$\rightarrow$ V_{RF}, $\omega_{RF}(t)$ sincrociclotrone

$\rightarrow$ $B(r)$ ciclotrone isocrono

Ex.: $^{12}C^+$ $R_{max} = 1m$ $B = 1.5\,T$ $V_0 \cong 10\,kV$

$E_{Kmax} = q^2 B^2 R_{max}^2 / 2m = 1.44 \cdot 10^{-12}\,J = 8.98\,MeV$

$n = E_{Kmax} / 2 n q V_0 = 449$ revolutions

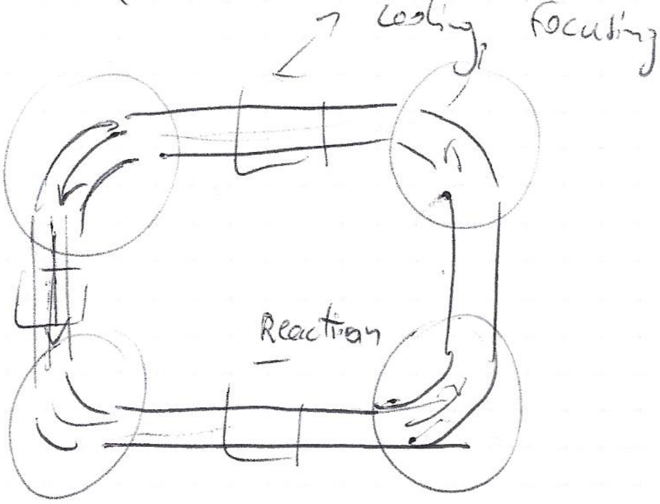
$\left(\frac{\omega}{2\pi} = \nu\right)$ $\nu_{RF} = \nu_c = 1.91\,MHz$ $\rightsquigarrow$ $T_c = 5.25 \cdot 10^{-7}\,s$

$t_{tot} = n T_c = 2.36 \cdot 10^{-4}\,s = 236\,\mu s$

$v_{max} = \sqrt{2 E_{max} / m} = 1.2 \cdot 10^{7}\,m/s$ $c = 3 \cdot 10^8\,m/s$

$\beta = 0.04$ $\gamma = 1.0008$

Sincrotrone



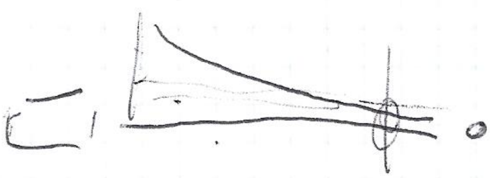
Adroterapia   terapia tumorali con Ioni (p^+, $^{12}C^+$)

CNAO (Pavia)

LINAC, Cycl. → paziente

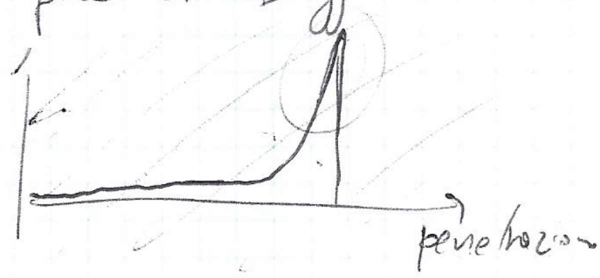
vs. radioterapia (X-rays)

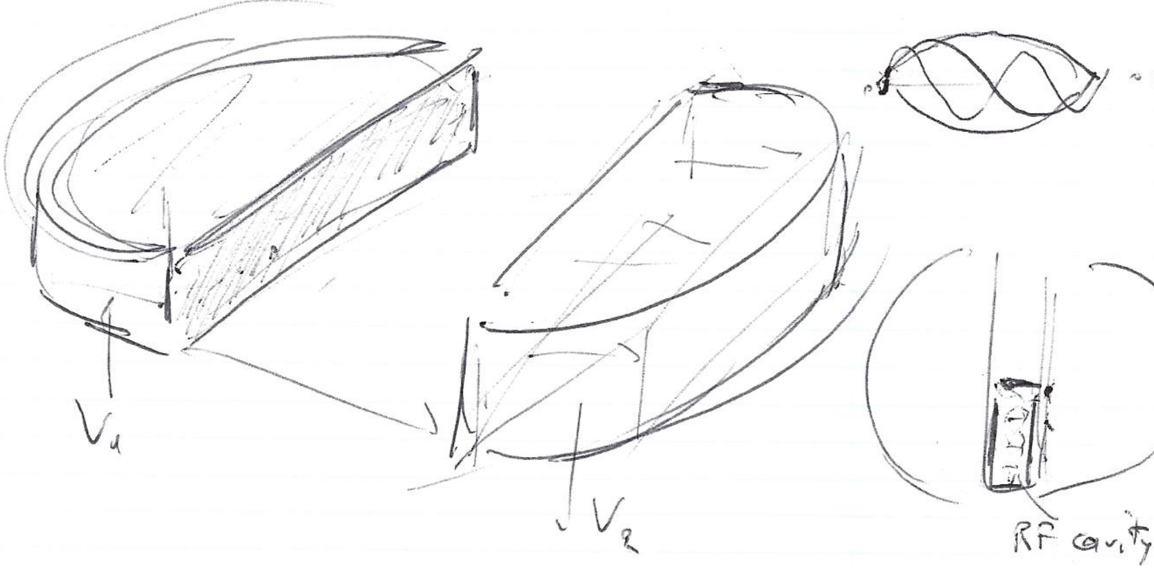
"auto che inchioda"



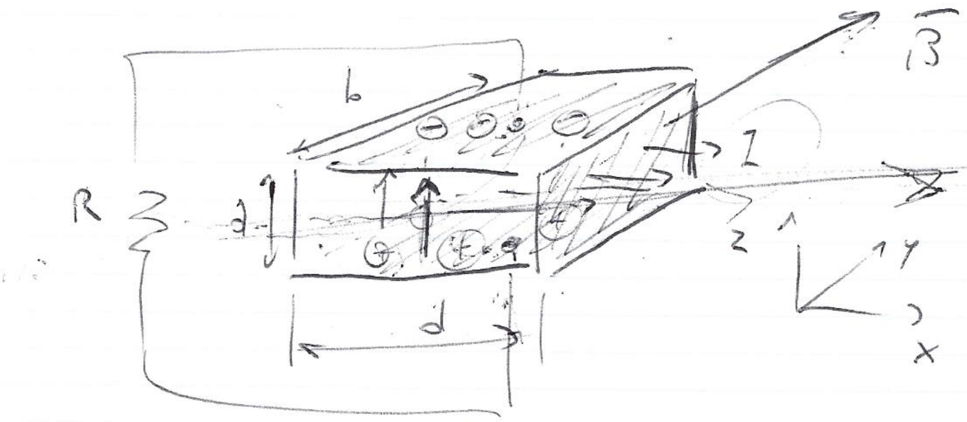
"auto a piena vel. contro un muro"

"picco di Bragg"





(13-6) Effetto Hall



$a = 5\ mm$

$b = 2\ mm$

$d = 30\ mm$

$I = 3A$

$B = 2T$

$R = 10^{-4}\ \Omega$

$\longrightarrow\ \oplus$

I

$\longleftarrow$

e^-

$$\bar{F}_L = q\bar{E} + q\bar{v}\times\bar{B}$$

$$\phi = (+) \begin{cases} \bar{F}_{Lm} \quad q\bar{v}_d \times \bar{B} = (-e)(-v_d\hat{e}_x)\times\bar{B} = e v_d B \underbrace{\boxed{\hat{e}_x \times \hat{e}_y}}_{\hat{e}_z} \\ \bar{F}_{Le} = (-e)\,E\hat{e}_z = -e E_0 E\hat{e}_z \end{cases}$$

$$e v_d B - e E_0 = 0 \qquad E = v_d B$$

potenziale di Hall $V_H = Ea = v_d B a$

$$I = J ab = q n v_d ab \qquad V_H = \frac{BI}{qnb} = R_H \frac{BI}{b}$$

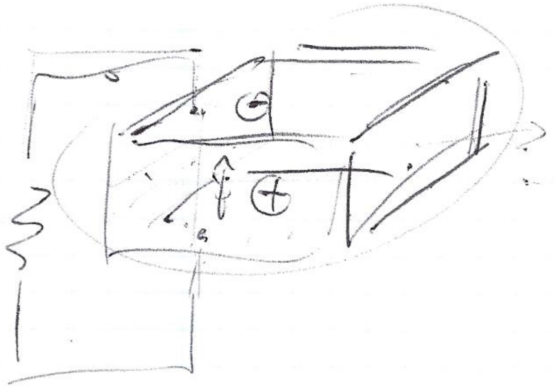
$R_H \doteq \frac{1}{qn}$ costante di Hall

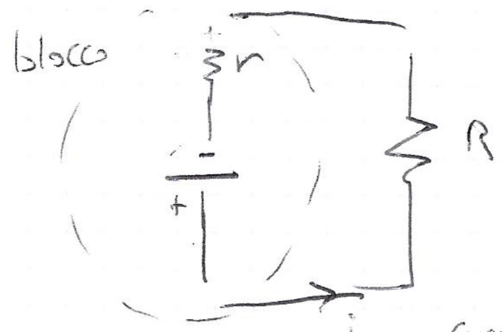
densità di carica dei portatori nel materiale

$[m^3/C]$

$n = 8.5 \cdot 10^{28}\ m^{-3}$ e^-

$V_H = -2.21 \cdot 10^{-7}\ V$





$V_H = (r+R)\, i_H$

i_H corrente di Hall

$r = \rho \frac{\ell}{S} = \rho \frac{a}{bd} = 1.39 \cdot 10^{-6}\ \Omega$

$\rho_{Cu} = 1.67 \cdot 10^{-8}\ \Omega \cdot m$

$i_H = \frac{V_H}{r+R} = 2.18 \cdot 10^{-3}\ A$

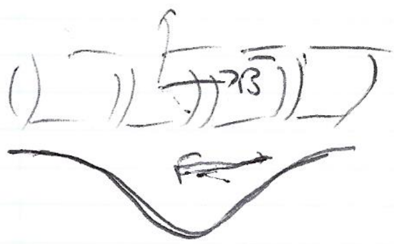
→ Sonde di Hall (Hall probe)

(13-5) Trappola di Penning (Penning trap) 6

$\vec{E}, \vec{B}$ apparato di CONFINAMENTO

(statici)

$\rightarrow \hat{z}$

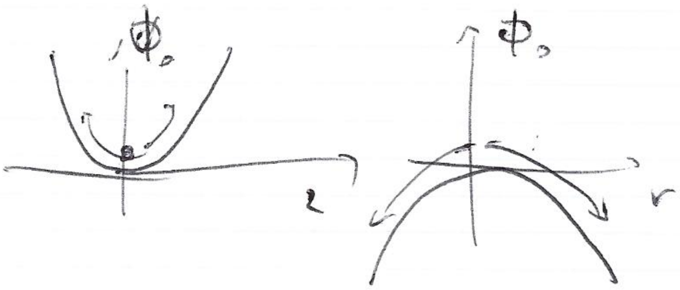


$$\begin{cases} \Phi_0(r,z) = \dfrac{V_0}{2d_0^2}\left(z^2 - \dfrac{1}{2}r^2\right) = \\ \qquad = \dfrac{V_0}{2d_0^2}\left(z^2 - \dfrac{1}{2}x^2 - \dfrac{1}{2}y^2\right) \\ \vec{B}_0 = B_0 \hat{e}_z \end{cases}$$

$r^2 = x^2 + y^2$

$$E_{0x,y} = -\frac{\partial \Phi_0}{\partial x,y} = \frac{V_0}{2d_0^2}\, x(y)$$

$$E_{0z} = -\frac{\partial \Phi_0}{\partial z} = -\frac{V_0}{d_0^2}\, z$$



q, m $\quad \vec{v} = v_x \hat{e}_x + v_y \hat{e}_y + v_z \hat{e}_z = \dot{x}\hat{e}_x + \dot{y}\hat{e}_y + \dot{z}\hat{e}_z$

$$\vec{F}_L = -q\vec{\nabla}\Phi_0 + q\vec{v}\times\vec{B}_0$$

$$\begin{cases} m\ddot{x} = \dfrac{qV_0}{2d_0^2}x + qB_0\dot{y} \\ m\ddot{y} = \dfrac{qV_0}{2d_0^2}y - qB_0\dot{x} \\ m\ddot{z} = -\dfrac{qV_0}{d_0^2}z \end{cases} \rightsquigarrow \begin{cases} \ddot{x} = \dfrac{\omega_z^2}{2}x + \omega_c\dot{y} \\ \ddot{y} = \dfrac{\omega_z^2}{2}y - \omega_c\dot{x} \\ \ddot{z} = -\dfrac{qV_0}{md_0^2}z = -\omega_z^2 z \end{cases}$$

$\omega_c = qB/m$ $\qquad$ $\omega_z = \sqrt{qV_0/md_0^2}$

$u \doteq x + iy$

$$\underbrace{\ddot{x} + i\ddot{y}}_{\ddot{u}} = \frac{\omega_z^2}{2}\underbrace{(x+iy)}_{u} + \omega_c(\dot{y} - i\dot{x}) \qquad -i(\dot{x} + i\dot{y}) = -i\dot{u}$$

$$\boxed{\ddot{u} + i\omega_c \dot{u} - \frac{\omega_z^2}{2} u = 0}$$

$$u \sim \exp(-i\omega t) \qquad \omega \in \mathbb{C}$$

$$(-i\omega)^2 \cancel{u} - i\omega \cdot i\omega_c \cancel{u} - \frac{\omega_z^2}{2}\cancel{u} = 0$$

$$\omega^2 - \omega_c\omega + \frac{\omega_z^2}{2} = 0$$

$$\omega = \frac{\omega_c \pm \sqrt{\omega_c^2 - 2\omega_z^2}}{2} \rightsquigarrow \omega_\pm = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

ω_+ = modified (reduced) cyclotron freq.

ω_- - magnetron frequency

$\Delta \geq 0$ $\qquad$ $\omega_c^2 \geq 2\omega_z^2$

$$\boxed{\omega_- \ll \omega_z \ll \omega_+, \omega_c} \qquad \omega_+ \to \omega_c$$

$$\omega_+ + \omega_- = \omega_c$$

$$\omega_+ \omega_- = \omega_z^2/2$$

$$\omega_- = \frac{\omega_z^2}{2\omega_+} \simeq \frac{\omega_z^2}{2\omega_c} = \frac{qV_0}{md_0^2} \cdot \frac{m}{qB_0} = \frac{V_0}{2B_0 d_0^2}$$

$$\boxed{\omega_c^2 = \omega_z^2 + \omega_+^2 + \omega_-^2}$$ Brown - Gabrielse invariance theorem

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$$u(t) = A_+ e^{-i\omega_+ t} + A_- e^{-i\omega_- t}$$

$$A_\pm = A_{0\pm} e^{-i\alpha_{0\pm} t}$$

$\longrightarrow R_\pm$ composizione di due moti circolari

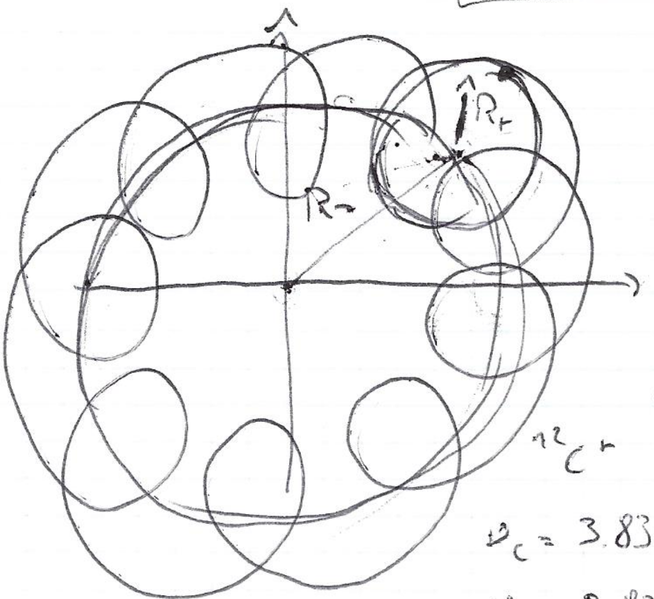
$u \longrightarrow x + iy$ $\qquad e^{ik} = \cos k + i \sin k$

$$u(t) = R_+ \exp[-i(\omega_+ t + \alpha_{0+})] + R_- \exp[-i(\omega_- t + \alpha_{0-})] =$$

$$= R_+ \cos(\omega_+ t + \alpha_{0+}) - i R_+ \sin(\omega_+ t + \alpha_{0+}) +$$

$$+ R_- \cos(\omega_- t + \alpha_{0-}) - i R_- \sin(\omega_- t + \alpha_{0-})$$

$$x(t) = R_+ \cos(\omega_+ t + \alpha_{0+}) + R_- \cos(\omega_- t + \alpha_{0-})$$
$$y(t) = -R_+ \sin(\quad) - R_- \sin(\quad)$$



epicicloide (trocoide)

$$\omega_c = \frac{qB}{m}$$

Ex: $V_0 = 12\,V$; $B = 3\,T$; $d_0 = 3\,mm$

$^{12}C^+$

$\nu_c = 3.835\ MHz$

$\nu_+ = 3.832\ MHz$

$\nu_z = 156.252\ kHz$

$\nu_- = 3.186\ kHz$

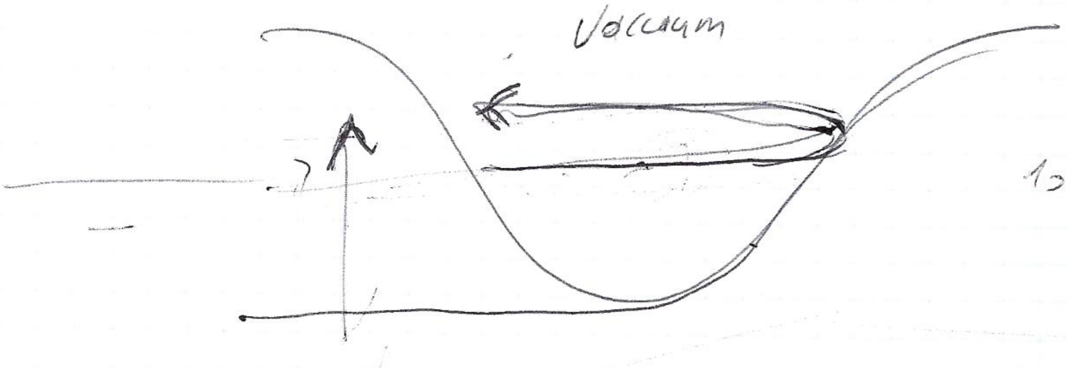
e^-

$\nu_c = 83.891\ GHz$

$\nu_+ = 83.891\ GHz$

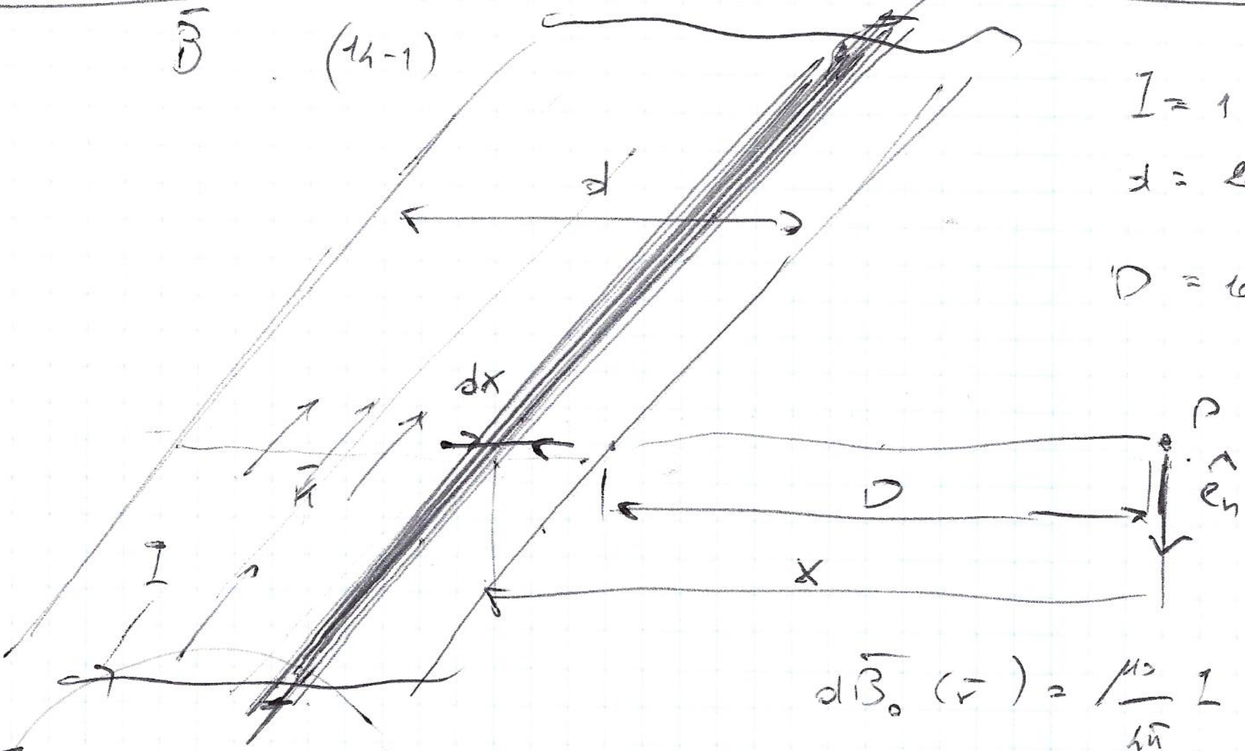
$\nu_z = 23.110\ MHz$

$\nu_- = 3.183\ kHz$



10^{-11} mbar

$\bar{B}$ (1h-1)

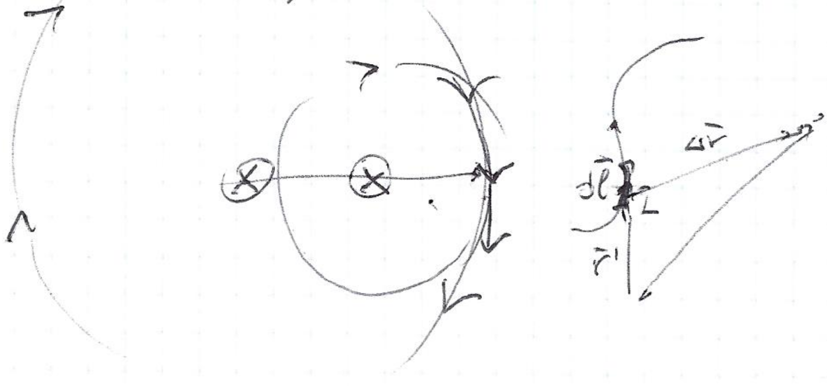


$I = 1\ A$

$d = 20\ mm$

$D = 100\ mm$

$$d\bar{B}_0(\bar{r}) = \frac{\mu_0}{4\pi} I \frac{d\bar{l} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

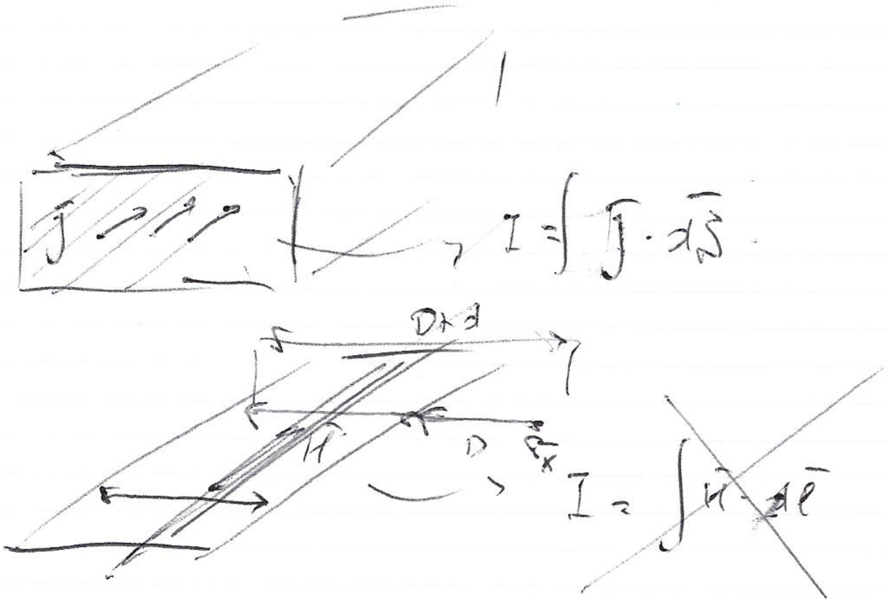


$\Delta\bar{r} = \bar{r} - \bar{r}'$

$$\bar{B}(\bar{r}) = \frac{\mu_0 I}{2\pi r} \hat{e}_\varphi$$

dI di una striscia

$\bar{K}$ $[A/m]$ $dI = K\,dx = \frac{I}{d}\,dx$



$\to I = \int \bar{J} \cdot d\bar{S}$

$\to I = \cancel{\int \bar{H} \cdot d\bar{l}}$

$$dB_{on} = \frac{\mu_0\, dI}{2\pi r} = \frac{\mu_0 I}{2\pi d} \frac{dr}{r}$$

$$B_{on} = \int dB_{on} = \frac{\mu_0 I}{2\pi d} \int_D^{D+d} \frac{dr}{r} = \frac{\mu_0 I}{2\pi d} \log\left(\frac{D+d}{D}\right)$$

$$B_{on}(D) = 1.82 \cdot 10^{-6}\,T$$