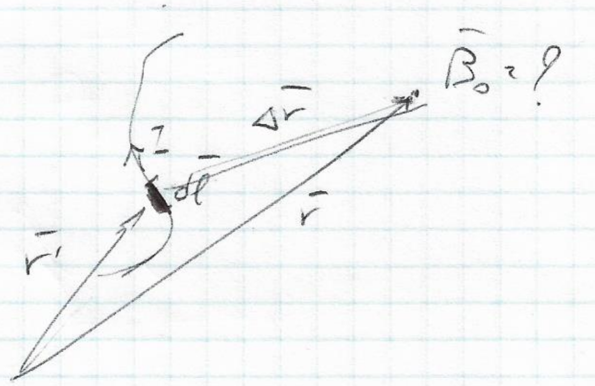
ES #19 Calcolo di $\bar{B}$ con legge di Biot-Savart e di Ampère

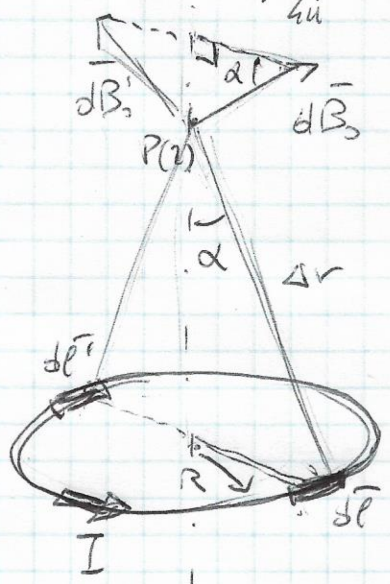
25/III/2021

(1)

$$d\bar{B}_0 = \frac{\mu_0}{4\pi}\frac{I\, d\bar{l}\times\Delta\bar{r}}{\Delta r^3}$$



(14-2)



$$dB_{0z} = dB_0 \sin\alpha$$

$$B_{0z}(z) = \int dB_{0z} = \frac{\mu_0 I}{4\pi}\int_\ell \frac{\overbrace{dl}^{R\,d\theta}}{\Delta r^2}\sin\alpha = \frac{\mu_0 I R \sin\alpha}{2\Delta r^2}$$

$$\Delta r = (R^2+z^2)^{\frac{1}{2}}$$

$$\sin\alpha = R/\Delta r = R/(R^2+z^2)^{\frac{1}{2}}$$

$$\boxed{B_{0z}(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}}$$

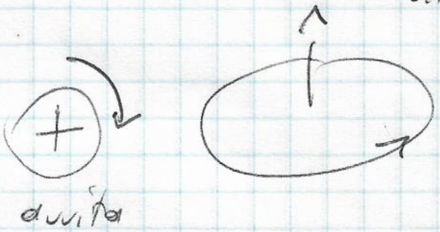
$R = 10$ cm ; $I = 100$ mA

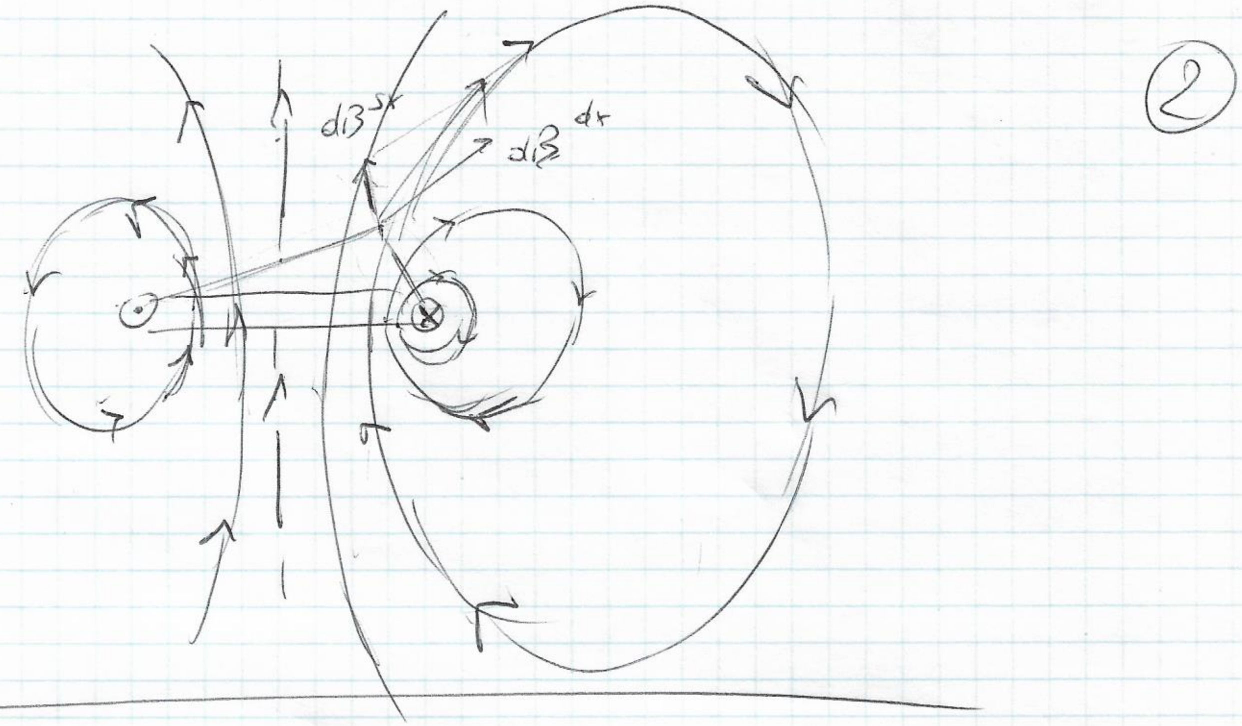
$B_{0z}(z=\emptyset) = 6.28\cdot 10^{-7}$ T

$\bar{m} = ?$ $\qquad \bar{m} = I S \hat{e}_z$

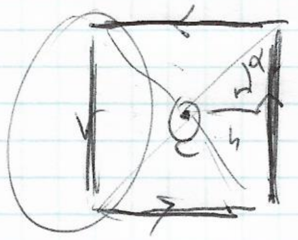
$m = I\pi R^2 = 3.14\cdot 10^{-3}$ A$\cdot$m^2

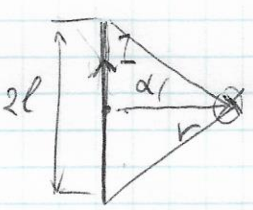
$$B_0(\bar{z}) = \frac{\mu_0}{2\pi}\frac{\bar{m}}{(R^2+z^2)^{3/2}}\hat{e}_z$$





(14-3) Spira quadrata, I, L lato $\bar{B}_0$ centro





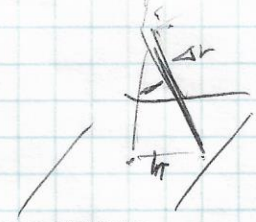
$$B_0(r) = \frac{\mu_0 I}{2\pi r} \sin\alpha = \cancel{\frac{\mu_0 I}{2\pi r}\frac{l}{(r^2+l^2)^{1/2}}}$$

$$= \frac{\mu_0 I}{2\pi r} \frac{l}{(r^2+l^2)^{\frac{1}{2}}}$$

$$r = h = L/2 = l; \quad \alpha = \pi/4$$

$$B_{0L}(C) = \frac{\mu_0 I}{2\pi h} \sin\alpha = \frac{\mu_0 I}{\pi L} \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \frac{\mu_0 I}{\pi L}$$

$$B_{0Z}(C) = 4 B_{0L} = \frac{2\sqrt{2}}{\pi} \frac{\mu_0 I}{L}$$



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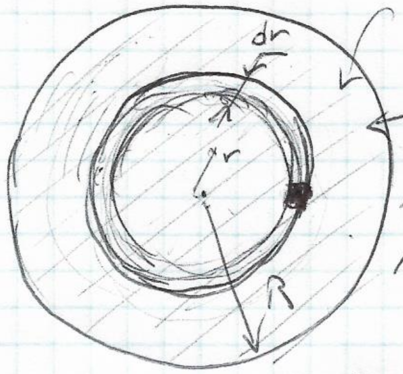
Generalizzare a spira = poligono regolare n lati

$$2\alpha = \frac{2\pi}{n} \Rightarrow \alpha = \pi/n \quad \cdot\ r = h = R\cos\alpha$$

$$B_{0Z}(C) = n B_{0L} = n \frac{\mu_0 I}{2\pi h} \sin\alpha = \frac{\mu_0 I}{2\pi R}\, n\, tg(\pi/n) = \frac{\mu_0 I}{2R} \frac{tg\, y}{y}$$

$$y = \pi/n \quad \lim_{y \to 0} tg\, y / y = (y + y^3/3 + \dots)/y = 1 \longrightarrow B_{0e}(C) = \mu_0 I / 2R$$

(14-4) Disco di Rowland



σ uniforme $= 10^{-6}\ C/m^2$; $R = 10\ cm$

dielettrico

$\omega = \dot{\vartheta} =$ costante $= 2\pi \cdot 10^{3}$ rad/s

$$dq = \sigma\, dS = \sigma\, 2\pi r\, dr \qquad T = \frac{2\pi}{\omega}$$

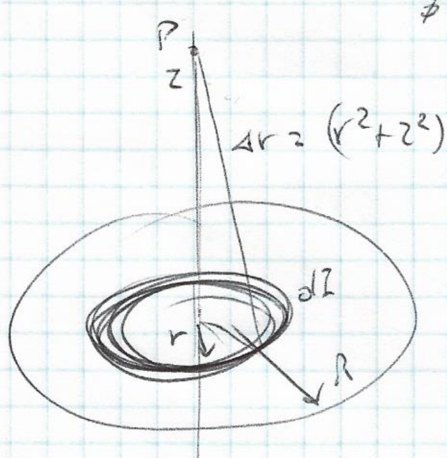
$$dI = \frac{dq}{\text{tempo}} = \frac{dq}{T} = dq\,\frac{\omega}{2\pi} =$$

$$= \sigma\, 2\pi r\, dr\, \frac{\omega}{2\pi} = \sigma \omega r\, dr$$

$$d\bar{m} = dI\, S\, \hat{e}_z = \sigma\omega r\, dr\, \pi r^2\, \hat{e}_z = \pi\sigma\omega r^3 dr\, \hat{e}_z$$

$$\bar{m} = \int d\bar{m} = \int_0^R \pi\sigma\omega\, r^3 dr\, \hat{e}_z = \sigma\omega\,\frac{R^4}{4}\,\hat{e}_z = \frac{Q\omega R^2}{4}\,\hat{e}_z = 4.93\cdot 10^{-7}\ A\cdot m^2$$

$$Q \doteq \sigma \pi R^2$$



$\Delta r = (r^2+z^2)^{\frac{1}{2}}$

$$dB_{0z}(z) = \left| \frac{\mu_0\, dI}{2}\, \frac{r^2}{(r^2+z^2)^{3/2}} \right| = \frac{\mu_0 \sigma\omega}{2}\, \frac{r^3\, dr}{(r^2+z^2)^{3/2}}$$

$$B_{0z}(z) = \int dB_{0z} = \frac{\mu_0\sigma\omega}{2} \int_0^R \frac{r^3}{(r^2+z^2)^{3/2}}\, dr$$

$$\int \frac{r^3}{(r^2+z^2)^{3/2}}\, dr = \int \underbrace{\frac{r}{(r^2+z^2)^{3/2}}}_{f'} \cdot \underbrace{r^2}_{g}\, dr = -\frac{1}{(r^2+z^2)^{\frac{1}{2}}} \cdot r^2 +$$

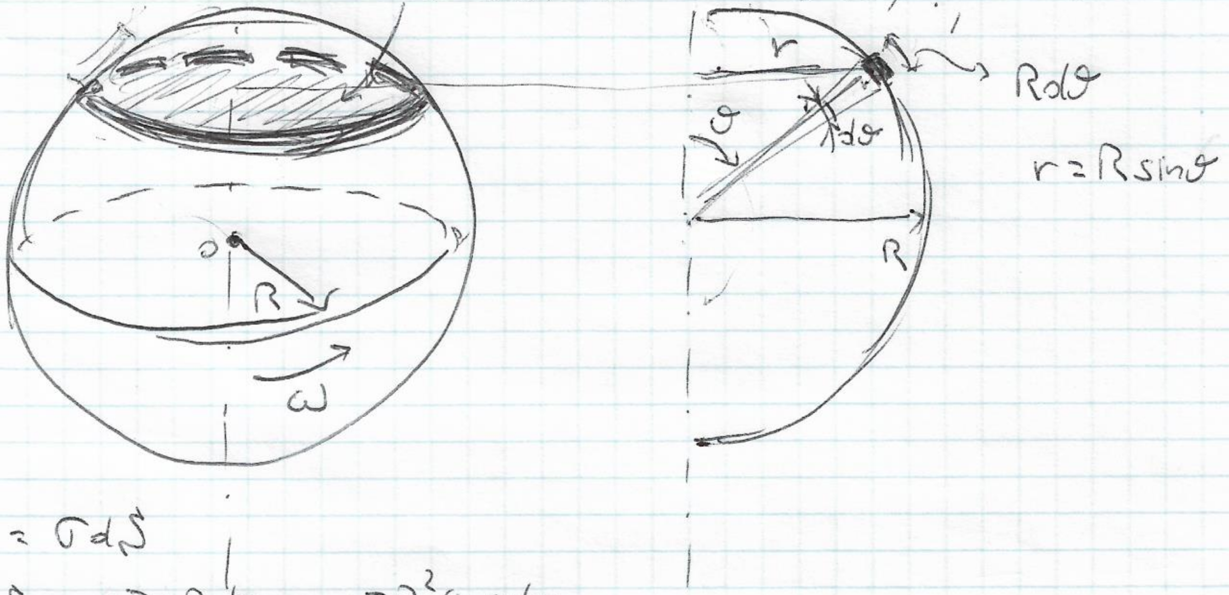
$$+\int \frac{1}{(r^2+z^2)^{\frac{1}{2}}}\, 2r\, dr = -\frac{r^2}{(r^2+z^2)^{1/2}} + 2\,(r^2+z^2)^{\frac{1}{2}} = \frac{r^2+2z^2}{(r^2+z^2)^{1/2}}$$

$$B_{0z}(z) = \frac{\mu_0 \sigma \omega}{2}\left[\frac{r^2+2z^2}{(r^2+z^2)^{1/2}}\right]_\phi^R = \frac{\mu_0\sigma\omega}{2}\left[\frac{R^2+2z^2}{(R^2+z^2)^{1/2}} - 2z\right]$$

$$B_{0z}(z=\phi) = \frac{\mu_0 \sigma \omega R}{2} = \mu_0 \frac{Q\omega}{2\pi R} = 4\pi^2 \cdot 10^{-11}\,T \sim 3.7\text{-}4 \cdot 10^{-10}\,T$$

(14-3) $\hat{e}_z$

$\vec{B}_0(0) = ?$ $\vec{m} = ?$



$dq = \sigma dS$

$dS = 2\pi r R d\vartheta = 2\pi R^2 \sin\vartheta d\vartheta$

$dq = 2\pi\sigma R^2 \sin\vartheta d\vartheta$

$dI = dq/T = dq\,\omega/2\pi = \sigma\omega R^2 \sin\vartheta d\vartheta$

$d\vec{m} = dI\, S\, \hat{e}_z = dI\, \pi r^2 \hat{e}_z = \pi\omega\sigma R^4 \sin^3\vartheta\, d\vartheta\, \hat{e}_z$

$$\vec{m} = \pi\omega\sigma R^4 \int_0^\pi \sin^3\vartheta d\vartheta = \pi\omega\sigma R^4 \int_\phi^\pi \underbrace{\sin\vartheta(1-\cos^2\vartheta)}_{\sin\vartheta\cdot\sin^2\vartheta} d\vartheta = \pi\omega\sigma R^4 \underbrace{\left[-\cos\vartheta + \tfrac{1}{3}\cos^3\vartheta\right]_\phi^\pi}_{4/3}$$

$$\vec{m} = \frac{4\pi}{3}\omega\sigma R^4 \hat{e}_z = \frac{Q\omega R^2}{3}\hat{e}_z$$

$Q = \sigma S_{sph} = \sigma 4\pi R^2$

$$dB_{0z}(\phi) = \frac{\mu_0\, dI}{2}\frac{r^2}{R^3} = \frac{\mu_0 I}{2}\frac{R^2\sin^2\vartheta}{R^3}\,\omega\sigma R^2 \sin\vartheta d\vartheta = \frac{\mu_0}{2}\omega\sigma R \sin^3\vartheta d\vartheta$$

$$B_{0z}(\phi) = \frac{\mu_0\omega\sigma R}{2}\int_\phi^\pi \sin^3\vartheta d\vartheta = \frac{2}{3}\mu_0\omega\sigma R \left(= \frac{\mu_0}{2\pi}\frac{m}{R^3}\right)$$

(14-6) Solenoide sottile (rettilineo)

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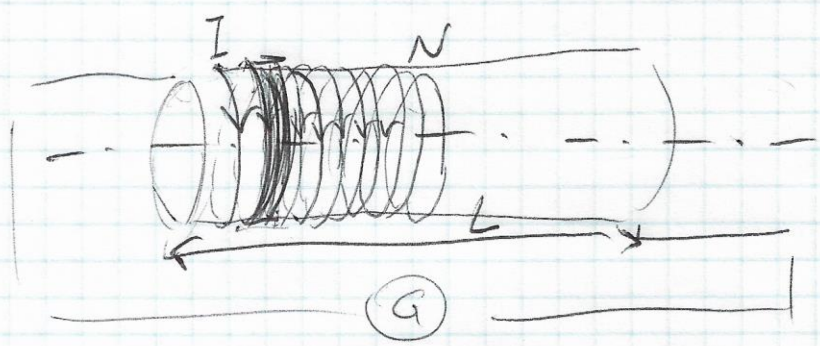
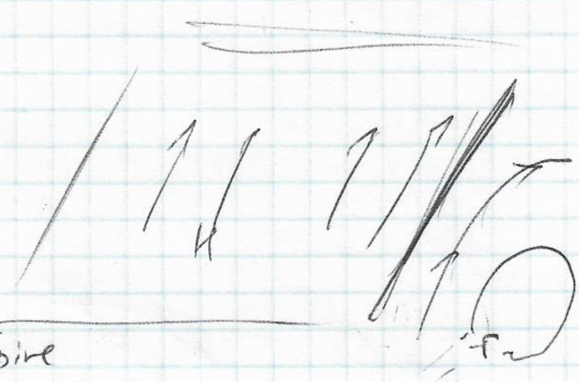
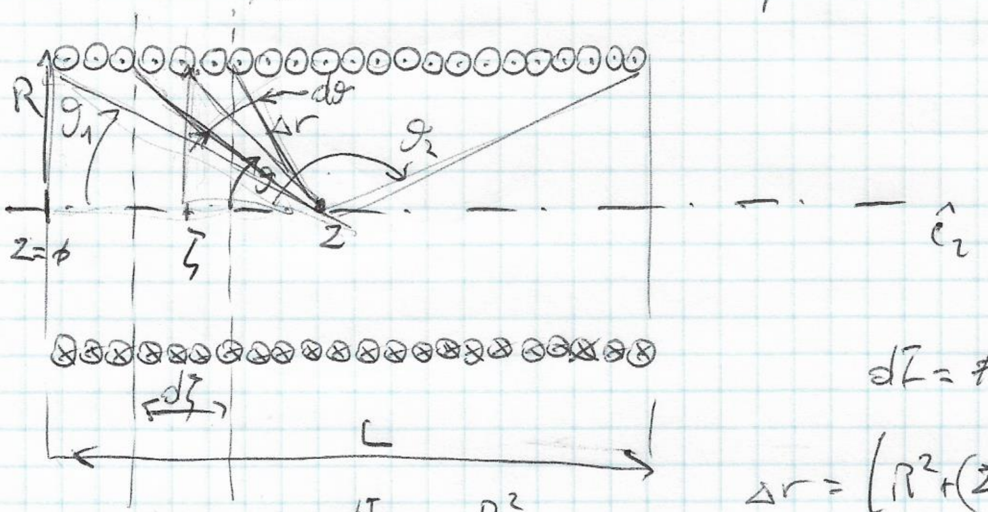




$n = N/L$ densità lineare di spire



$$dI = \#_{spire} \cdot I = n\,d\zeta \cdot I$$

$$\Delta r = \left[R^2+(z-\zeta)^2\right]^{1/2}$$

$$dB_{0z}(z) = \frac{\mu_0\,dI}{2}\,\frac{R^2}{(R^2+(z-\zeta)^2)^{3/2}} = n\,d\zeta\,\frac{\mu_0 I}{2}\,\frac{R^2}{\left[R^2+(z-\zeta)^2\right]^{3/2}}$$

$$R = (z-\zeta)\,tg\,\vartheta \quad \rightsquigarrow \quad z-\zeta = R/tg\,\vartheta$$

$$d\zeta = \frac{R}{tg^2\vartheta}\,\frac{1}{\cos^2\vartheta}\,d\vartheta = \frac{R\,d\vartheta}{\sin^2\vartheta}$$

$$dB_{0z}(z) = \frac{\mu_0 n I R^2}{2}\,\frac{1}{\underbrace{\left[R^2+R^2/tg^2\vartheta\right]}_{R^2/\sin^2\vartheta}{}^{3/2}}\,\frac{R\,d\vartheta}{\sin^2\vartheta} = \frac{\mu_0 n I}{2}\,\sin\vartheta\,d\vartheta$$

$$B_{0z}(z) = \int_{\vartheta_1}^{\vartheta_2} \frac{\mu_0 n I}{2}\,\sin\vartheta\,d\vartheta = \frac{\mu_0 n I}{2}\,(\cos\vartheta_1 - \cos\vartheta_2)$$

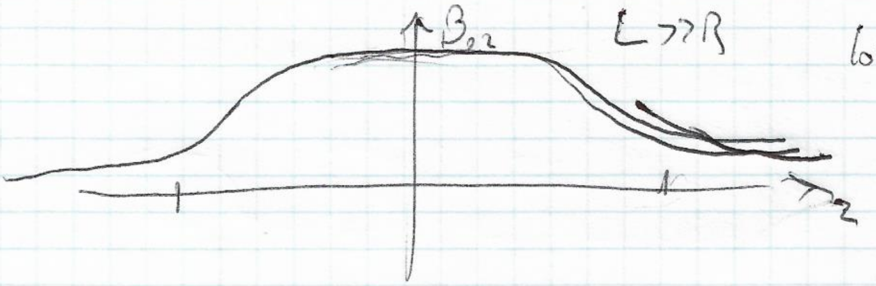
$z = L/2$

$\cos\vartheta_2 = -\cos\vartheta_1$

in $z=L/2$ $\quad B_{0z} = \mu_0 n I \cos\vartheta_1 = \mu_0 n I\,\dfrac{L/2}{(R^2+L^2/4)^{1/2}}$

$L \to \infty$

$B_{0z} = \mu_0 n I = \mu_0 K$



$L >> R$

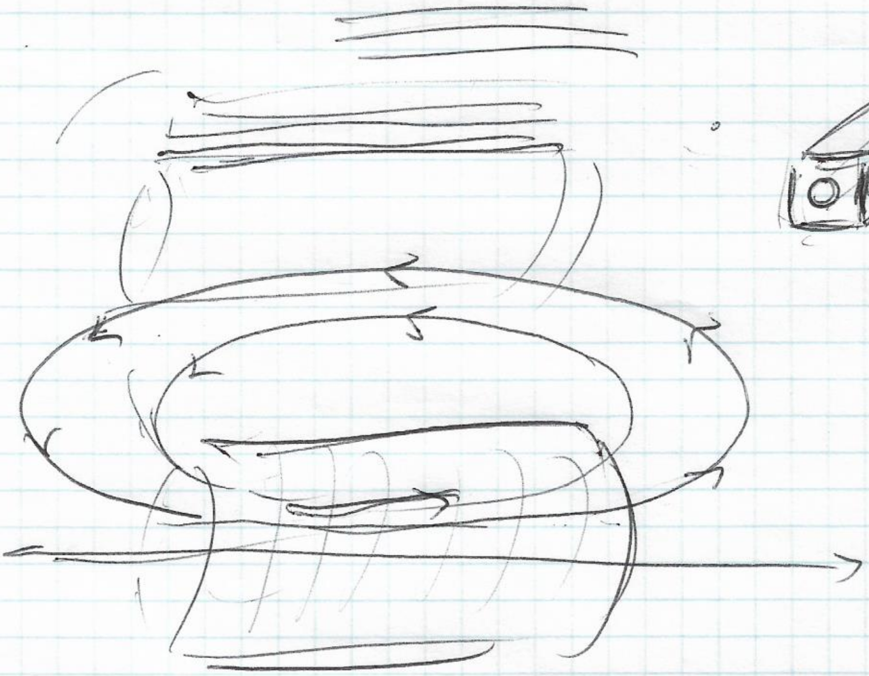
long region of uniformity

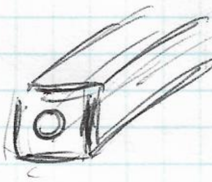
$L \sim R$

$B = \mu_0 n I$

$4\pi \cdot 10^{-7}$

$10^{-2} - 10^{-1}$ T

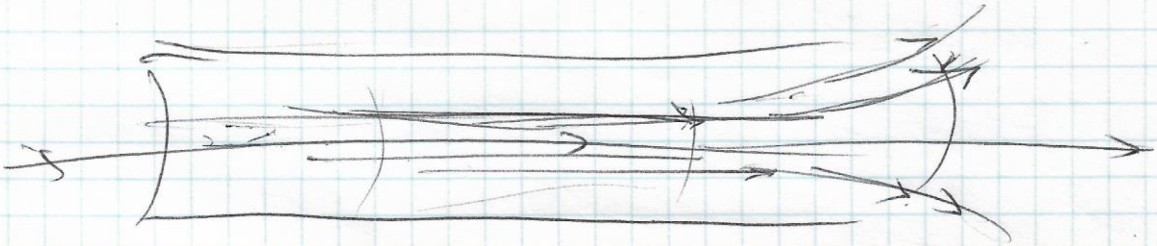




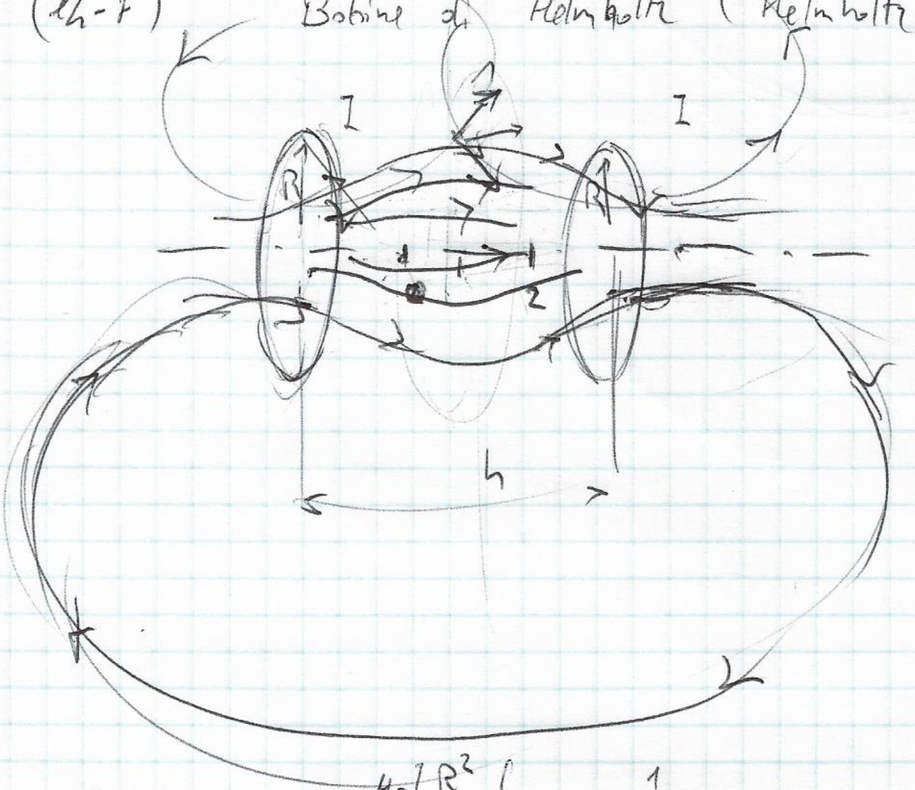
$I < 600$ A

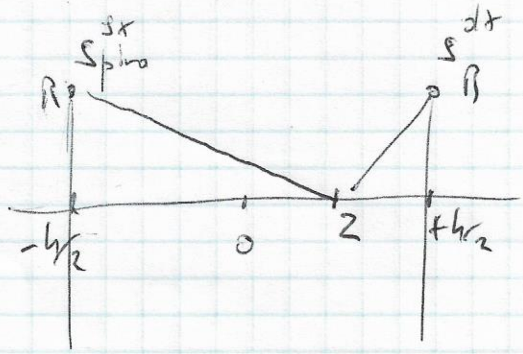
$\sim 1\,\Omega$

$P_J = RI^2 = IV$



(lh-7) Bobine di Helmholtz (Helmholtz coil)





$$B_{0z}(z) = \frac{\mu_0 I R^2}{2}\left\{\frac{1}{[R^2+(z+h/2)^2]^{3/2}} + \frac{1}{[R^2+(z-h/2)^2]^{3/2}}\right\}$$

$$B_{0z}(\phi) = \mu_0 I R^2 / [R^2 + h^2/4]^{3/2}$$

Sviluppo di $B_{0z}(z)$ intorno a $z=\phi$; Taylor

$$B_{0z}(z) = B_{0z}(\phi) + \cancel{\left.\frac{\partial B_{0z}}{\partial z}\right|_{z=\phi} z} + \frac{1}{2}\left.\frac{\partial^2 B_{0z}}{\partial z^2}\right|_{z=\phi} z^2 + \cancel{\frac{1}{3!}\left.\frac{\partial^3 B_{0z}}{\partial z^3}\right|_{z=\phi} z^3} + \frac{1}{4!}\left.\frac{\partial^4 B_{0z}}{\partial z^4}\right|_{z=\phi} z^4 \dots$$

Derivate dispari nulle in $z=\phi$

$$B_{0z}(-z) = B_{0z}(z)$$

$$\left.\frac{\partial^2 B_{0z}}{\partial z^2}\right|_{z=\phi} = \phi \quad \text{imposizione \underline{nostra} per uniformità}$$

$$\frac{\partial B_0}{\partial z} = -\frac{3\mu_0 I R^2}{2}\left\{\frac{z+h/2}{[R^2+(z+h/2)^2]^{5/2}} + \frac{z-h/2}{[R^2+(z-h/2)^2]^{5/2}}\right\}$$

in $z=\phi$ è nulla

$$\frac{\partial^2 B_{0z}}{\partial z^2} = -\frac{3\mu_0 I R^2}{2}\left\{\frac{R^2-4(z+h/2)^2}{\left[R^2+(z+h/2)^2\right]^{7/2}}+\frac{R^2-4(z-h/2)^2}{\left[R^2+(z-h/2)^2\right]^{7/2}}\right\}$$

$$\left.\frac{\partial^2 B_{0z}}{\partial z^2}\right|_{z=0} = 0 \qquad R^2-h^2+R^2-h^2=0$$

$$R^2 = h^2 \qquad h = R$$

$$B_{0z}(0) = \mu_0 \frac{8}{5\sqrt{5}}\frac{I}{R}$$

$$\frac{\partial^3 B_{0z}}{\partial z^3} \quad \dots \quad ; \quad \left.\frac{\partial^4 B_{0z}}{\partial z^4}\right|_{z=0,\ h=R}$$

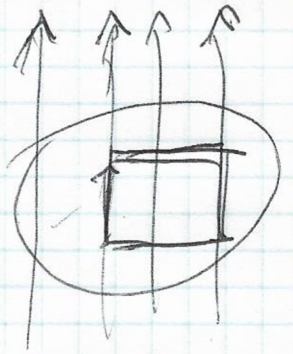
$$B_{0z}(z) = B_{0z}(0) + \frac{1}{24}\left.\frac{\partial^4 B_{0z}}{\partial z^4}\right|_{z=0,\ h=R}\cdot z^4 = \mu_0\frac{8}{5\sqrt{5}}\frac{I}{R}\left[1-\frac{144}{125}\frac{z^4}{R^4}\right]$$

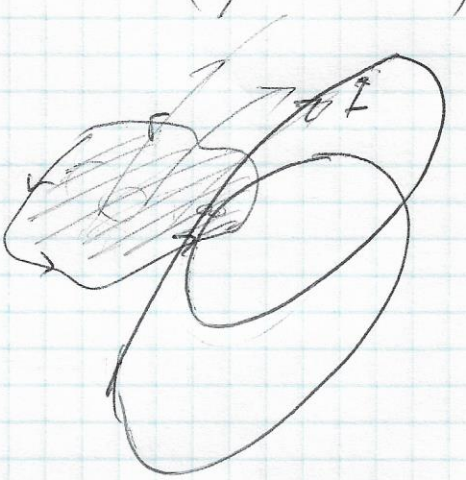
$$B_{0z}(z=\pm h/2=\pm R/2) = B_{0z}(0)\left[1-0.072\right]$$

Legge di Ampère

$$\vec{\nabla} \times \vec{B}_0 = \mu_0 \vec{J} \qquad \text{(stazionario; magnetostatica)}$$

$$\oint_{\mathcal{C} = \partial S} \vec{B}_0 \cdot d\vec{\ell} = \mu_0 \iint_S \vec{J} \cdot d\vec{S} = \left(\mu_0 \sum I_{conc} \right)$$





scelta del circuito: ragionata sulla base delle simmetrie (direzione, dipendenza di $\vec{B}$ dalle coord.)