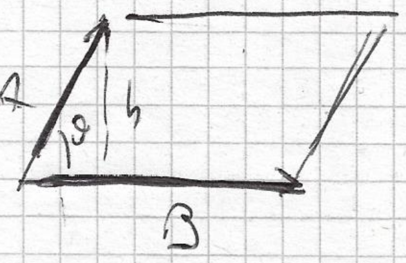
ES #2 Calcolo vettoriale, 07/X/2020

Forza di Coulomb, campo elettrostatico

(0-7)



$$A = |\bar{A} \times \bar{B}|$$

$$A = Bh = BA\sin\theta$$

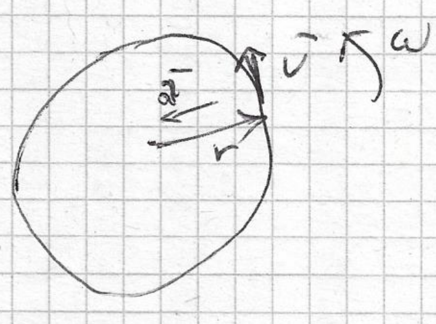
$$|\bar{A} \times \bar{B}| = AB\sin\theta$$

(0-10) $\bar{r} = \cos(\omega t)\hat{e}_x + \sin(\omega t)\hat{e}_y$, ω = costante

a) $\bar{v} \perp \bar{r}$

b) $\bar{a} \parallel -\bar{r}$ $\quad a \propto r$

c) $\bar{r} \times \bar{v}$ vettore costante



a) ℓ parametro t

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{dx}{dt}\hat{e}_x + \frac{dy}{dt}\hat{e}_y = -\omega\sin(\omega t)\hat{e}_x + \omega\cos(\omega t)\hat{e}_y$$

$$\bar{r}\cdot\bar{v} = r_x v_x + r_y v_y = -\omega\cos(\omega t)\sin(\omega t) + \omega\sin(\omega t)\cos(\omega t) = \emptyset$$

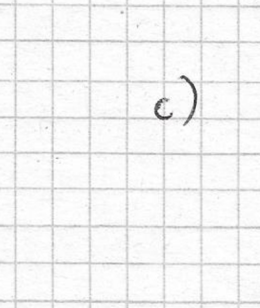
b) $$\bar{a} = \frac{d^2\bar{r}}{dt^2} = \frac{d\bar{v}}{dt} = -\omega^2\cos(\omega t)\hat{e}_x - \omega^2\sin(\omega t)\hat{e}_y = -\omega^2\bar{r}$$

acc. centripeta

c) $$\bar{r}\times\bar{v} = (\cos(\omega t)\hat{e}_x + \sin(\omega t)\hat{e}_y)\times(-\omega\sin(\omega t)\hat{e}_x + \omega\cos(\omega t)\hat{e}_y) =$$

$$\hat{e}_x \times \hat{e}_x = \emptyset \qquad \hat{e}_y \times \hat{e}_y = \emptyset$$

$$= \omega\cos^2(\omega t)\underbrace{\hat{e}_x\times\hat{e}_y}_{\hat{e}_z} - \omega\sin^2(\omega t)\underbrace{\hat{e}_y\times\hat{e}_x}_{-\hat{e}_z} = \omega\underbrace{(\cos^2+\sin^2)}_{1}\hat{e}_z = \omega\hat{e}_z$$



$\hat{e}_x \; \hat{e}_y \; \hat{e}_z$

$$\hat{e}_x \times \hat{e}_y = \hat{e}_z \qquad \hat{e}_x \times \hat{e}_z = -\hat{e}_y$$

$$\hat{e}_y \times \hat{e}_z = \hat{e}_x \qquad \hat{e}_z \times \hat{e}_y = -\hat{e}_x$$

$$\hat{e}_z \times \hat{e}_x = \hat{e}_y \qquad \hat{e}_y \times \hat{e}_x = -\hat{e}_z$$

(O-11)

Particella m $\bar{r}$ rispetto O sdr

$\bar{F}$ su particella

$\bar{M} = \bar{r} \times \bar{F}$ mom. della forza

$\bar{H} = \bar{r} \times m\bar{v}$ " angolare

$$\bar{M} = \frac{d\bar{H}}{dt} \qquad \bar{M} = \bar{r} \times \bar{F} = \bar{r} \times \frac{d}{dt}(m\bar{v})$$

$$\frac{d\bar{H}}{dt} = \frac{d}{dt}(\bar{r} \times m\bar{v}) = \underbrace{\frac{d\bar{r}}{dt} \times m\bar{v}}_{\bar{v} \times m\bar{v} = 0} + \bar{r} \times \frac{d}{dt}(m\bar{v}) = \bar{r} \times \frac{d}{dt}(m\bar{v}) = \bar{r} \times \bar{F} = \bar{M}$$

$\bar{r}_k \qquad k = 1, \dots, N$

$$\bar{H} = \sum_k m_k \bar{r}_k \times \bar{v}_k \; ; \; \bar{M} = \sum_k \bar{r}_k \times \bar{F}_k$$

$$\bar{M} = d\bar{H}/dt$$

(O-12)

$$\bar{a} = \frac{d\bar{v}}{dt} = 12\cos(2t)\hat{e}_x - 8\sin(2t)\hat{e}_y + 16t\,\hat{e}_z \qquad \forall t \geq 0$$

$\bar{r}(0) = 0$; $\bar{v}(0) = 0$ $\rightarrow$ $\bar{r}(t)$, $\bar{v}(t)$ $\forall t$

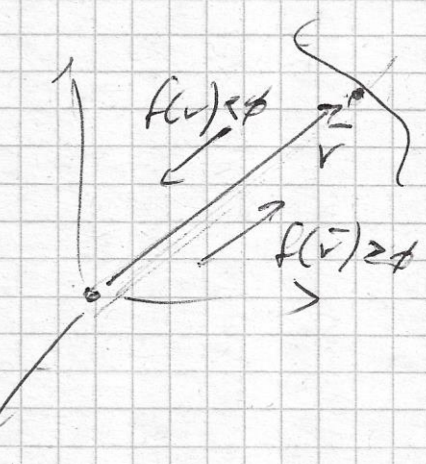
$$\bar{v}(t) = \int \bar{a}\,dt = \hat{e}_x \int a_x\,dt + \hat{e}_y \int \dots$$

$$= \dots + c_1 \qquad \bar{v}(t=0) = 0$$

$$\bar{r}(t) = \int \bar{v}\,dt \qquad \bar{r}(t=0) = 0$$

(0-13) Part. m

$$m\frac{d^2\bar r}{dt} = f(r)\hat r_1$$



$\bar r$ pos particella O sdr

$\hat r_1$ versore $\bar r/r$

f(r) funzione della sola distanza r

a) $\bar r \times \frac{d\bar r}{dt} = \bar C$ vettore costante

b) $f(r) \gtrless 0$

c) Discutere a) geometricamente

d) " moto pianeta attorno al Sole

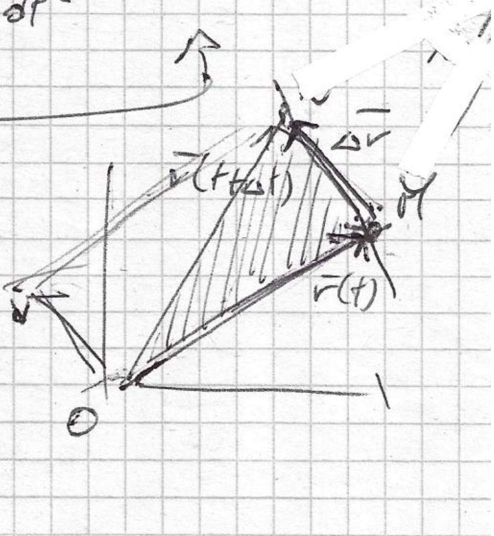
a) $\bar r \times$

$$\underbrace{m\bar r \times \frac{d^2\bar r}{dt^2}}_{\Rightarrow\ \hookrightarrow\ =0} = f(r)\cancel{\bar r \times \hat r_1} = 0$$

$$\frac{d}{dt}\left(\bar r \times \frac{d\bar r}{dt}\right) = \cancel{\frac{d\bar r}{dt} \times \frac{d\bar r}{dt}} + \bar r \times \frac{d^2\bar r}{dt^2} = 0$$

$\Rightarrow$ $\bar r \times d\bar r/dt = \bar C$ costante

b) $f(r)\hat r_1$ CENTRALE



c) A spazzata OMN

$$A_{OMN} = \frac{1}{2}\left|\bar r \times \Delta\bar r\right|$$ A in Δt

$$\frac{A}{\Delta t} = \frac{1}{2}\left|\vec{r} \times \frac{\Delta \vec{r}}{\Delta t}\right|$$

$$\lim_{\Delta t \to 0} \frac{1}{2}\left|\vec{r} \times \Delta\vec{r}/\Delta t\right| = \frac{1}{2}\left|\vec{r} \times d\vec{r}/dt\right| = \frac{1}{2}\left|\vec{r} \times \vec{v}\right|$$

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{r} \times \vec{v} = \text{vett. costante}$$

areal velocity = constant

d) $\bar{F}$ pianeta – stella $\bar{F}_g$ CENTRALE

(Kepler)

(O-14) Particella C xy $(z=0)$ $C_e(0;0;0)$ raggio $= 3$

$$\bar{F}(x,y,z) = (2x-y-z)\hat{e}_x + (x+y-z)\hat{e}_y + (3x-2y+4z)\hat{e}_z$$

$$\mathcal{L} = \int_\gamma \bar{F}\cdot d\bar{r} = \int_C \bar{F}\cdot d\bar{r}$$

$$d\bar{r} = dx\,\hat{e}_x + dy\,\hat{e}_y \qquad x = 3\cos(t) \quad y = 3\sin(t) \qquad t \in [0, 2\pi]$$

$$\frac{dx}{dt} = -3\sin t \to dx = -3\sin(t)\,dt$$

$$dy = 3\cos(t)\,dt$$

$$\mathcal{L} = \int_C \bar{F}\cdot d\bar{r} = \int_C F_x\,dx + \int_C F_y\,dy = \int_C (2x-y)\,dx + \int (x+y)\,dy =$$

$$= \int_0^{2\pi} (6\cos t - 3\sin t)\cdot\underbrace{(-3\sin t)\,dt} + \int_0^{2\pi} (3\cos t + 3\sin t)\,3\cos t\,dt =$$

$$= \int_0^{2\pi} 9 - 9\cos(t)\sin(t)\,dt = 9\left[t - \cancel{\tfrac{1}{2}\sin^2(t)}\right]_0^{2\pi} = 18\pi$$

(0-15) $\quad \mathcal{C}$ curva $\begin{cases} x = t^2 \\ y = 2t \\ z = t^3 \end{cases} \quad t \in [0;1]$

$\phi(x,y,z) = 2xyz^2$

$\bar{F}(x,y,z) = xy\,\hat{e}_x - z\,\hat{e}_y + x^2\,\hat{e}_z$

a) $\int_{\mathcal{C}} \phi\, d\bar{r} \quad ; \quad \int_{\mathcal{C}} \bar{F} \times d\bar{r}$

a) $\phi = 2xyz^2 = 2t^2 \cdot 2t \cdot t^6 = 4t^9$

$\bar{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z = t^2\hat{e}_x + 2t\hat{e}_y + t^3\hat{e}_z$

$$\left.\begin{aligned} dx &= 2t\,dt \\ dy &= 2\,dt \\ dz &= 3t^2\,dt \end{aligned}\right) \quad \overline{dr} = dx\,\hat{e}_x + dy\,\hat{e}_y + dz\,\hat{e}_z = (2t\,\hat{e}_x + 2\hat{e}_y + 3t^2\hat{e}_z)\,dt$$

$$\int_{\mathcal{C}} \phi\, d\bar{r} = \int_0^1 4t^9 (2t\hat{e}_x + 2\hat{e}_y + 3t^2\hat{e}_z)\,dt =$$

$$= \left[\frac{8}{11} t^{11}\hat{e}_x + \frac{8}{10} t^{10}\hat{e}_y + t^{12}\hat{e}_z \right]_0^1 = \frac{8}{11}\hat{e}_x + \frac{4}{5}\hat{e}_y + \hat{e}_z$$

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b) $\int_{\mathcal{C}} \bar{F} \times d\bar{r} \qquad \bar{F} = 2t^3\hat{e}_x - t^3\hat{e}_y + t^4\hat{e}_z$

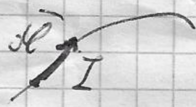
$$\bar{F} \times d\bar{r} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ F_x & F_y & F_z \\ dx & dy & dz \end{vmatrix} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 2t^3 & -t^3 & t^4 \\ 2t & 2 & 3t^2 \end{vmatrix} dt =$$

$$\bar{F} \times d\bar{r} = \underbrace{\left[\left(-3t^3 - 2t^4\right)\hat{e}_x + \left(2t^5 - 6t^5\right)\hat{e}_y + \left(4t^3 + 2t^4\right)\hat{e}_z \right]}_{\bar{Q}} dt =$$

$$\int_{\ell} \bar{F} \times d\bar{r} = \int_{\phi}^{1} \bar{Q}\, dt = \left[\left(-\frac{1}{2}t^6 - \frac{2}{5}t^5\right)\hat{e}_x - \frac{2}{3}t^6 \hat{e}_y + \left(t^4 + \frac{2}{5}t^5\right)\hat{e}_z \right]_{\phi}^{1} =$$

$$= -\frac{9}{10}\hat{e}_x - \frac{2}{3}\hat{e}_y + \frac{7}{5}\hat{e}_z$$

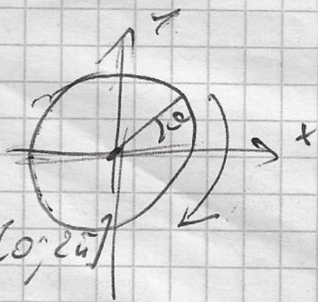
(O-16) $\qquad d\bar{F} = I\, d\bar{\ell} \times \bar{B}$



$d\bar{\ell}$ el. di cammino ∞esimo

ℓ raggio a, centro $C(x_c, \phi, \phi)$ xy $(z=1)$

$$\bar{B} = B_0 \frac{x}{x_0}\hat{e}_z \qquad \bar{F} = \int_{\ell} d\bar{F}$$



$$\bar{r} = x_c\hat{e}_x + a(\cos\vartheta\, \hat{e}_x - \sin\vartheta\, \hat{e}_y)$$

$\vartheta \in [0, 2\pi]$

$$d\bar{\ell} = -a(\sin\vartheta\, \hat{e}_x + \cos\vartheta\, \hat{e}_y)\, d\vartheta$$

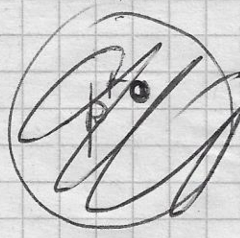
$$\bar{B} = B_0 \frac{x_c + a\cos\vartheta}{x_0}\hat{e}_z$$

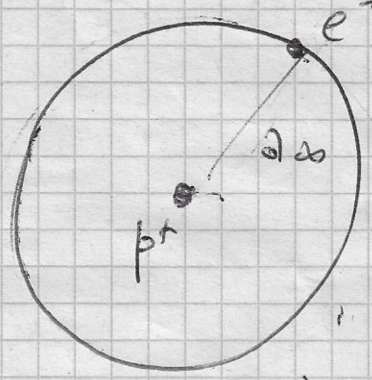
$$d\bar{F} = I d\bar{\ell} \times \bar{B} = \left[\frac{aIB_0}{x_0}\left(x_c \sin\vartheta + a\sin\vartheta\cos\vartheta\right)\hat{e}_y + - \frac{aIB_0}{x_0}\left(x_c\cos\vartheta + a\cos^2\vartheta\right)\hat{e}_x \right] d\vartheta$$

$$[\ \ ] = \bar{\alpha}$$

$$\bar{F} = \int_{\ell} d\bar{F} = \int_{\phi}^{2\pi} \bar{\alpha}\, d\vartheta = -\frac{a^2 I B_0}{x_0}\pi\hat{e}_x \qquad \int \cos^2\vartheta\, d\vartheta = \frac{\vartheta}{2} + \frac{\sin(2\vartheta)}{4}$$

(1-1) H





S.I.

$q_p = 1.60 \cdot 10^{-19}\ C = e$

$q_e = -1.60 \cdot 10^{-19}\ C = -e$

p^+, e^- pt formi

$m_p = 1.67 \cdot 10^{-27}\ kg$

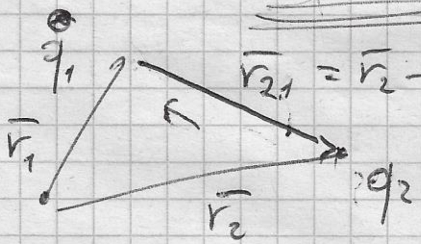
$m_e = 9.11 \cdot 10^{-31}\ kg$

$G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2$

$a_0 = 0.53 \cdot 10^{-10}\ m = 0.53\ \text{Å}$ $\qquad 10^{-10}\ m = 1\ \text{Å}$

Ångstrom

$\varepsilon_0 = \underline{8.854 \cdot 10^{-12}}\ C^2/Nm^2$



$$\bar{F}_{21} = \bar{F}_1(\bar{r}_2) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\bar{r}_2 - \bar{r}_1|^3} (\bar{r}_2 - \bar{r}_1)$$

$$\frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|^3} = \frac{1}{|\bar{r}_2 - \bar{r}_1|^2} \cdot \underbrace{\frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}}_{\text{centrale}}$$

$$\propto 1/(\Delta r)^2$$

$$F_C = \frac{1}{4\pi\varepsilon_0} \frac{|q_p q_e|}{a_0^2} = \frac{e^2}{4\pi\varepsilon_0 a_0^2}$$

$F_C = 8.19 \cdot 10^{-8}\ N$ attrattiva (−)

$$F_G = G \frac{m_p m_e}{a_0^2} = 3.61 \cdot 10^{-47}\ N$$

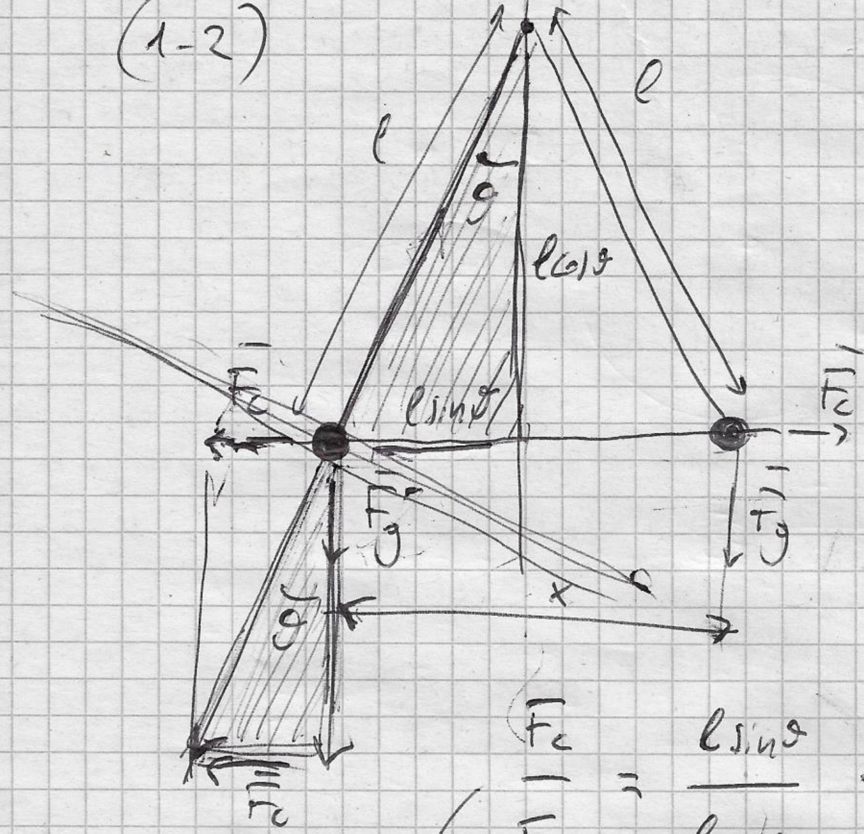
$$F_C / F_G = 2.27 \cdot 10^{39}$$

(1-2)

$m = 10\,g$

$l = 1\,m$

$q = 1.6 \cdot 10^{-7}\,C$



Pos. equilibrio!

$$\frac{\overline{F_c}}{\overline{F_g}} = \frac{l \sin\vartheta}{l \cos\vartheta} = tg\,\vartheta$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}\cdot\frac{1}{mg} = tg\,\vartheta = \frac{x/2}{(l^2 - x^2/4)^{1/2}}$$

$$\frac{q^2}{4\pi\varepsilon_0 m g x^2} = \frac{x}{2(l^2 - x^2/4)^{1/2}} \quad \text{in } x$$

Hyp: ϑ piccolo $\quad tg\,\vartheta \simeq \sin\vartheta \quad (\cos\vartheta \to 1$, $\vartheta \to 0)$

$$tg\,\vartheta \simeq \sin\vartheta = \frac{x/2}{l}$$

$$\frac{q^2}{4\pi\varepsilon_0 m g x^2} = \frac{x}{2l} \leadsto x = \left(\frac{l q^2}{2\pi\varepsilon_0 m g}\right)^{\frac{1}{3}} = 16.74\ cm$$

$$\sin\vartheta = x/2l = 0.0837 \leadsto \vartheta = 4.80^\circ$$

$$tg\,\vartheta = 0.084 \qquad 3.6‰$$