ES. #20 Legge di Ampère - 29/11/2021

Forza magnetica su circuiti in corrente

(1)

$$\bar{\nabla} \times \bar{B}_0 = \mu_0 \bar{J} \rightsquigarrow \oint_{C=\partial S} \bar{B}_0 \cdot d\bar{l} = \mu_0 \int_S \bar{J} \cdot d\bar{S}$$

(15.1) Filo rettilineo ∞ R finito, I = cost. invasato

(r, ϑ, z) $\quad \bar{B}_0 \quad B_{0r} \; B_{0\vartheta} \; B_{0z}$ $(r, \cancel{\vartheta}, \cancel{z})$ $\} \to B_{0\vartheta}(r)$

ϑ - invariante

z - invariante

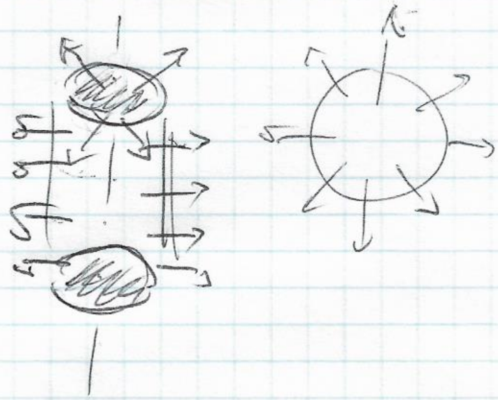
$(dB \propto I d\bar{l} \times \Delta\bar{r})$

$\hookrightarrow \cancel{B_{0z}}$

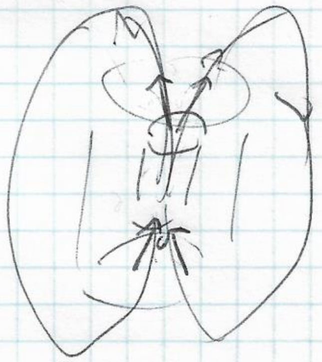
$\cancel{B_{0r}(r)}$, $B_{0\vartheta}(r)$

$\bar{\nabla} \cdot \bar{B}_0 = 0$

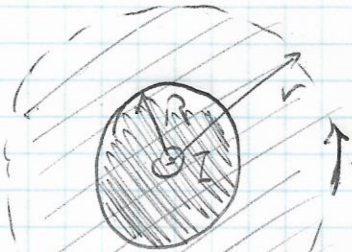
$\oint \bar{B} \cdot d\bar{S} = 0$



$$\boxed{\bar{B}_0 = B_{0\vartheta}(r)\, \hat{e}_\vartheta}$$



$r \geq R$

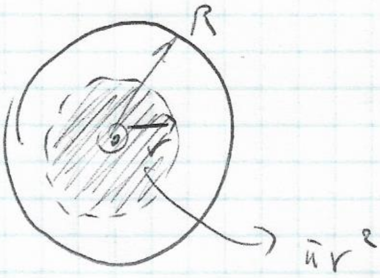


$\uparrow d\bar{l} = r\, d\vartheta\, \hat{e}_\vartheta$

$$\oint \bar{B}_0 \cdot d\bar{l} = \int_0^{2\pi} B_{0\vartheta}(r)\, r\, d\vartheta = 2\pi r\, B_{0\vartheta}(r) =$$

$$= \mu_0 \int \bar{J}_0 \cdot d\bar{S} = \mu_0 I \quad \to \bar{B}_0(r) = \frac{\mu_0 I}{2\pi r}\, \hat{e}_\vartheta$$

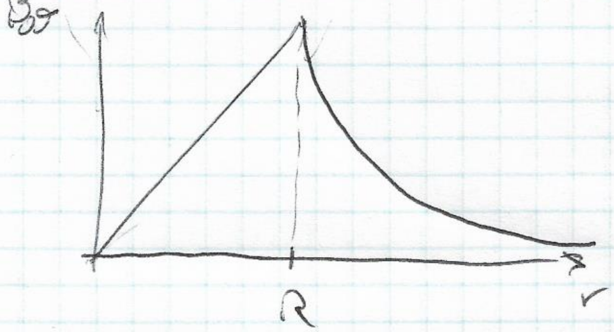
$r \le R$

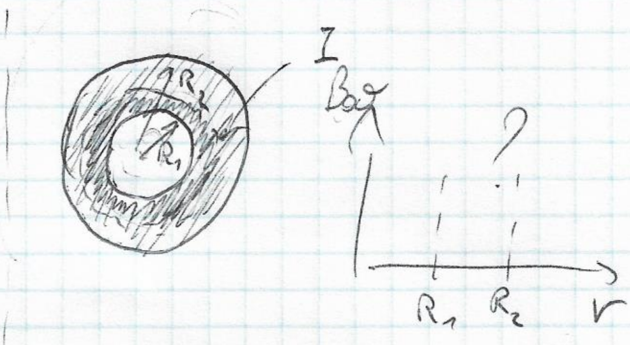


$$\oint \bar B_\theta \cdot d\bar l = 2\pi r B_{0\theta}(r) = \mu_0 \int \bar J \cdot d\bar S =$$

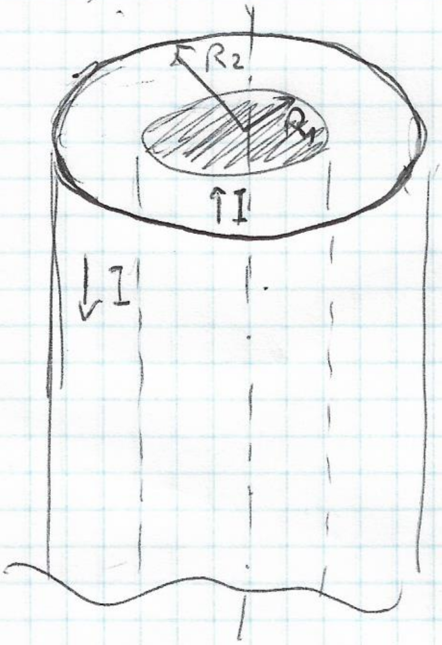
$$I = J\pi R^2 \qquad J = \frac{I}{\pi R^2} \qquad = \mu_0 I \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2}$$

$$\bar B_0(r) = \frac{\mu_0 I}{2\pi R^2} r \hat e_\theta$$

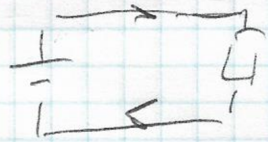




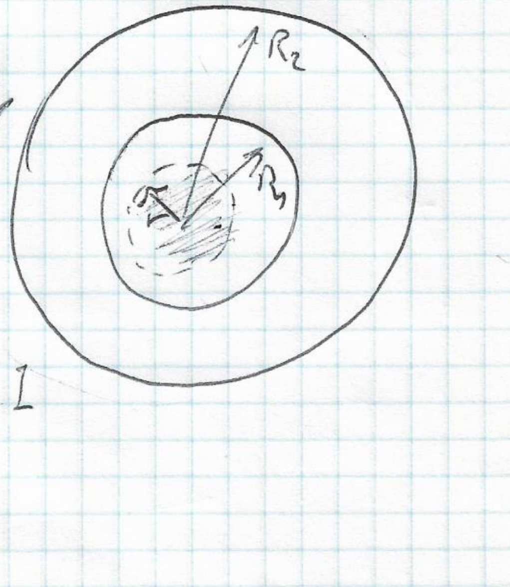
(15-2) Cavo coassiale rettilineo ∞



$\underline{B_{0\theta}(r)}$



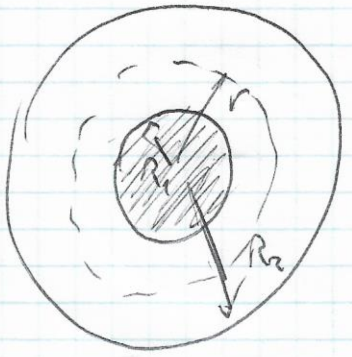
$r < R_1$



$$\int \bar B_0 \cdot d\bar l = 2\pi r B_{0\theta}(r) = \mu_0 \frac{r^2}{R_1^2} I$$

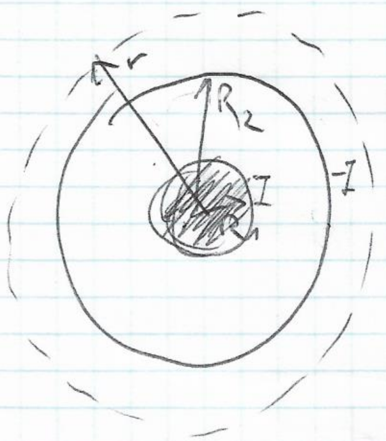
$$B_{0\theta}(r) = \frac{\mu_0 I}{2\pi R_1^2} r$$

$R_1 < r < R_2$

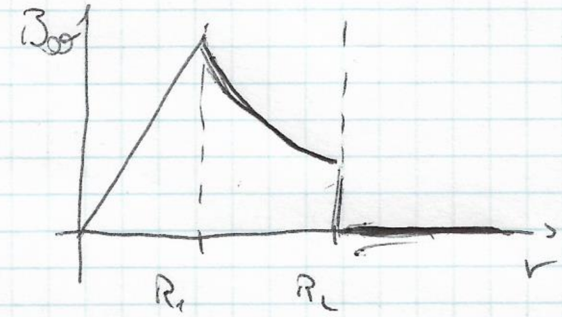


$$2\pi B_{0\vartheta}(r) = \mu_0 I \rightsquigarrow B_{0\vartheta}(r) = \frac{\mu_0 I}{2\pi r}$$

$r > R_2$



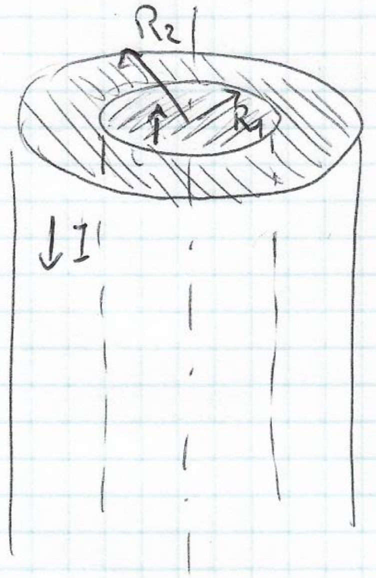
$$2\pi r B_{0\vartheta}(r) = \emptyset \qquad B_{0\vartheta}(r) = \emptyset$$



$$H = \frac{I}{2\pi R_2}$$

$$\Delta B = \mu_0 H = \frac{I}{2\pi R_2} - \emptyset$$

(15-3) Cavo coassiale n2 rettilineo ∞



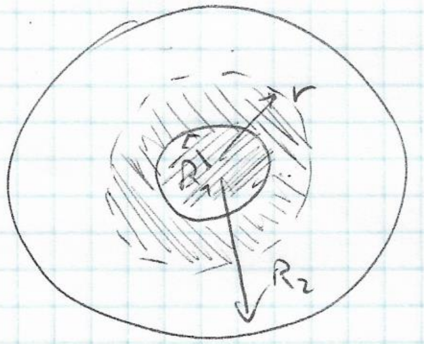
Intercapedine sottile isolante

$B_{0\theta}(r)$

$r<R_1$

$$2\pi r B_{0\theta}(r) = \mu_0 \frac{r^2}{R_1^2} I \Rightarrow B_{0\theta} = \frac{\mu_0 I}{2\pi R_1^2} r$$

$R_1<r<R_2$



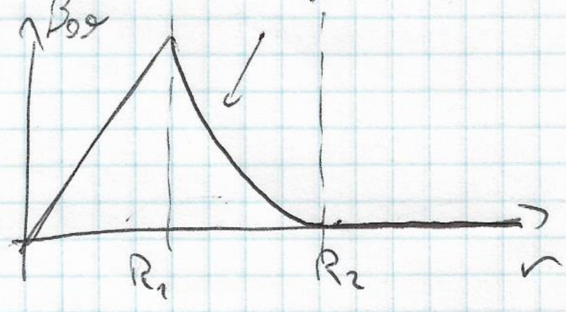
$$2\pi r B_{0\theta}(r) = \mu_0 I_c$$

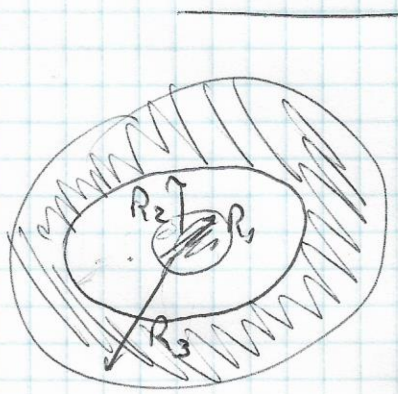
$$I_{out} = -\frac{\pi(r^2-R_1^2)}{\pi(R_2^2-R_1^2)} I$$

$$I_c = I_{in} + I_{out} = I - \frac{r^2-R_1^2}{R_2^2-R_1^2} I = \frac{R_2^2-r^2}{R_2^2-R_1^2} I$$

$$B_{0\theta}(r) = \frac{\mu_0 I}{2\pi} \frac{R_2^2-r^2}{r(R_2^2-R_1^2)}$$

?

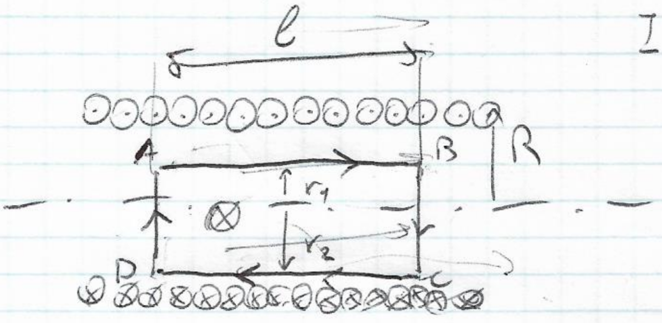




(15-4) Solenoide rettilineo ∞

I $\quad n = \#$ spire/ud lunghezza

(1)



$\bar{B}_0(r)$

$B_{0z}(r)$ $\quad \cancel{B_{0r}(r)}$

$\bar{\nabla} \cdot \bar{B}_0 = \emptyset$

$r_1, r_2 < R$

$$\oint \bar{B}_0 \cdot d\bar{\ell} = \int_A^B \bar{B}_0 \cdot d\bar{\ell} + \cancel{\int_B^C \bar{B}_0 \cdot \underbrace{d\bar{\ell}}_{\parallel \hat{e}_r}} + \int_C^D \bar{B}_0 \cdot d\bar{\ell} + \cancel{\int_D^A \bar{B}_0 \cdot \underbrace{d\bar{\ell}}_{\parallel \hat{e}_r}} = \emptyset$$

$\overline{AB} = \overline{CD} = \ell \qquad B_{0z}(r_1)\,\ell - B_{0z}(r_2)\,\ell = \emptyset \Rightarrow B_{0z}(r_1) = B_{0z}(r_2)$

B_{0z} uniforme $r < R$

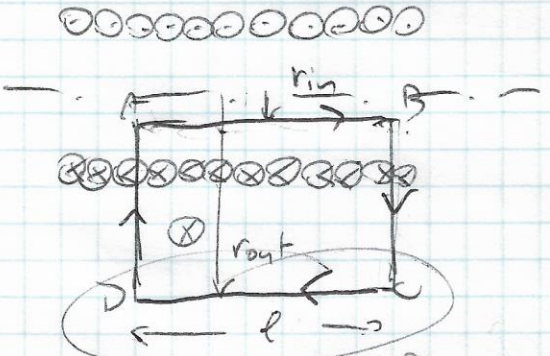
$$A \xrightarrow[d\bar{\ell} \to]{\overbrace{\longrightarrow \bar{B}_{0z}}^{\ell}} B$$

$$D \xleftarrow[d\bar{\ell}]{} C$$

Att.: circuito tutto ext

B_{0z} uniforme $= \emptyset$

(2)

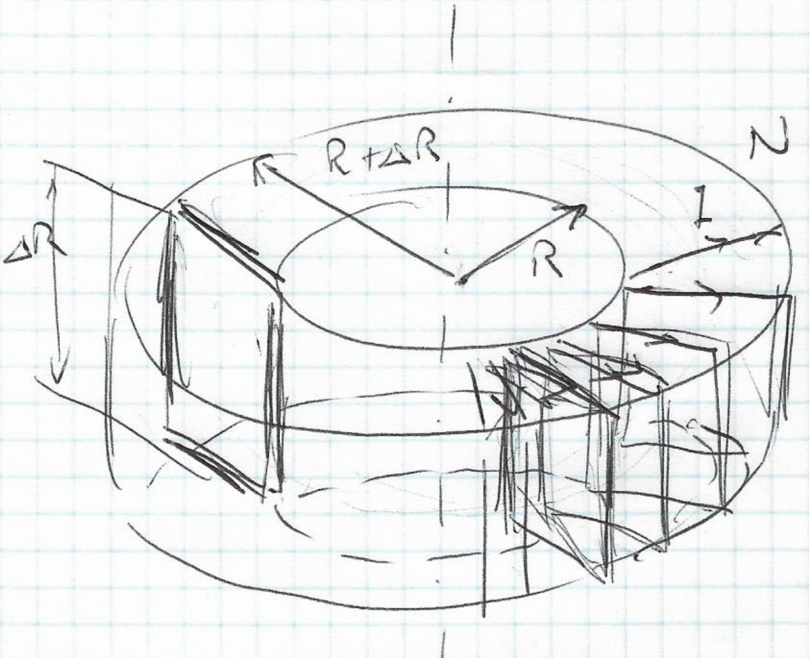


$$\oint \bar{B}_0 \cdot d\bar{\ell} = \underbrace{\int_A^B \bar{B}_0 \cdot d\bar{\ell}}_{B_{0z}\ell} + \cancel{\int_B^C}^{\perp} + \underbrace{\int_C^D}_{B_{0z} = \emptyset} + \cancel{\int_D^A}^{\perp}$$

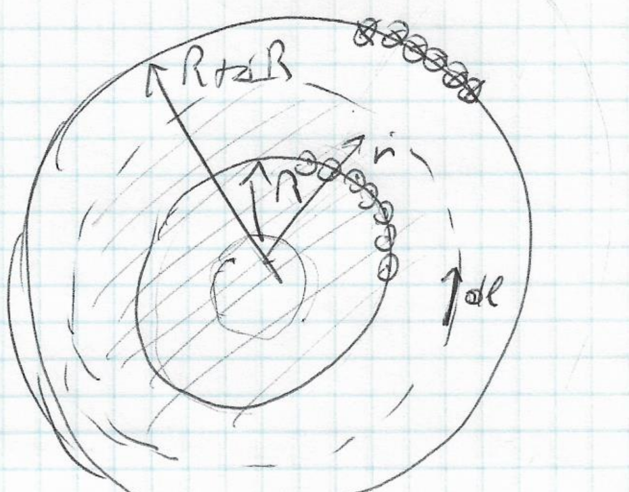
$B_{0z}\,\ell = \mu_0\, n \ell I \Rightarrow B_{0z} = \mu_0\, n I$

(15-5) Toroide a sezione quadrata

→ Solenoide chiuso a ciambella



$\bar{B}_0 = B_{0\theta}(r)\hat{e}_\theta$



$$\oint \bar{B}_0 \cdot d\bar{l} = 2\pi r\, B_{0\theta}(r) = \mu_0 N I$$

$$B_{0\theta}(r) = \frac{\mu_0 N I}{2\pi r}$$

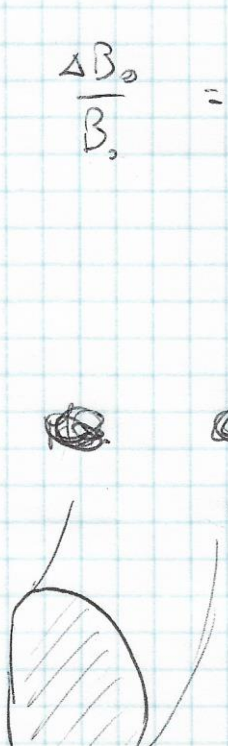
$$\frac{\Delta B_0}{B_0} = \frac{B_0(R) - B_0(R+\Delta R)}{B_0(R)} = R\left[\frac{1}{R} - \frac{1}{R+\Delta R}\right] =$$

$$= \frac{\Delta R}{R+\Delta R} \simeq \frac{\Delta R}{R} \qquad \Delta R \ll R$$

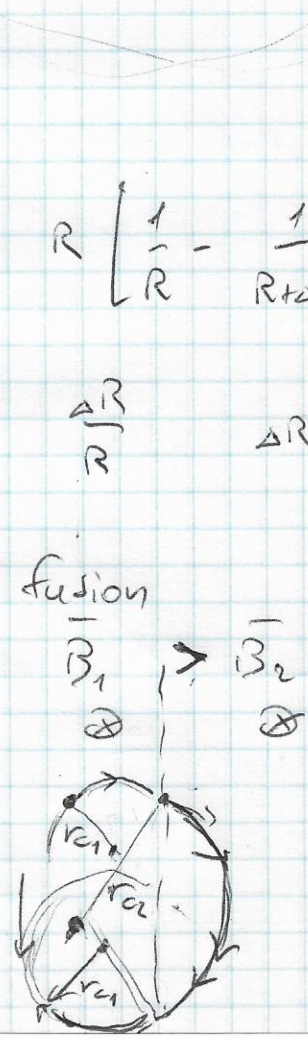
Magnetic confinement fusion

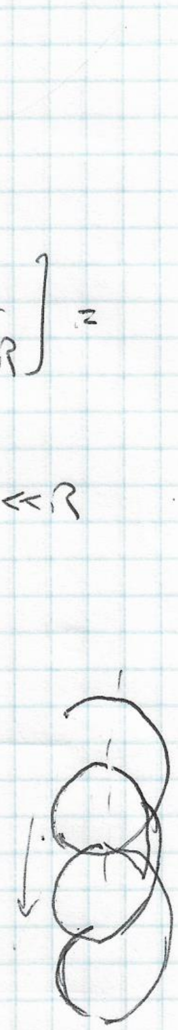
Tokamak / Stellarator

$$\bar{v}_d = \frac{r_c v_\perp}{2} \frac{\bar{B} \times \nabla B}{B^2}$$

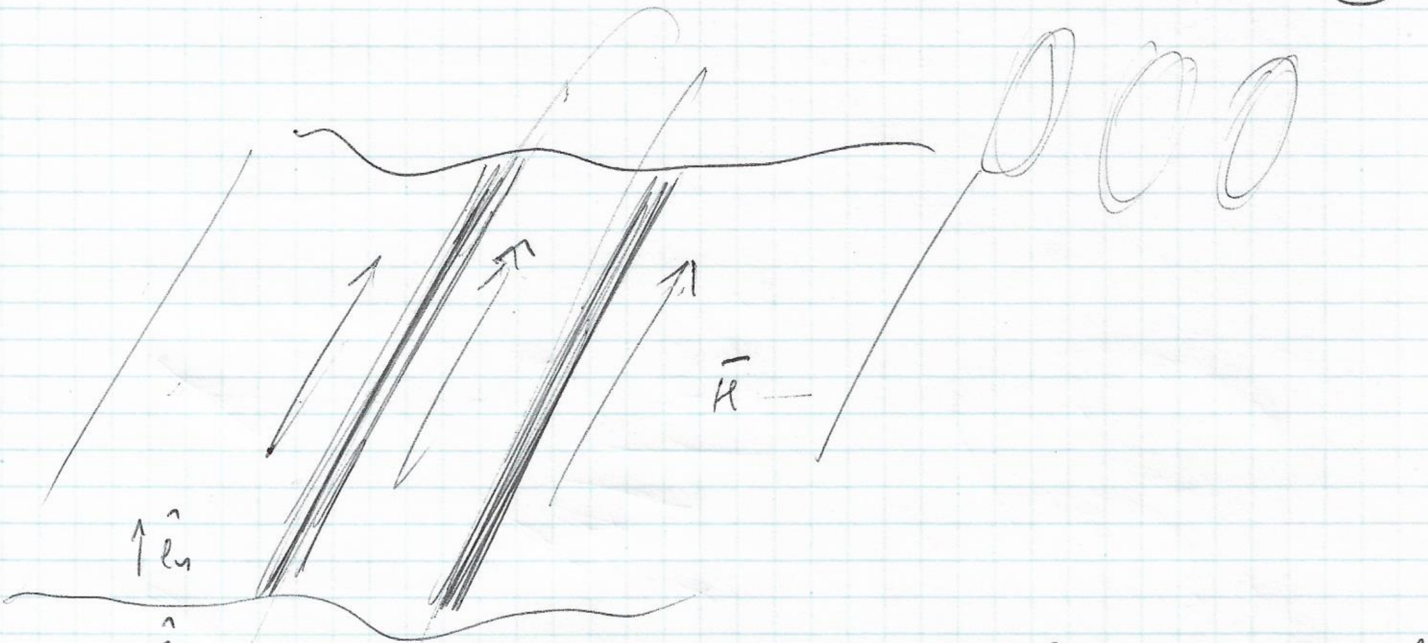


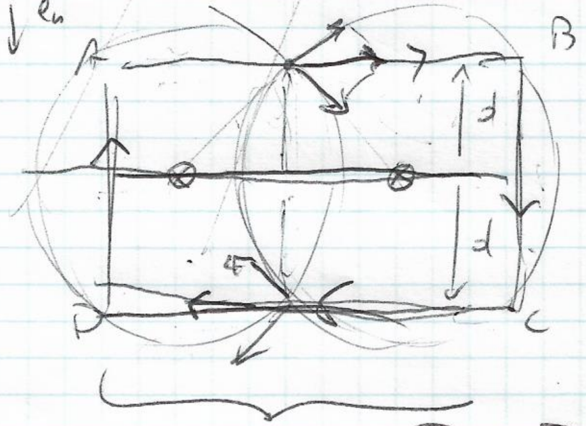
$\bar{B}_1 > \bar{B}_2$





(15-6) Corrente piana indefinita





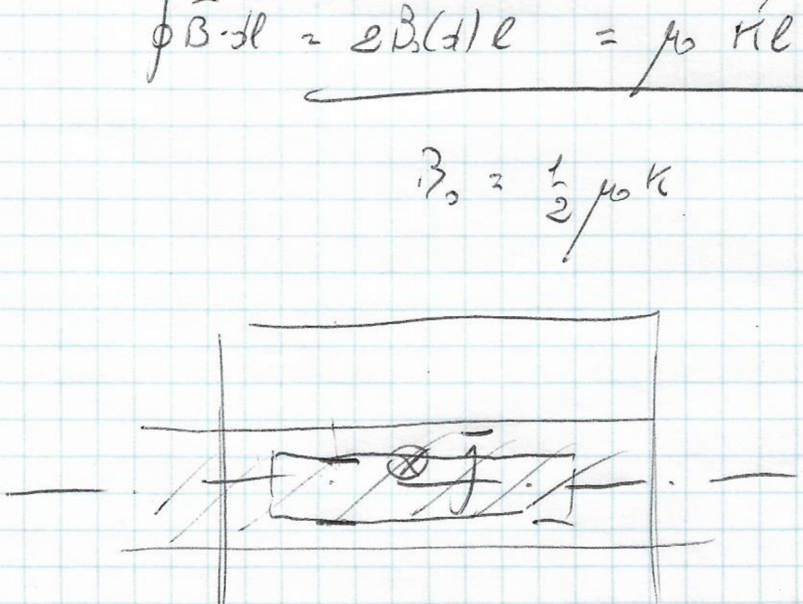
$\ell = \overline{AB} = \overline{CD}$

$$\oint \bar{B}_0 \cdot d\bar{l} = \underbrace{\int_A^B}_{B_0(d)\ell} + \cancel{\int_B^C} + \underbrace{\int_C^D}_{+B_0(d)\ell} + \cancel{\int_D^A} =$$

$K = [A/m]$

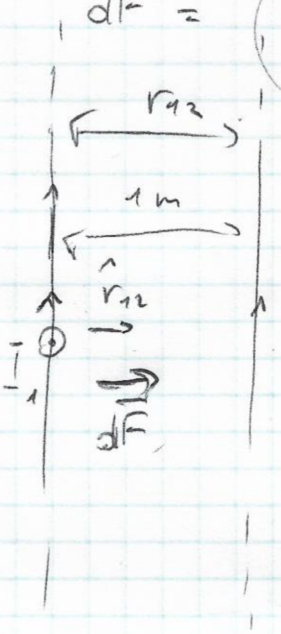
$$\oint \bar{B} \cdot d\bar{l} = 2B_0(d)\ell = \mu_0 K \ell$$

$$B_0 = \frac{1}{2}\mu_0 K \qquad \bar{B}_0 = \frac{1}{2}\mu_0 \bar{K} \times \hat{e}_n$$



Forza magnetica

$$d\bar F = I d\bar l \times \bar B_2$$



$$\bar F / L = \mu_0 / 2\pi \cdot 2 \cdot 10^{-7} \ N/m$$

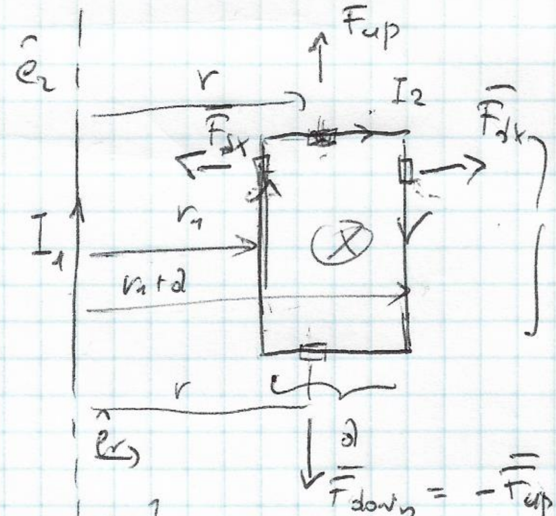
$$d\bar F = I_1 dl_1 \frac{\mu_0 I_2}{2\pi r_{12}} \hat r_{12}$$

$$\frac{dF}{dl_1} = \frac{\mu_0 I_1 I_2}{2\pi r_{12}} = \frac{\mu_0}{2\pi}$$

$I_1 = I_2 = 1\,A$

$r_{12} \to 1\,m$

(15-7)



$a = 5\,cm$

$b = 20\,cm$

$r_1 = 5\,cm$

$I_1 = 10\,A$

$I_2 = 1\,A$

$$I d\bar l \times \bar B$$

$$B_{01} = \frac{\mu_0 I_1}{2\pi r}$$

$$\bar F_{sx} = \int_{l_{sx}} I_2 d\bar l \times \bar B_1(r_1) = -I_2 b B_1(r_1) \hat e_r$$

$$\hat e_z \times \hat e_\theta = -\hat e_r$$

$$\bar F_{dx} = \int_{l_{dx}} I_2 d\bar l \times \bar B_1(r_1+a) = I_2 b B_1(r_1+a) \hat e_r$$

$$-\hat e_z \times \hat e_\theta = \hat e_r$$

$$\bar{F} = \bar{F}_{sx} + \bar{F}_{dx} = \frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{1}{r_1+a} - \frac{1}{r_1}\right]\hat{e}_r =$$

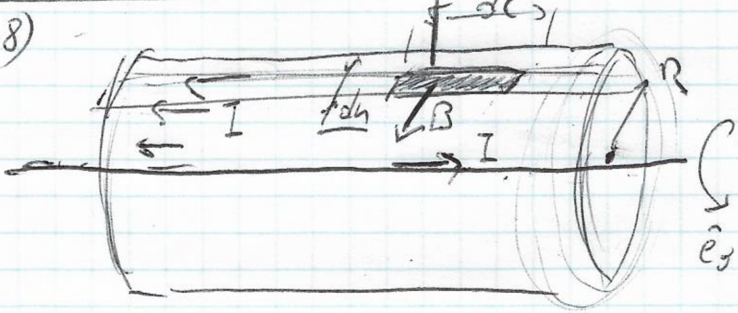
(9)

$$B_1(r_1) = \frac{\mu_0 I_1}{2\pi r_1}$$

$$= -\frac{\mu_0 I_1 I_2 a l_0}{2\pi r_1 (r_1+a)}\hat{e}_r = 4 \cdot 10^{-6} N$$

$$B_1(r_1+a) = \frac{\mu_0 I}{2\pi(r_1+a)}$$

(15-8)



$$dF = \underbrace{dI}\, dl\, B_0 = \frac{I\, dh}{2\pi R}\, dl\, \frac{\mu_0 I}{2\pi R}$$

$$dI = I\frac{dh}{2\pi R}$$

$$dS = dl\, dh$$

$$B_0 = \frac{\mu_0 I}{2\pi R}$$

$$\frac{dF}{dS} = \mu_0\left(\frac{I}{2\pi R}\right)^2$$