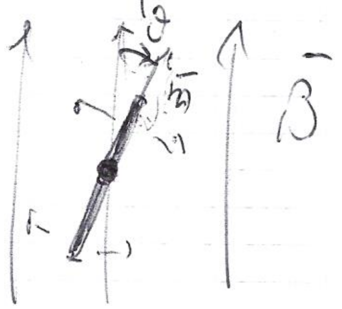
ES. #21 Forze magnetiche su dipoli — potenziale vettore

12/11/2021

(1)

(15-9)



$$\bar{U} = -\bar{m}\cdot\bar{B}$$

$$\bar{F} = -\bar{\nabla}U = \bar{\nabla}(\bar{m}\cdot\bar{B})$$

$$\bar{M} = \bar{m}\times\bar{B}$$

$$M = -mB\sin\vartheta$$

I momento di inerzia

$$M = I\frac{d\omega}{dt} = I\frac{d^2\vartheta}{dt^2}$$

$$\omega = \dot{\vartheta}$$

$$I\frac{d^2\vartheta}{dt^2} = -mB\sin\vartheta = -mB\vartheta$$

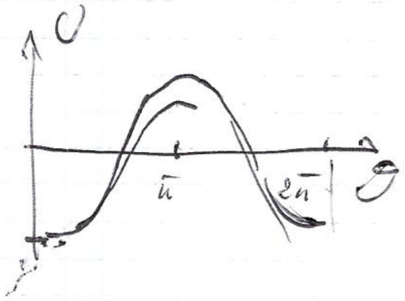
hyp.: piccole oscillazioni $\sin(\vartheta) \sim \vartheta$

$$\frac{d^2\vartheta}{dt^2} = -\omega^2\vartheta \rightarrow \omega = \sqrt{mB/I}$$

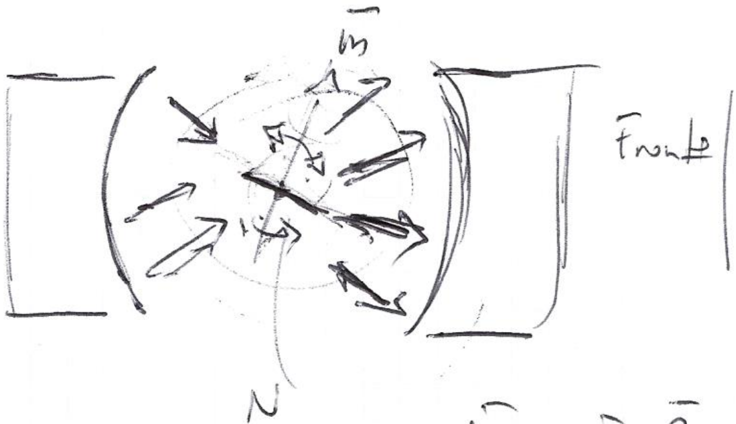
$$\vartheta(t) = \vartheta_{max}\sin(\omega t + \varphi)$$

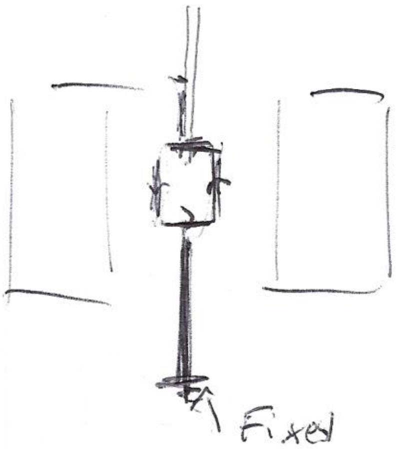
$$U = -\bar{m}\cdot\bar{B} = -mB\cos\vartheta$$

$$M = -\frac{dU}{d\vartheta}$$



(15-10) Galvanometro di D'Arsonval

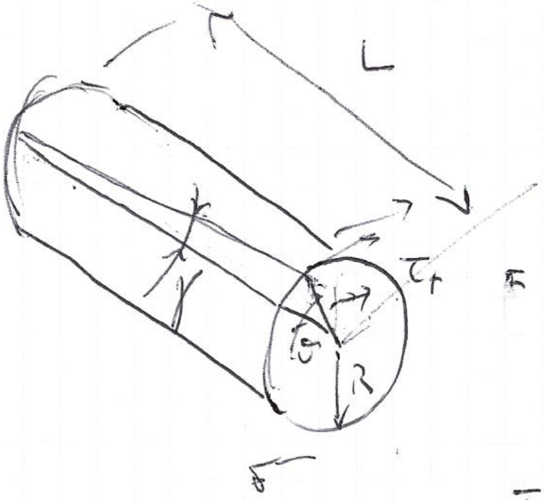




$$\bar{M} = \bar{m} \times \bar{B}$$

$$m = NIS = NId^2$$

$$M_m = mB = NId^2B$$



$$\tau_t = G\gamma$$

$$\gamma L = \theta R$$

$$\tau_{t\,max} = \tau_t(r=R) = G\theta R/L$$

$$\tau_t(r) = \tau_{t\,max}\, r/R$$

$$M_t = \int \tau_t\, r\, dA = \frac{\tau_{t\,max}}{R} \underbrace{\int r^2 dA}_{J_z} = \frac{\tau_{t\,max}}{R} \frac{\pi R^4}{2}$$

A = area della sezione circolare

J_z second moment of inertia

$dA = r\,dr\,d\theta$

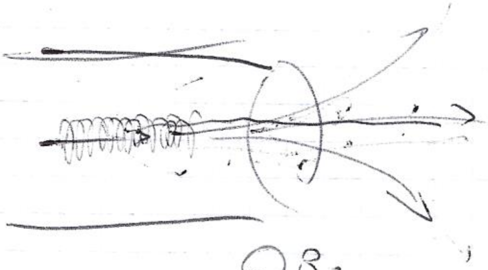
equ.

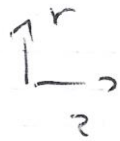
$$M_t = \frac{\tau_{t\,max}}{R} J_z = \frac{\pi}{2} \frac{G R^4 \theta}{L} = M_m = NId^2B$$

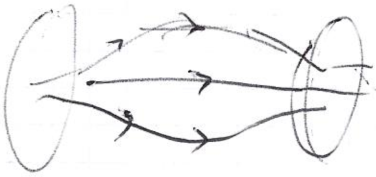
$d = 15$ mm; $N = 20$; $B = 0.2$ T; $L = 150$ mm; $R = 50\ \mu m$; $G = 5 \cdot 10^8\ N/m^2$

$I = 0.1\ \mu A \rightarrow \theta = 2.75$ mrad ($\simeq 0.16°$)

(15-11) Magnetic mirror → magnetic bottle







$$\frac{\partial B_z}{\partial z}$$

$$\bar{\nabla}\cdot\bar{B} = 0$$

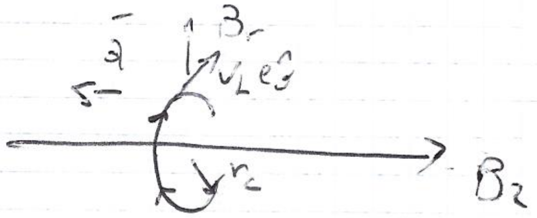
$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \underbrace{\frac{\partial B_z}{\partial z}}_{\neq 0} = 0$$

$$\frac{\partial}{\partial r}(rB_r) = -r\frac{\partial B_z}{\partial z}$$

$$rB_r = -\frac{1}{2}r^2\frac{\partial B_z}{\partial z} \rightsquigarrow B_r = -\frac{1}{2}r\frac{\partial B_z}{\partial z}$$

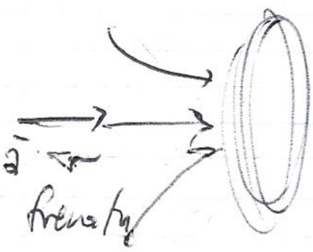
e^-

$$\bar{a} = \underbrace{-\frac{e}{m}v_\perp\hat{e}_\theta \times \bar{B}_r}_{\bar{F}_L/m} = -\frac{e}{m}v_\perp\hat{e}_\theta \times \left(-\frac{1}{2}r_c\frac{\partial B_z}{\partial z}\hat{e}_r\right) =$$



$$= -\frac{ev_\perp r_c}{2m}\frac{\partial B_z}{\partial z}\hat{e}_z \cdot \frac{B_0}{B_0} =$$

$$= -\frac{v_\perp\omega_c r_c}{2}\frac{1}{B_0}\frac{\partial B_z}{\partial z}\hat{e}_z \Rightarrow$$



$r_c = v_\perp/\omega_c$

$$\bar{a} = -\frac{v_\perp^2}{2}\underbrace{\frac{1}{B_0}\frac{\partial B_z}{\partial z}}_{1/\ell_c}\hat{e}_z = -\frac{v_\perp^2}{2\ell_c}\hat{e}_z$$

4

$-e$

$T = \frac{2\pi r_c}{v_\perp}$

$I = -e v_\perp / 2\pi r_c \rightsquigarrow \mu$

$\mu = IS = I \pi r_c^2 = -\frac{e v_\perp}{2} r_c = -\frac{m v_\perp^2}{2B}$

$r_c = \frac{m v_\perp}{eB} \left(= \frac{v_\perp}{\omega_c} \right)$

$$\boxed{\mu = + \frac{m v_\perp^2}{2B}}$$

$$F_z = m a_z = -\frac{m v_\perp^2}{2} \frac{1}{B_0} \frac{\partial B_z}{\partial z} = -\mu \frac{\partial B_z}{\partial z}$$

$$\bar{F} = \bar{\nabla}(\bar{\mu} \cdot \bar{B}) = \bar{\nabla}(\mu B_z) = \mu \bar{\nabla} B_z \approx -\mu \frac{\partial B_z}{\partial z} \hat{e}_z$$

if μ = const.

μ adiabatic invariant

↳ almost a constant of motion

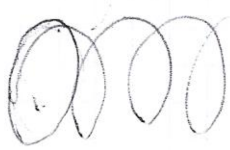
for periodic motion $\oint$ $J = \oint \bar{p} \cdot d\bar{q} = \oint \bar{p} \cdot d\bar{l} =$

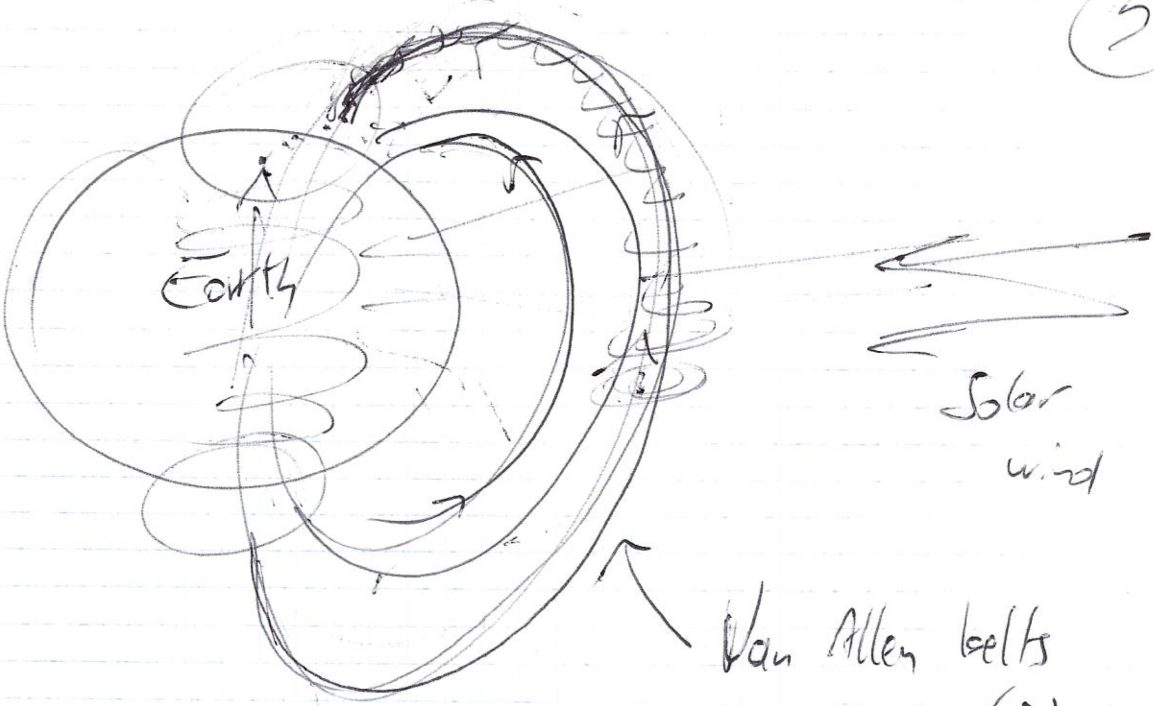
$(p = p_\perp = m v_\perp)$ $= \oint m v_\perp^2 \, dl / v_\perp = \frac{2}{2} \oint \underbrace{\left(m v_\perp^2 \frac{m}{qB} \right)}_{\to \mu} d\theta =$

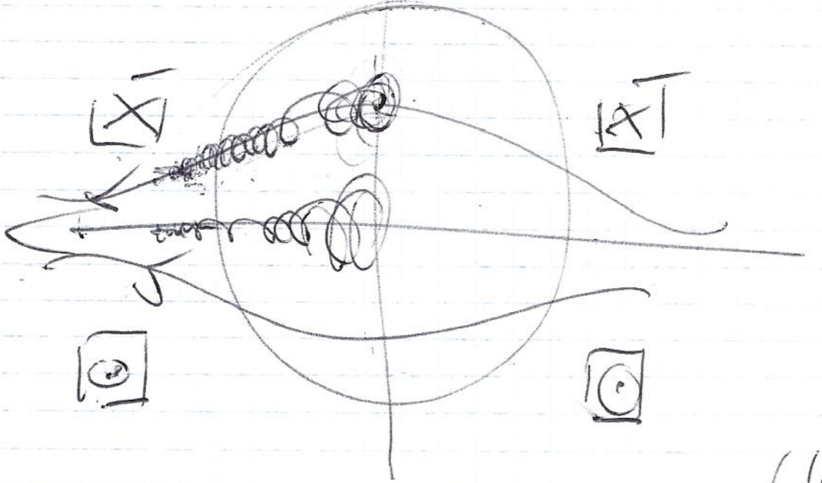
$dl = r_c \, d\theta$

$v_\perp = r_c \omega_c$

$= 2 \frac{m}{q} \oint \mu \, d\theta \propto \langle \mu \rangle$





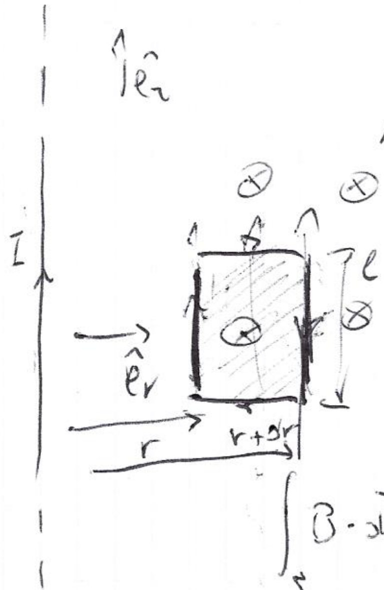


magnetic bottle

fusion?

(loss cone)

(15-13)



$$\bar A_0(\bar r) = \frac{\mu_0}{4\pi}\int \frac{\bar J(\bar r')}{|\bar r-\bar r'|}\, d\tau'$$

$dS'\, dl'$

anche $= \frac{\mu_0}{4\pi}\int \frac{I\, d\bar l'}{|\bar r-\bar r'|}$

$$\bar B_0 = \bar\nabla\times\bar A_0$$

$$\int_S \bar B\cdot d\bar S = \int_S (\bar\nabla\times\bar A_0)\cdot d\bar S = \oint_{\mathcal{C}=\partial S} \bar A_0\cdot d\bar l$$

$$\bar A_0(\bar r) = A_{0z}(\bar r)\,\hat e_z = A_{0z}(r)\,\hat e_z$$

$$\int_r^{r+dr} B_0(r)\, l\, dr = l\,\underbrace{\frac{\mu_0 I}{2\pi r}}_{B_0(r)}\, dr = A_{0z}(r)\, l - A_{0z}(r+dr)\, l = l\left[A_{0z}(r) - \left(A_{0z}(r) + dA_{0z}\right)\right] = -\, l\, dA_{0z}$$

$$dA_{0z} = -\frac{\mu_0 I}{2\pi r}\, dr$$

$$\cancel{A_{0z}(r) - A_{0z}(r_0) = \int dA_{0z} = -\frac{\mu_0 I}{2\pi}\log\left(\frac{r}{r_0}\right)}$$

$$A_{0z}(r) - A_{0z}(r_0) = \int_{r_0}^{r} dA_{0z} = -\frac{\mu_0 I}{2\pi}\log\left(\frac{r}{r_0}\right)$$

(15-14)

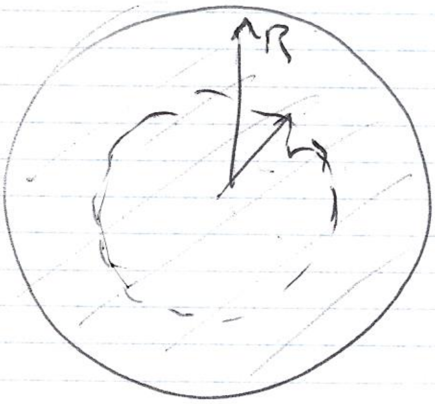
I, $n = N/L$

$\hat{e}_r$

$A_{0\theta}(r)$

$$\int_S \bar{B}_0 \cdot d\bar{S} = \oint_{\partial S} \bar{A}_0 \cdot d\bar{l}$$

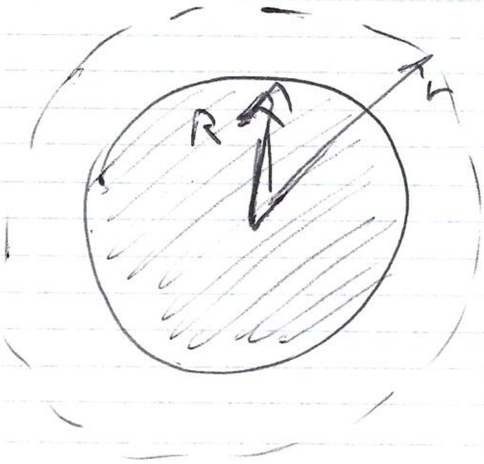
$\boxed{r < R}$



$$B_{0z}\,\pi r^2 = A_{0\theta}(r)\,2\pi r$$

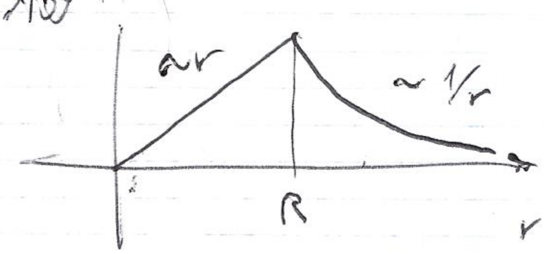
$$A_{0\theta}(r) = \frac{1}{2} B_{0z}\, r = \frac{1}{2}\mu_0 n I r$$

$\boxed{r > R}$



$$B_{0z}\,\pi R^2 = A_{0\theta}(r)\,2\pi r$$

$$A_{0\theta}(r) = \frac{1}{2} B_{0z}\, R^2/r = \frac{1}{2}\mu_0 n I R^2 \frac{1}{r}$$



$$\bar{J} \times \left(\frac{1}{r}\,\bar{e}_\theta \right) = \phi$$

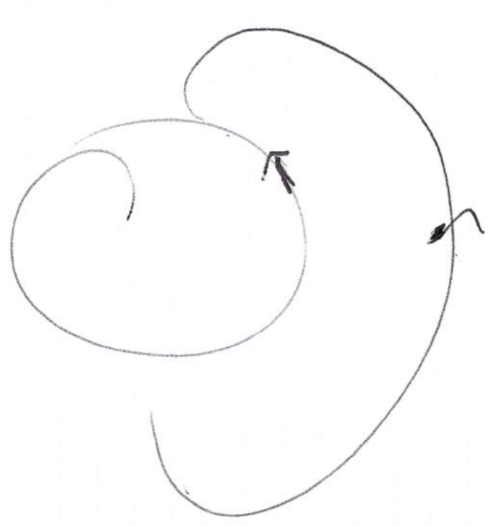
(15-12)

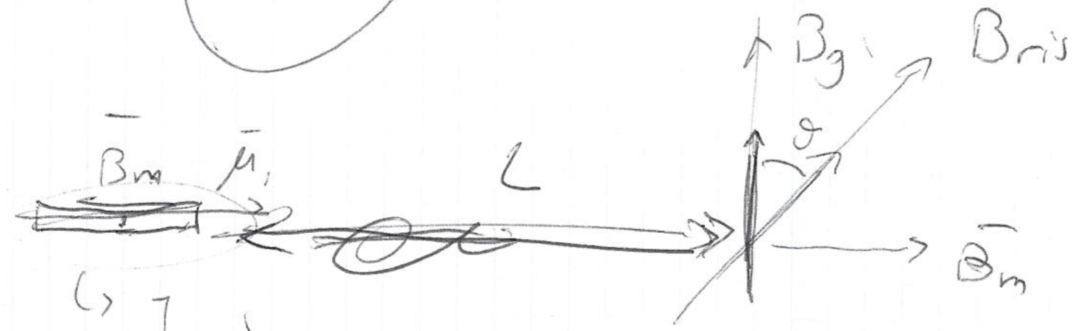
B_g = comp. orizz. del $\bar{B}$ terrestre

Magnete a sbarra

+

Ago magnetico





$$\omega = \sqrt{\frac{\mu B_g}{I}} \rightarrow \mu = \omega^2 I / B_g$$

$$B_g = B_{ris} \cos\vartheta = \frac{B_m}{\sin\vartheta} \cos\vartheta = B_m / \mathrm{tg}\,\vartheta$$

$$B_m = \frac{\mu_0}{2\pi} \frac{\mu}{L^3}$$

$$\bar{B}_{dip}(z) = \frac{\mu_0}{2\pi} \frac{\bar{m}}{\underbrace{\sqrt{R^2+z^2}^{3/2}}_{L^3}}$$

$$B_g = \frac{1}{\mathrm{tg}\,\vartheta} \frac{\mu_0}{2\pi} \frac{\mu}{L^3} = \frac{1}{\mathrm{tg}\,\vartheta} \frac{\mu_0}{2\pi L^3} \frac{\omega^2 I}{B_g}$$

$$B_g = \left(\frac{\mu_0 \, \omega^2 I}{2\pi L^3 \, \mathrm{tg}\,\vartheta} \right)^{1/2}$$

$I = 3.17 \cdot 10^{-2}$ Kg m^2

$\omega = 2\pi\nu \Rightarrow \nu = 0.1$ Hz

$L = 1$ m $\Rightarrow \vartheta = 45°$

$B_g = 5 \cdot 10^{-5}$ T =

= 0.5 Gauss