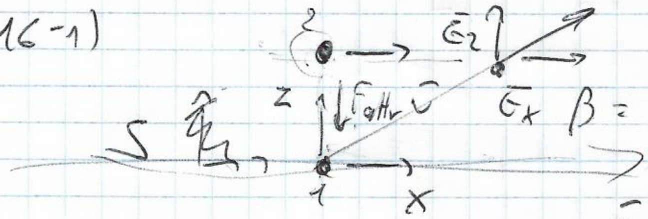
# ES. #22 – Moto relativistico di cariche; induzione elettromagnetica

19/IV/2021

(1)

(16-1)




$\beta = v/c$

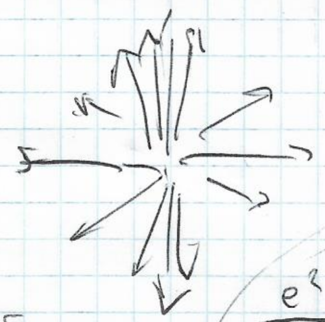
$\bar{E}\ (S) \rightarrow$ LRF $S'$

$\leftarrow -v$

$$E'_x = E_x = \frac{Q}{4\pi\varepsilon}\frac{x}{(x^2+z^2)^{3/2}} = \frac{Q}{4\pi\varepsilon_0}\frac{\gamma x'}{((\gamma x')^2+z'^2)^{3/2}}$$

$$E'_z = \gamma E_z = \gamma\frac{Q}{4\pi\varepsilon_0}\frac{z}{(x^2+z^2)^{3/2}} = \frac{Q\,\gamma z'}{((\gamma x')^2+z'^2)^{3/2}}$$

$$\frac{E_x}{E_z} = \frac{E'_x}{E'_z}$$



distanza $r$

$$S' \quad \gamma\frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow K \quad \longleftarrow \quad S \quad \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow K$$

$$F'_\perp = \frac{1}{\gamma}F_\perp = \frac{1}{\gamma}K \quad \text{forza totale}$$

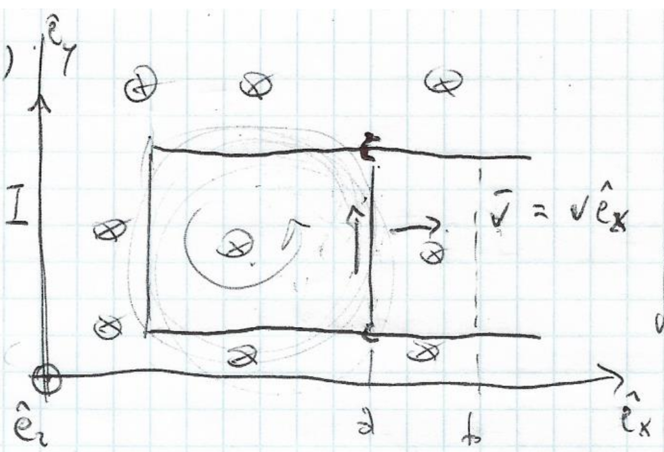
$$\frac{1}{\gamma}K = \gamma K - F_{attr} \qquad \gamma = 1/\sqrt{1-\beta^2}$$

$$F_{attr} = \left(\gamma - \frac{1}{\gamma}\right)K = \beta^2\gamma K \quad \text{origine magnetica}$$

$$F_{attr} = \gamma\beta^2\frac{e^2}{4\pi\varepsilon_0 r^2} = \underbrace{e\beta c\,\underbrace{\left(\frac{\beta}{c}\frac{\gamma e}{4\pi\varepsilon_0 r^2}\right)}_{B}}_{evB}$$

(Purcell)

(17-1)




$$f_i = -\frac{d\phi(\bar B)}{dt}$$ 2

$$\bar E_i = \bar v \times \bar B$$

Metodo (A) : $\bar E_i = \bar v \times \bar B_0 = -\frac{\mu_0 I v}{2\pi x} \underbrace{\hat e_x \times \hat e_z}_{-\hat e_y} = \frac{\mu_0 I v}{2\pi x}\hat e_y$

$$\bar B_0(x) = -\frac{\mu_0 I}{2\pi x}\hat e_z$$

$$f_i = \int \bar E_i \cdot \underbrace{d\bar l}_{dy\,\hat e_y} = \int_0^l (\bar v \times \bar B_0)\cdot dy\,\hat e_y = \frac{\mu_0 I v l}{2\pi x}$$

Metodo (B) : Faraday-Neumann

$$f_i = -\frac{d\phi(\bar B)}{dt} \qquad \phi = \int \bar B \cdot d\bar S$$


$$d\phi(B) = B_0(x)\,l\,dx$$

$$\frac{d\phi}{dt} = B_0(x)\,l\,\frac{dx}{dt} = B_0(x)\,l\,v$$

$$f_i = -d\phi/dt = -B_0(x)\,l\,v = -\frac{\mu_0 I l v}{2\pi x}$$

$$i_{ind} = \frac{f_i}{R} = \frac{\mu_0 I l v}{2\pi R x}$$

Sulla barretta mobile $d\bar F = i_{ind}\, d\bar l \times \bar B \rightsquigarrow \bar F = i_{ind}\,\bar l \times \bar B =$

$$= i\,l\,B_0(x)\underbrace{\hat e_y \times (-\hat e_z)}_{-\hat e_x} = -i_{ind}\,l\,B_0\,\hat e_x = -\frac{B_0^2 l^2 v}{R}\hat e_x =$$

$$= -\left(\frac{\mu_0 I l}{2\pi x}\right)^2 \frac{v}{R}\hat e_x$$

$$\bar{F}_{ext} = -\bar{F}$$

$$d\mathcal{L} = \bar{F}_{ext} \cdot d\bar{l} = F_{ext}\, dx$$

$$\mathcal{L} = \int_a^b \bar{F}_{ext}\, dx = \left(\frac{\mu_0 I l}{2\pi}\right)^2 \frac{v}{R} \int_a^b \frac{dx}{x^2} = \left(\frac{\mu_0 I l}{2\pi}\right)^2 \frac{v}{R}\, \frac{b-a}{ab}$$

$$\hookrightarrow \left[-\frac{1}{x}\right]_a^b$$

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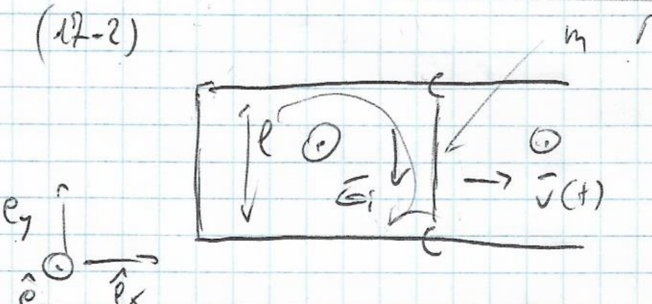
(17-2)




$$\bar{B}_0 = B_0 \hat{e}_z$$

impulso $\bar{J} = m\bar{v}_0$

$$\bar{E}_i = \bar{v} \times \bar{B}_0 = v B_0\, \hat{e}_x \times \hat{e}_z = -vB\hat{e}_y$$

$$f_i = \int \bar{E} \cdot d\bar{l} = -vB_0 l \quad \rightsquigarrow \quad i_{ind} = f_i/R = -v_0 B l/R$$

$$\hookrightarrow l\hat{e}_y$$

$$\bar{F} = i l \hat{e}_y \times \bar{B}_0 = -v B_0^2\, l^2/R\; \hat{e}_x$$

$v(t)$

$$F_x = m\frac{dv}{dt} \quad (v_x)$$

$$\downarrow$$

$$-\frac{B_0^2 l^2}{R} v = m\frac{dv}{dt} \quad \rightsquigarrow \quad \int_{v_0}^{v} \frac{dv'}{v'} = -\frac{B_0^2 l^2}{mR}\int_0^t dt'$$

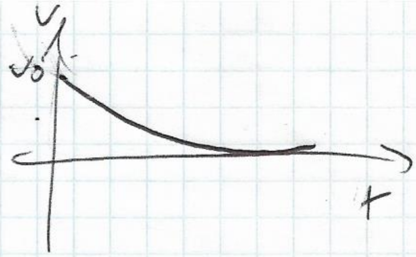
$$v(t) = v_0\, e^{-t/\tau}$$

$$\tau = mR/B_0^2 l^2$$

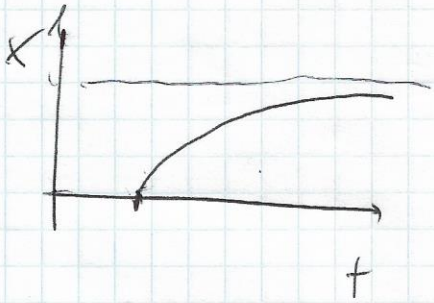
$$v = dx/dt \qquad \int_{x_0}^{x} dx' = \int_0^t v\,dt' = v_0 \int_0^t e^{-t'/\tau}\, dt' = -v_0\tau \left[e^{-t/\tau}\right]_0^t$$

$$x - x_0 = v_0\tau\left[1 - e^{-t/\tau}\right]$$

$$x = x_0 + v_0\tau\left(1 - e^{-t/\tau}\right)$$




$$x_\infty = x_0 + v_0\tau = x_0 + \frac{mRv_0}{B_0^2 l^2}$$




$$G = \int_0^{+\infty} P_J\, dt = \int_0^{+\infty} R\, i^2\, dt =$$

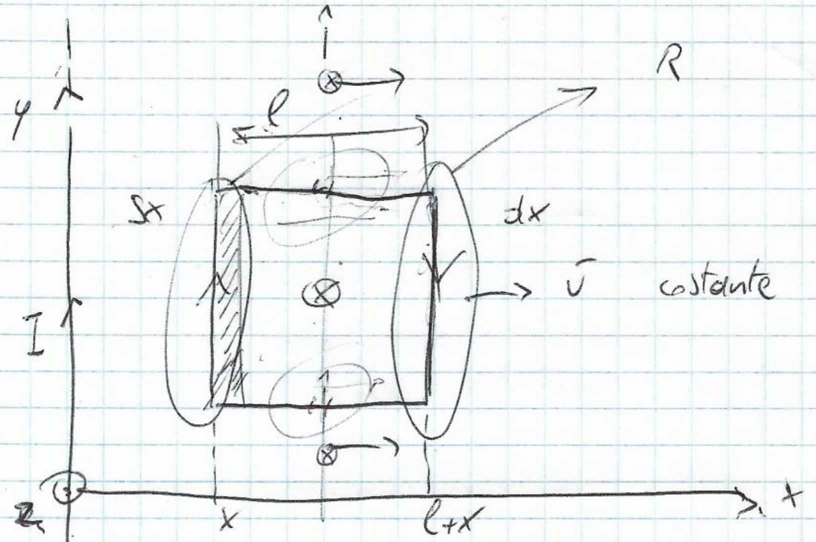
$$i(t) = -v(t) B_0 l / R = -\frac{B_0 l v_0}{R} e^{-t/\tau}$$

$$\frac{B_0^2 l^2 v_0^2}{R}\int_0^{+\infty} e^{-2t/\tau}\, dt = -\frac{B_0^2 l^2 v_0^2}{R}\underbrace{\frac{\tau}{2}}_{\frac{mR}{2B_0^2 l^2}}\underbrace{\left[e^{-2t/\tau}\right]_0^{+\infty}}_{-1} = \frac{1}{2} m v_0^2 = E_{K,iniz}$$

(17-3)



$f_i \Rightarrow i_{ind} = \frac{f_i}{R}$




$\bar v$ costante $\qquad P_J$

Metodo A: campo elettromotore $\qquad \bar{E}_i = \bar v \times \bar B_0$

$$f_i = f_{i,sx} + f_{i,dx} = \underbrace{\left[\bar v \times \overbrace{\bar B_0(x)}^{\hat e_y}\right]}_{\bar E_{i,sx}} \cdot l \hat e_y + \underbrace{\left[\bar v \times \overbrace{\bar B_0(l+x)}^{\hat e_y}\right]}_{\bar E_{i,dx}} \cdot l(-\hat e_y) =$$

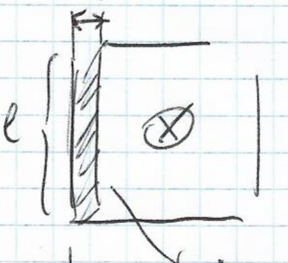
$$f_i = \oint \bar E_i \cdot d\bar l$$

$$f_i = vl \frac{\mu_0 I}{2\pi x} - vl \frac{\mu_0 I}{2\pi(l+x)} = \frac{\mu_0 I l^2 v}{2\pi x(l+x)}$$

$$i_{ind} = f_i / R$$

Metodo B:

$$f_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_x^{l+x} B_0(x') l\, dx' = -\frac{\mu_0 I}{2\pi}\frac{d}{dt}\left[\log\left(\frac{l+x}{x}\right)\right]$$



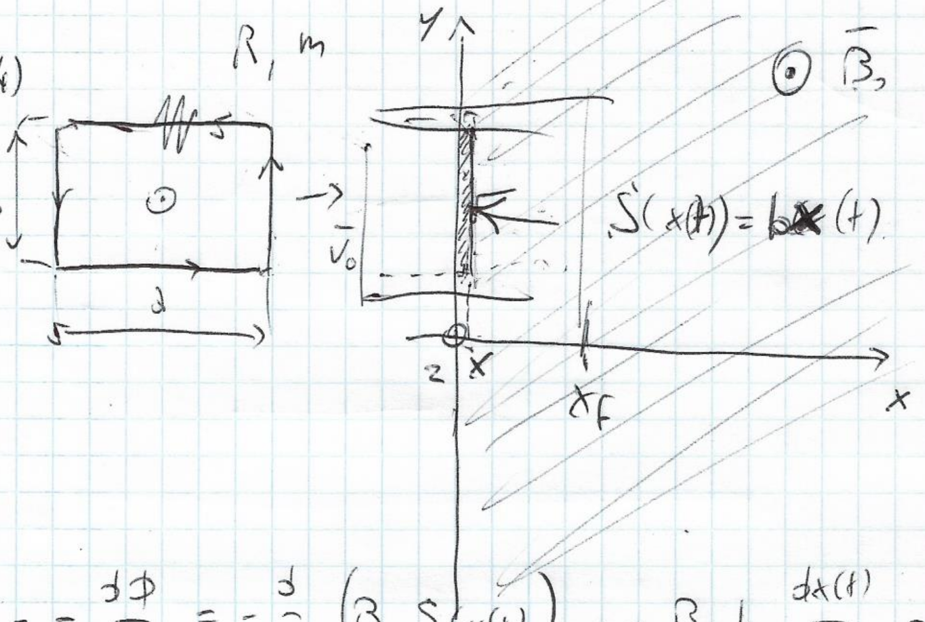

$$\frac{d}{dt} = \frac{d}{dx}\frac{dx}{dt} = v\frac{d}{dx}$$

$dS = l\,dx \quad \bar B_0(x)$

$$f_i = -\frac{\mu_0 I l v}{2\pi}\underbrace{\frac{d}{dx}\left[\log\left(\frac{x+l}{x}\right)\right]}_{= l/x(x+l)} = \frac{\mu_0 I l^2 v}{2\pi x(x+l)}$$

$$P_J = R I^2 = f_i^2 / R$$

(17-4)    R, m



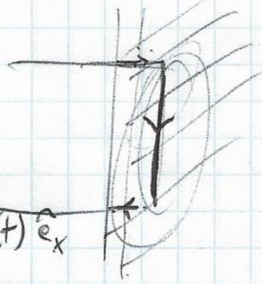

$S(x(t)) = b\,x(t)$

$v(t)$

$\bar E_i = \bar v \times \bar B$

$$f_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(B_0 S(x(t))\right) = -B_0 b \frac{dx(t)}{dx} = -B_0 b v(t)$$

$$i_{ind} = f_i / R = -B_0 b v / R$$



$$\bar F = i_{ind}\, b\, \underbrace{(-\hat e_y) \times B_0 \hat e_z}_{-\hat e_x} = -i_{ind} b B_0 \hat e_x = -\frac{B_0^2 b^2}{R} v(t)\, \hat e_x$$

$\bar F = m \bar a \rightarrow$ lungo $\hat e_x$

$$F_x = m \frac{dv}{dt}$$

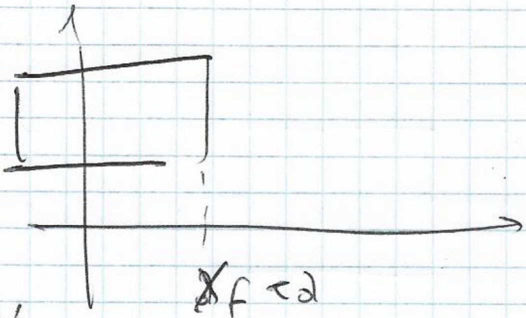
$$-\frac{B_0^2 b^2}{R} v(t) = m \frac{dv(t)}{dt} \qquad dx = v\,dt$$

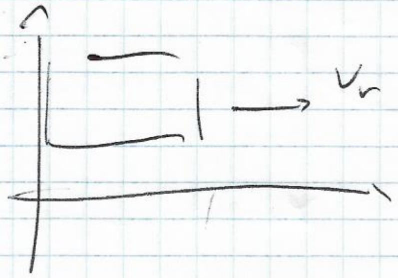
$$dv = -\frac{B_0^2 b^2}{mR} dx \rightsquigarrow \int_{v_0}^{v} dv' = -\frac{B_0^2 b^2}{mR} \int_0^x dx'$$

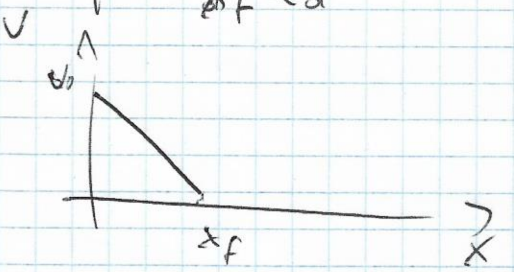
$$v = v_0 - \frac{B_0^2 b^2}{mR} x$$

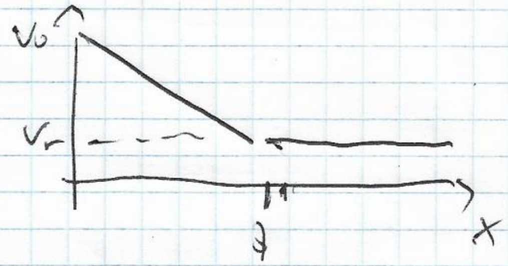
$x_{finale} = ? \rightsquigarrow v_{finale} = 0 \rightsquigarrow x_f = \frac{mRv_0}{B_0^2 b^2}$ se $x_f < d$

$$\frac{mRv_0}{B_0^2 b^2} < d \qquad v_r = v_0 - \frac{B_0^2 b^2}{mR} d > 0$$


xf < a


vr


v
v0
xf
x


v0
vr
x