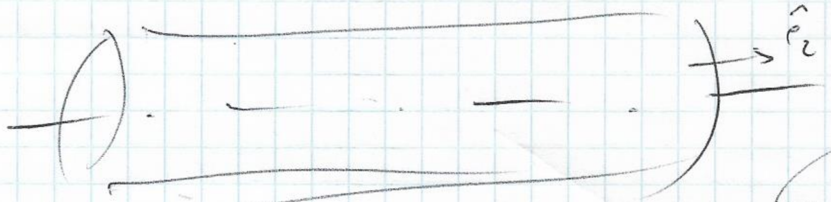
ES. #23 - induzione elettromagnetica / autoinduttanza

26/11/2021

(1)

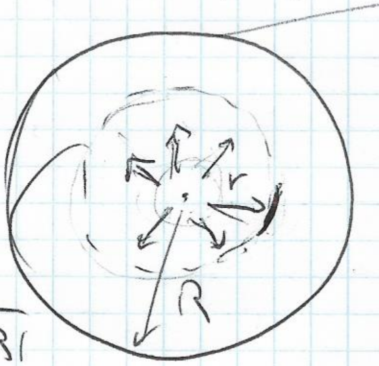
(12-5)



$i(t) = I_0 \sin(\omega t)$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(r, z, ϑ)



$\vec{E} = \vec{E}(r) \qquad \vec{E} \perp \hat{e}_z$

$\downarrow$

$E_\vartheta(r)$

$\boxed{r < R}$

$$\oint_{C = \partial S} \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$d\vec{l} = r\, d\vartheta\, \hat{e}_\vartheta$

$$\int_0^{2\pi} E_\vartheta(r)\, r\, d\vartheta = 2\pi r E_\vartheta(r)$$

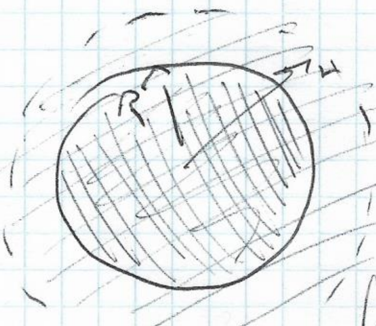
$\vec{B}_0 = \mu_0 n\, i(t)\, \hat{e}_z = \mu_0 n I_0 \sin(\omega t)\, \hat{e}_z$

$$-\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\mu_0 n I_0 \omega \cos(\omega t)\, \pi r^2$$

uniforme

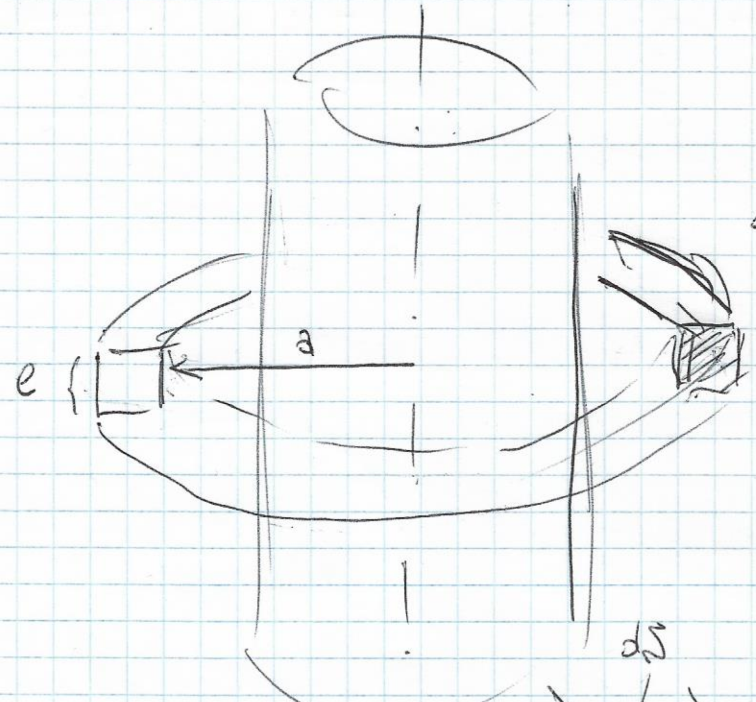
$$\boxed{E_\vartheta(r, t) = -\frac{1}{2} \mu_0 n I_0 r\, \omega \cos(\omega t)}$$

$\boxed{r > R}$



$$\oint \vec{E} \cdot d\vec{l} = 2\pi r E_\vartheta(r) = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial B}{\partial t} \pi R^2$$

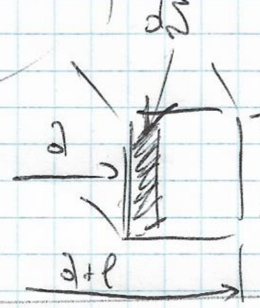
$$\boxed{E_\vartheta(r, t) = -\frac{R^2}{2r} \mu_0 n I_0 \omega \cos(\omega t)} \quad \sim \frac{1}{r} \hat{e}_\vartheta$$



$$\bar{J} = \sigma \bar{E} = \frac{1}{\rho} \bar{E}$$

$$I_a(t) = \int_S \bar{J} \cdot d\bar{S} =$$

$$= \int_a^{a+l} J(r)\, l\, dr =$$

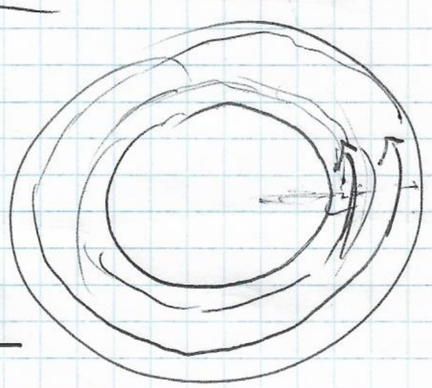


$$= \frac{1}{\rho} \frac{R^2}{2} \mu_0 n I_0 \omega \cos(\omega t)\, l \int_a^{a+l} \frac{dr}{r} \Rightarrow$$

$$\boxed{I_a(t) = \frac{R^2}{2\rho} \mu_0 n I_0\, l \log\left(\frac{a+l}{a}\right) \omega \cos(\omega t)}$$

$P_J = ?$

$$P_J = \int_{vol} \bar{E} \cdot \bar{J}\, dvol$$



$a >> l$

$$P_{J\,approx} \cong \hat{R}\, I_a^2$$

$$\left(R = \frac{\rho L}{\Sigma}\right) \qquad \hat{R} = \rho \frac{2\pi (a + l/2)}{l^2}$$

~~$R_{sol} = 80$ mm ; $n = 600$ sp/m ;~~

$R_{sol} = 80$ mm ; $n = 600$ sp/m ; $I_0 = 150$ A $\rightsquigarrow B_{0\,max} = 0.113$ T

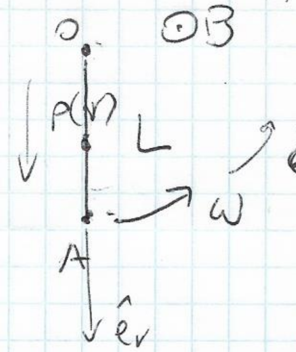
$\omega = 2\pi \cdot 50$ rad/s ; $a = 160$ mm ; $l = 10$ mm ; $\rho = 1.68 \cdot 10^{-8}\ \Omega \cdot m$

$\hat{R} = 1.76 \cdot 10^{-4}\ \Omega$; $I_{a\,max} = 814$ A ; $P_{J\,max} = \hat{R} I_a^2 \simeq 116$ W

$E_\theta (a + l/2)_{max} \simeq 0.14$ V/m

(17-6)

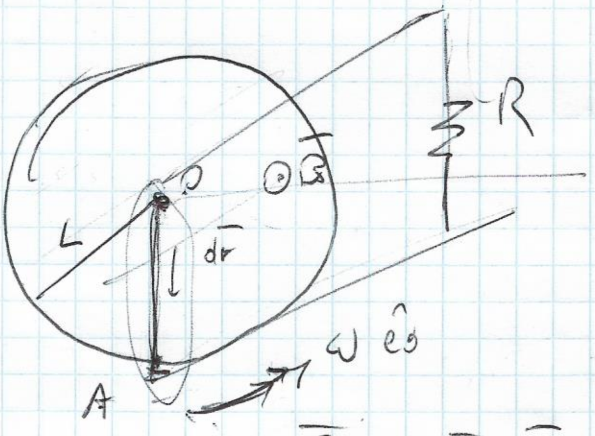
3



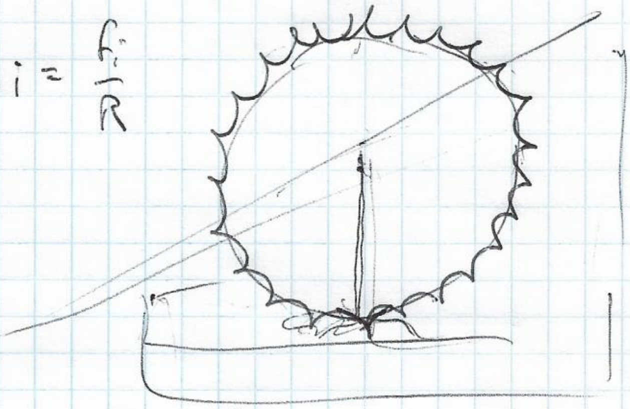
$$\bar{E}_i = \bar{v} \times \bar{B} = \omega r \hat{e}_\vartheta \times B \hat{e}_z = \omega r B \hat{e}_r$$

$$f_i = \int \bar{E}_i \cdot d\bar{l} = \int_\emptyset^L \omega r B \, dr = \frac{\omega B L^2}{2}$$

Disco di Barlow



$$i = \frac{f_i}{R}$$



$$d\bar{F} = i \, d\bar{r} \times \bar{B} = -iB \, dr \, \hat{e}_\vartheta$$

$$d\bar{M} = \bar{r} \times d\bar{F} = -iBr \, dr \, \hat{e}_r \times \hat{e}_\vartheta = -iBr \, dr \, \hat{e}_z$$

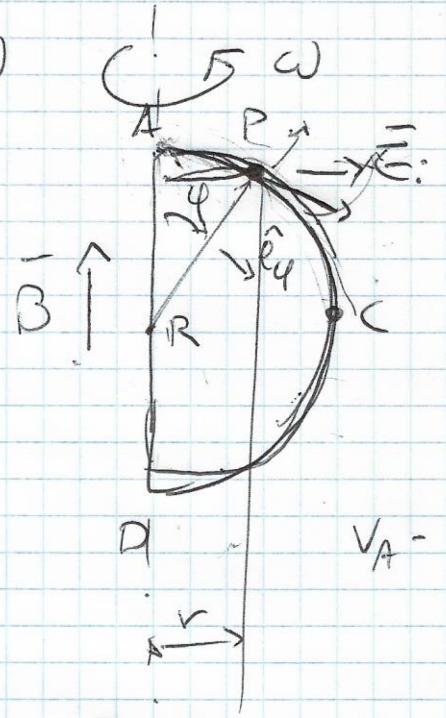
$$\bar{M}_m = -iB \int_\emptyset^L r \, dr \, \hat{e}_z = -iB \frac{L^2}{2} \hat{e}_z = -\frac{\omega B^2 L^4}{4R} \hat{e}_z$$

$$\bar{M}_{ext} = -\bar{M}_m = \frac{\omega B^2 L^4}{4R} \hat{e}_\vartheta$$

$$P_{mecc} = \omega M_{ext} = \omega^2 B^2 L^4 / 4R$$

$$P_J = R i^2 = P_{mecc}$$

(17-7)



$$\bar{E}_i = \bar{v} \times \bar{B}$$

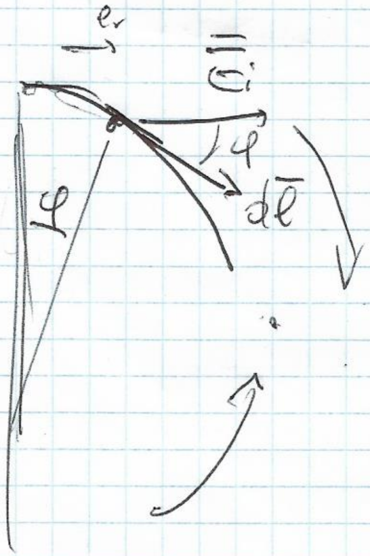
$$\bar{E}_i = R\sin\varphi\, \omega B$$

$$v = r\omega = R\sin\varphi\, \omega$$

$$V_A - V_C = \int_A^C \underbrace{\bar{E}_i \cdot d\bar{l}}_{R\,d\varphi\, \hat{e}_\varphi} =$$

$$= \int_0^{\pi/2} \omega B R \sin\varphi \cdot R\,d\varphi \cos\varphi = \omega B R^2 \int_0^{\pi/2} \sin\varphi \cos\varphi\, d\varphi =$$

$$= \omega B R^2 \left[\frac{\sin^2\varphi}{2} \right]_0^{\pi/2} = \frac{\omega B R^2}{2}$$



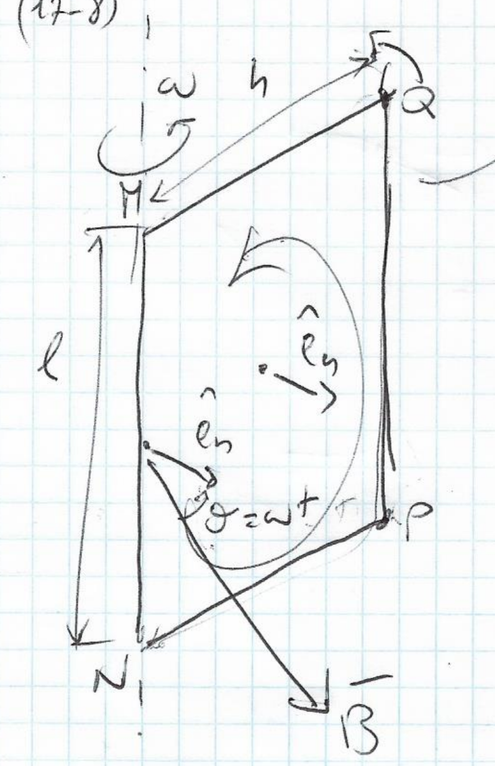
$$\cos\varphi = \hat{e}_r \cdot \hat{e}_\varphi$$

$$V_A = V_D$$

$$V_D - V_C = V_A - V_C$$

$$\Rightarrow V_D = V_A$$

(17-8)



$\rho = 1.68 \cdot 10^{-8}\ \Omega \cdot m$

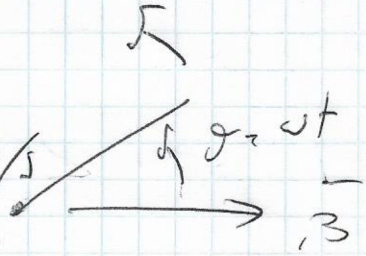
$\Sigma = 1\ mm^2$

$l = 10\ cm$

$h = 5\ cm$

$\omega = 150\ rad/s$

$B = 0.1\ T$



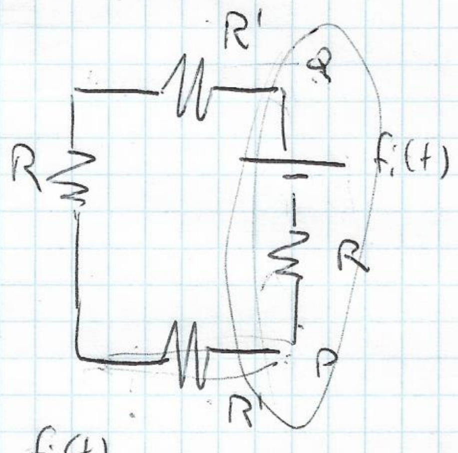
$V_P - V_Q = ?$

$\Phi(\bar{B}) = \bar{B} \cdot \bar{\Sigma}(t) = B l h \cos(\omega t)$

$\bar{B} // \hat{e}_n \quad t = 0$

$\vartheta \sim \omega t + \cancel{\varphi}$

$f_i = -\dfrac{d\Phi}{dt} = B l h \omega \sin(\omega t)$



$R' = \rho h / \Sigma$

$R = \rho l / \Sigma$

$R_{tot} = 2(R + R') = 2\rho(l+h)/\Sigma$

$i(t) = \dfrac{f_i(t)}{R_{tot}} = B l h \omega \sin(\omega t) \cdot \Sigma / 2\rho(l+h)$

$i_{max} = 29.76\ A$

$A - B \qquad V_A - V_B + \sum_{A \to B} f_{el} = I \sum_{A \to B} R_K$

$\downarrow$

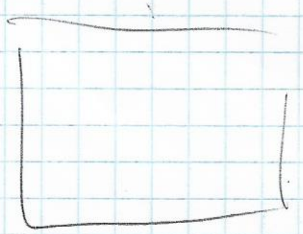
$P - Q \qquad V_P - V_Q + f_i(t) = i(t) \dfrac{\rho l}{\Sigma}$

$$V_P - V_Q = -Blh\omega\sin(\omega t) + Blh\omega\sin(\omega t)\cdot\frac{\Sigma}{2\rho(l+h)}\cdot\frac{\rho l}{\Sigma} =$$

$$= Blh\omega\sin(\omega t)\left[\frac{l}{2(l+h)} - 1\right]$$

$$V_P - V_Q = -Blh\omega\,\frac{2l+h}{2(l+h)}\,\sin(\omega t)$$

$$(V_P - V_Q)_{max} = \downarrow = 0.1\ V$$



Autoinduzione

quasi stazionarietà

'auto' fenomeno

$$U_m = \frac{1}{2} L I^2$$

$$\bar{B}_0(\bar{r}) = \frac{\mu_0}{4\pi}\oint I\,\frac{d\bar{l}\times\Delta\bar{r}}{\Delta r^3} \propto I$$

$$\Phi(\bar{B}) = \int \bar{B}\cdot d\bar{S} \propto \bar{B} \propto I$$

$$\boxed{\Phi(\bar{B}) = L\,I}$$

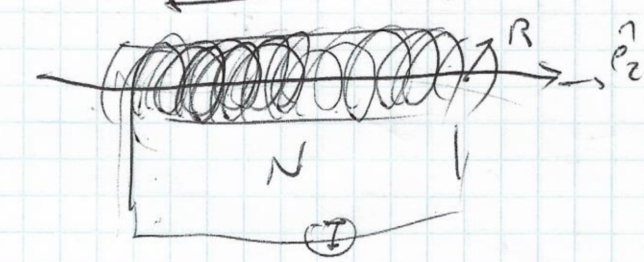
L coeff. di autoinduzione (induttanza)

$$[L] = \frac{[Wb]}{[A]} = \frac{[V\cdot s]}{[A]} = [\Omega\cdot s] = [H]$$

Henry u.d.m. di L

(18-1)

$l \gg R$



$$\bar{B}_0 = \mu_0 n I \hat{e}_z = \mu_0 \frac{N}{l} I \hat{e}_z$$

$$\Phi_{sp}(\bar{B}) = B_0 S = B_0 \pi R^2$$

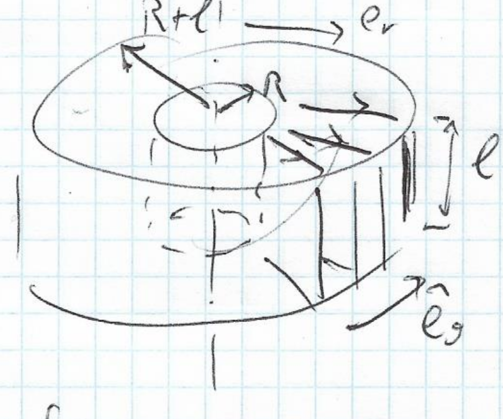
$$\Phi = N \Phi_{sp} = N B_0 \pi R^2 = \mu_0 \frac{N^2 I}{l} \pi R^2$$

$$L_0 = \frac{\Phi}{I} = \mu_0 \frac{N^2}{l} \pi R^2 = \mu_0 n^2 l \pi R^2$$

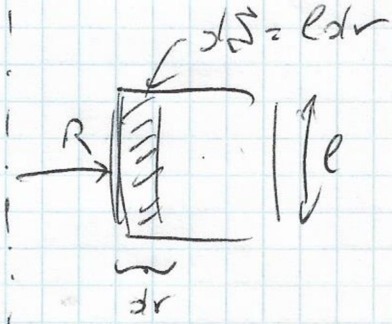
$n = N/l$

$$\left.\begin{array}{l} l = 10 \text{ cm} \\ R = 1 \text{ cm} \\ N = 1000 \end{array}\right\} L_0 = 3.95 \text{ mH}$$

(18-2)



$$\bar{B}_0(r) = \mu_0 \frac{NI}{2\pi r} \hat{e}_\theta$$



$$\Phi(\bar{B}_0) = N \int \bar{B}_0 \cdot d\bar{S} =$$

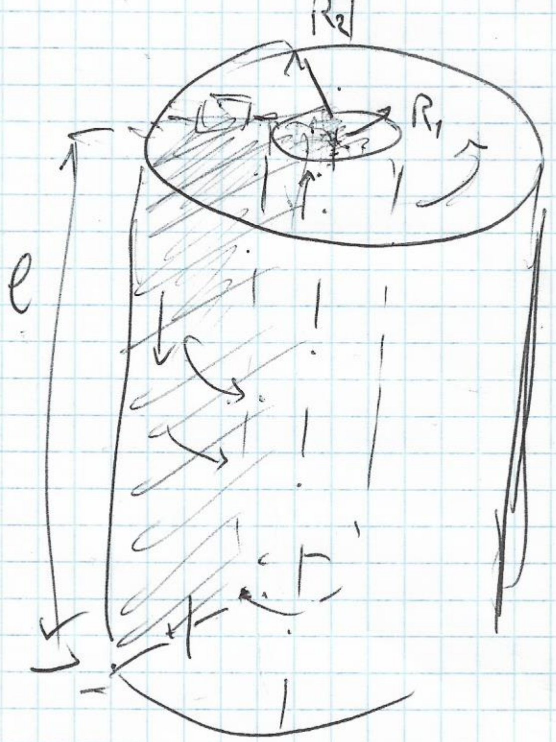
$$= N \int_R^{R+l} \underbrace{\frac{\mu_0 N I}{2\pi r}}_{B(r)} \underbrace{l\,dr}_{dS} = \frac{\mu_0 N^2 I l}{2\pi} \log\left(\frac{R+l}{R}\right)$$

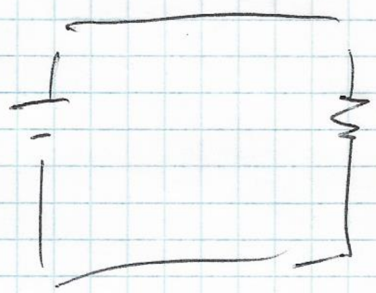
$$L_0 = \frac{\Phi}{I} = \frac{\mu_0 N^2 l}{2\pi} \log\left(\frac{R+l}{R}\right) = 2.77 \cdot 10^{-5} \text{ H} \left\{\begin{array}{l} R = 5 \text{ mm} \\ l = 5 \text{ mm} \\ N = 200 \end{array}\right.$$

ferro- solido (lineare)

$$\mu_0 \to \mu = \mu_0 \mu_r \hookrightarrow \sim 100 - 1000$$

(18-4) Cavo coassiale





$$\bar{B}_0(\bar{r}) = \frac{\mu_0 I}{2\pi r}\hat{e}_\varphi$$

$$\Phi(\bar{B}_0) = \int_{R_1}^{R_2} \frac{\mu_0 I}{2\pi r}\, l\, dr = \frac{\mu_0 I l}{2\pi}\log\left(\frac{R_2}{R_1}\right)$$

$$L_{0,u} = \frac{\Phi}{l I} = \frac{\mu_0}{2\pi}\log\left(\frac{R_2}{R_1}\right)$$

induttanza "esterna"

$r < R_1 \qquad I\frac{r^2}{R_1^2}$