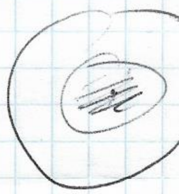
ES. #24 Induttanza 27/11/2021

(1)

(18-4) reprise

$B \neq \emptyset$ in R_1 e in R_2

Cond. centrale pieno $\quad I \to J\pi R_1^2$



$$B_\theta(r) = \frac{\mu_0 I}{2\pi R_1^2} r \cdot u_\theta \quad \forall r < R_1$$

$r < R_1$

$$d\Phi_{in}' = B(r)\, \ell\, dr$$

$$L = \frac{\Phi}{I}$$

$\hookrightarrow$ no I ma $I\left(\frac{r}{R_1}\right)^2$

$$d\Phi_{in} = B(r)\, dr \cdot \left(\frac{r}{R_1}\right)^2$$

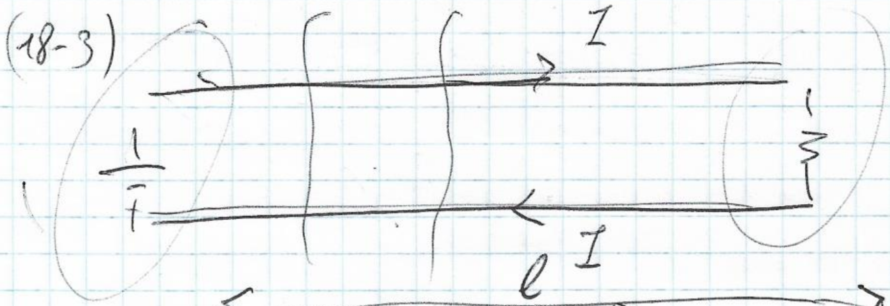
$$\Phi_{in} = \int d\Phi_{in} = \int_0^{R_1} \frac{\mu_0 I}{2\pi R_1^4} r^3 \ell\, dr = \frac{\mu_0 I \ell}{2\pi R_1^4} \left.\frac{r^4}{4}\right|_0^{R_1} \Rightarrow$$

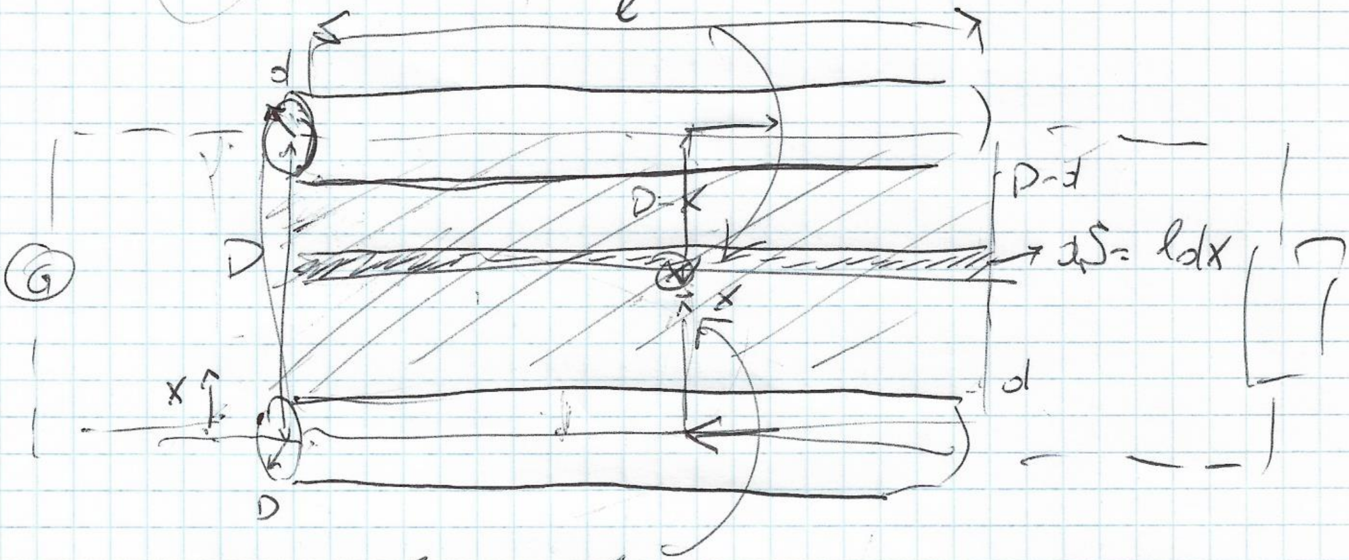
$$\boxed{\Phi_{in} = \frac{\mu_0 I \ell}{8\pi}}$$

$$L_{in,u} = \frac{\Phi_{in}}{\ell I}$$

$$L_{\text{qu}} = \frac{\Phi_{in}}{\ell I} + \frac{\Phi_{out}}{\ell I} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log\frac{R_2}{R_1}\right]$$

(18-3)





$$B_0(x) = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(D-x)} \qquad d\Phi = B_0(x)\, dS = B_0(x)\, l\, dx$$

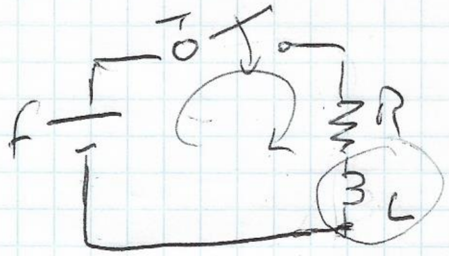
$$\Phi(\vec{B}_0) = \frac{\mu_0 I}{2\pi}\int_d^{D-d}\left[\frac{1}{x} + \frac{1}{D-x}\right] l\, dx = \frac{\mu_0 I l}{2\pi}\Big[\log x + \log(D-x)\Big]_d^{D-d} =$$

$$= \dots = \frac{\mu_0 I l}{\pi}\log\left(\frac{D-d}{d}\right) \underset{D \gg d}{\simeq} \log\left(\frac{D}{d}\right)$$

$$L_{0,u} = \frac{\Phi}{l I} = \frac{\mu_0}{\pi}\log\left(\frac{D-d}{d}\right)$$

Circuiti con L (3)

Transitorio di chiusura di circ. RL (18-5)



$t=0$ toggle on

$$\Phi_a = LI$$

$$f_a = -\frac{d\Phi_a}{dt} = -L\frac{dI}{dt}$$

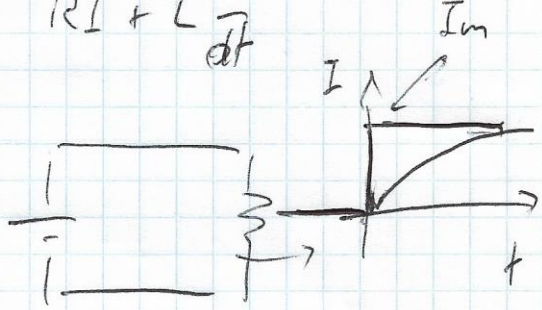
$f + f_a = RI$

$\hookrightarrow -L\, dI/dt$

$$f - L\frac{dI}{dt} = RI$$

$$f = RI + L\frac{dI}{dt}$$

$$\tau = \frac{L}{R} \qquad I_m = \frac{f}{R}$$



$$\frac{dI}{I-I_m} = -\frac{dt}{\tau}$$

$d(I - I_m) = dI$

$$\log(I - I_m) = -t/\tau + \log k$$

$$I(t) - I_m = k\, e^{-t/\tau} \qquad I(0) = 0$$

$$0 - I_m = k \qquad k = -I_m$$

$$I(t) = I_m(1 - e^{-t/\tau}) = \frac{f}{R}(1 - e^{-t/\tau})$$

$$f_a(t) = -L\frac{dI}{dt} = -f e^{-t/\tau}$$

$$f_a(0) = -f \qquad \tau = L/R$$

④

$$f = RI + L\frac{dI}{dt}$$ energeticamente?

$$d\mathcal{L} = f\,dQ = f I\,dt$$

$$P = \frac{d\mathcal{L}}{dt} = fI$$

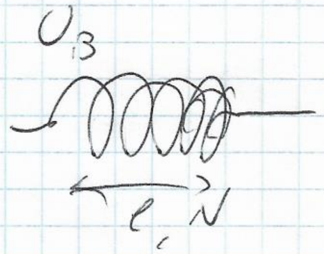
$$fI = \underbrace{RI^2}_{P_J} + \underbrace{LI\frac{dI}{dt}}_{P_L}$$

$$d\mathcal{L}_G = fI\,dt = RI^2\,dt + LI\,dI = dU_R + dU_L$$

$$\mathcal{L}_G = \int_0^t fI\,dt'$$

$$U_R = \int_0^t RI^2\,dt'$$

$$U_L = \int_0^{I(t)} LI'\,dI' = \frac{1}{2}LI^2 \sim$$



$$L\frac{dI}{dt} = \frac{d\Phi}{dt} = NS\frac{dB}{dt}$$

$$\Phi = NSB$$

$$dU_L = LI\frac{dI}{dt} = I\frac{d\Phi}{dt} = NIS\,dB = \frac{S\ell}{\mu_0}B\,dB$$

$S\ell$

$$B = \mu_0\frac{N}{\ell}I$$

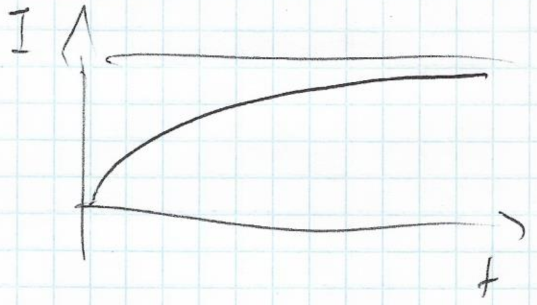
$$du_L = \frac{dU_L}{S\ell} = \frac{1}{\mu_0}B\,dB$$

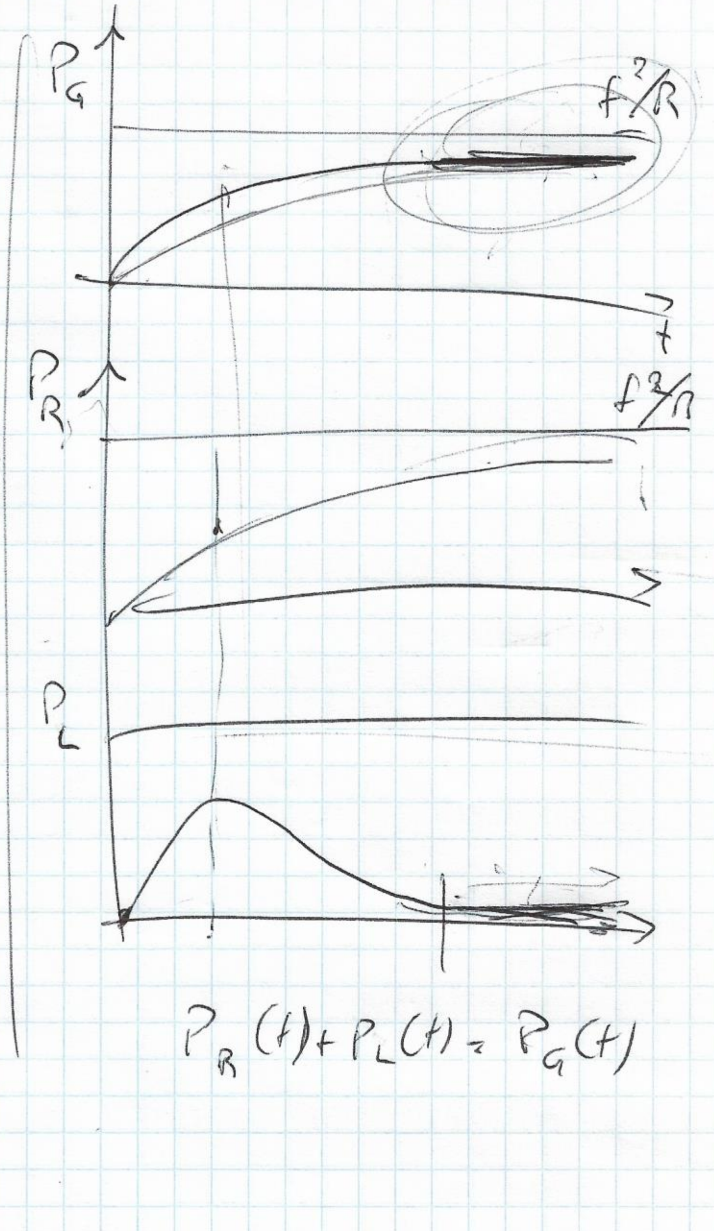
$$u_m = u_L = \int\frac{1}{\mu_0}B\,dB = \frac{1}{2\mu_0}B^2$$

$\vec{B}\cdot\vec{B}$

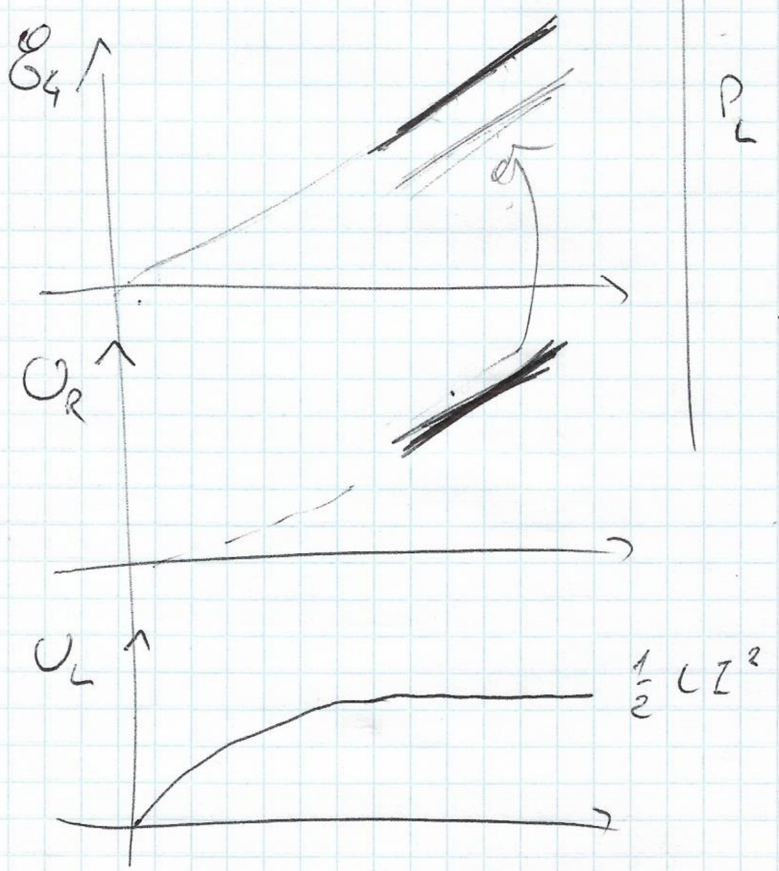
$$\Phi = \int \bar{B} \cdot d\bar{S} = \int (\bar{\nabla} \times \bar{A}) \cdot d\bar{S}$$

$$U_m = \frac{1}{2\mu_0} \int B^2 \, d\tau = \frac{1}{2} \int_\tau \bar{A} \cdot \bar{J} \, d\tau$$





$$R I^2(\infty) = R \frac{f^2}{R^2} = \frac{f^2}{R}$$

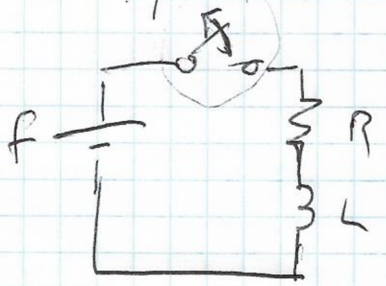


$$P_R(t) + P_L(t) = P_G(t)$$

(18-6)

⑥

Apertura RL $\rightsquigarrow$ $R \rightsquigarrow R'$



$$f - L\frac{dI}{dt} = R'I \qquad \tau = L/R'$$

$$I(t) = \frac{f}{R'} - k e^{-t/\tau}$$

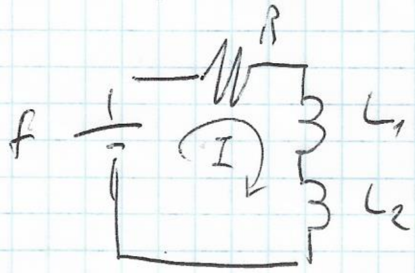
$$I(0) = I_0 = \frac{f}{R} = \frac{f}{R'} - k \rightsquigarrow k = f\left(\frac{1}{R'} - \frac{1}{R}\right) = \frac{f}{R'} - I_0$$

$$I(t) = \frac{f}{R'} - \frac{f}{R'}e^{-t/\tau} + I_0 e^{-t/\tau} = \frac{f}{R'}\left(1 - e^{-t/\tau}\right) + I_0 e^{-t/\tau}$$

$$I_0 \to \frac{f}{R}$$

$$I(t) \simeq I_0 e^{-t/\tau} = \frac{f}{R} e^{-t/\tau}$$

Serie



$$f_{a1} = -L_1 \frac{dI}{dt} = -f\frac{L_1}{L_1+L_2} e^{-t/\tau}$$

$$f_{a2} = -L_2 \frac{dI}{dt} = -f\frac{L_2}{L_1+L_2} e^{-t/\tau}$$

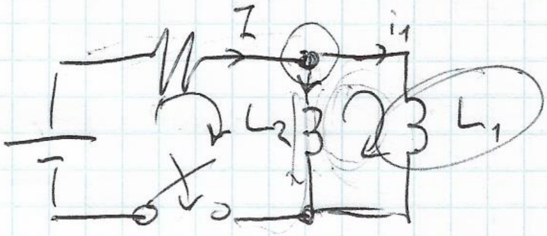
$$f + f_{a1} + f_{a2} = RI$$

$$f - \underbrace{(L_1+L_2)}_{L_{eq}}\frac{dI}{dt} = RI \rightsquigarrow f - L_{eq}\frac{dI}{dt} = RI$$

$$I(t) = \frac{f}{R}\left(1 - e^{-t/\tau}\right) \qquad L_{eq\,series} = \sum_k L_k$$

$$\tau = L_{eq}/R = (L_1+L_2)/R$$

Parallelo di induttanze



$$\begin{cases} f - L_2 \frac{d}{dt}(I - i_1) = RI & (1) \\ -L_1 \frac{di_1}{dt} - L_2 \frac{d}{dt}(i_1 - I) = \phi & (2) \end{cases}$$

$$(2) \quad (L_1 + L_2)\frac{di_1}{dt} = L_2 \frac{dI}{dt}$$

$$\frac{di_1}{dt} = \frac{L_2}{L_1+L_2}\frac{dI}{dt}$$

sost. in (1):

$$f - L_2 \frac{dI}{dt} + \frac{L_2^2}{L_1+L_2}\frac{dI}{dt} = RI$$

$$f = RI + \frac{L_1 L_2}{L_1+L_2}\frac{dI}{dt}$$

$\frac{L_1 L_2}{L_1+L_2} \to L_{eq}$

$$\frac{1}{L_{eq\parallel}} = \sum_k \frac{1}{L_k}$$

$$\tau \doteq L_{eq}/R$$

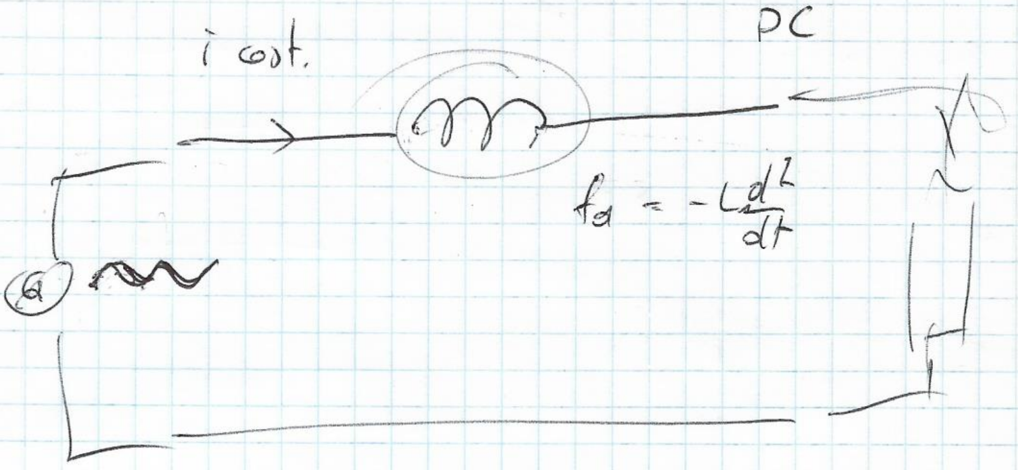
$$I(t) = \frac{f}{R}\left(1 - e^{-t/\tau}\right)$$

$$i_1(t) = \frac{L_2}{L_1+L_2} I(t) + k \qquad k \to \phi$$

$$i_1(t) = \frac{f}{R}\frac{L_2}{L_1+L_2}\left(1 - \exp(-t/\tau)\right)$$

in L_2 passa $I - i_1 = \frac{f}{R}\frac{L_1}{L_1+L_2}\left(1 - \exp(-t/\tau)\right)$

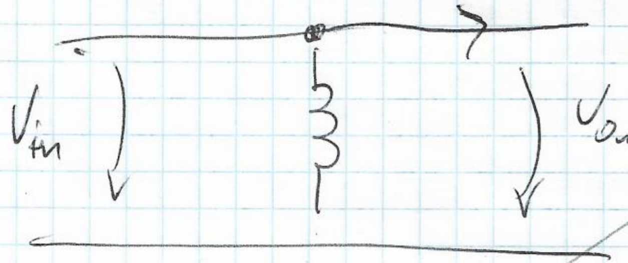
L, C

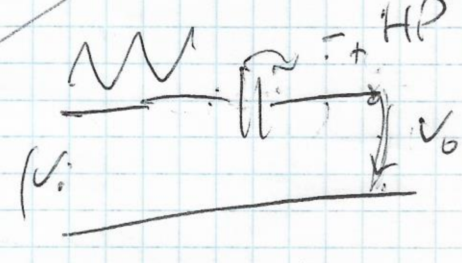


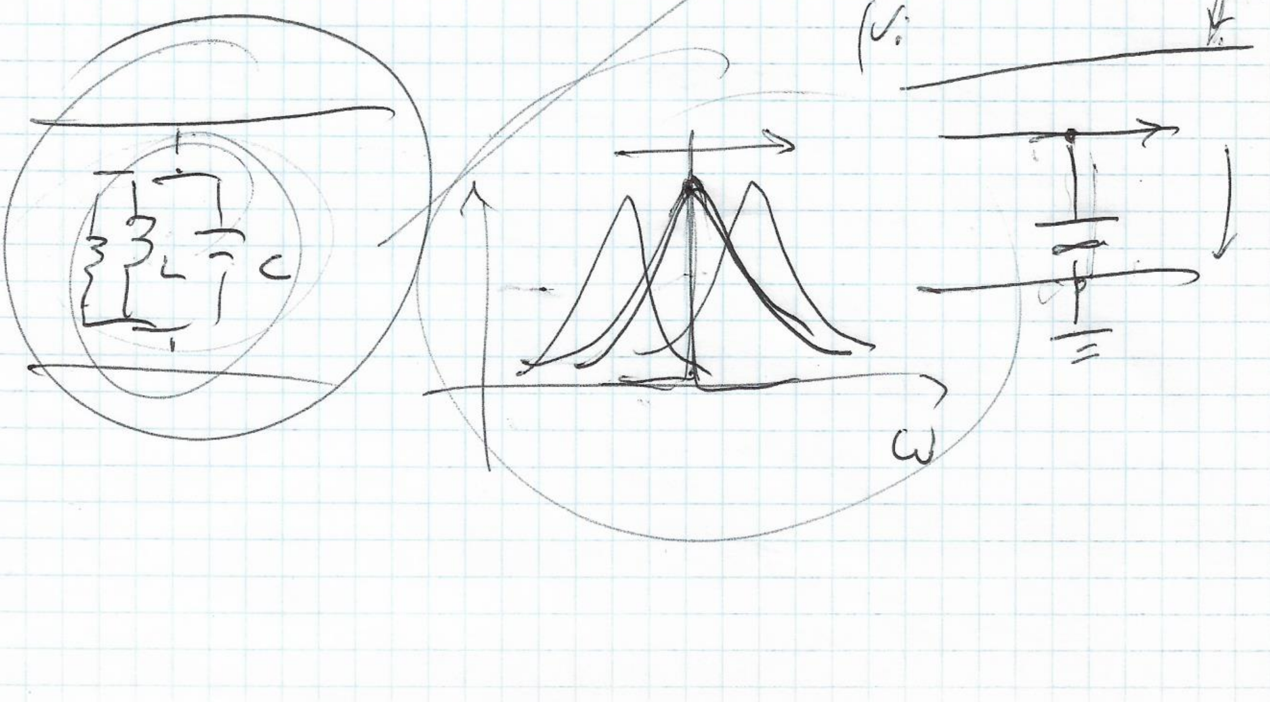
$$\frac{d}{dt}\sin(\omega t) \sim \omega\cos(\omega t)$$

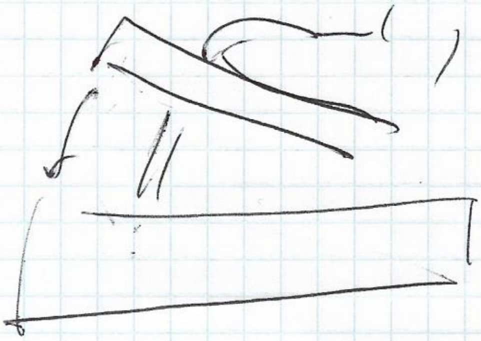
ripple Lowpass filter

choke









Wah pedal, e.g. Crybaby (youtube documentary)

'Cry Baby - The pedal that rocks the world'

also 'I found the first wah demo ever! (1967)'

on the JHS Pedals youtube channel

Tech expl.: 'The technology of wah pedals' on www.geofex.com