ES. #25 Induttanza - mutua induttanza - macchine elettriche 03/11/2021

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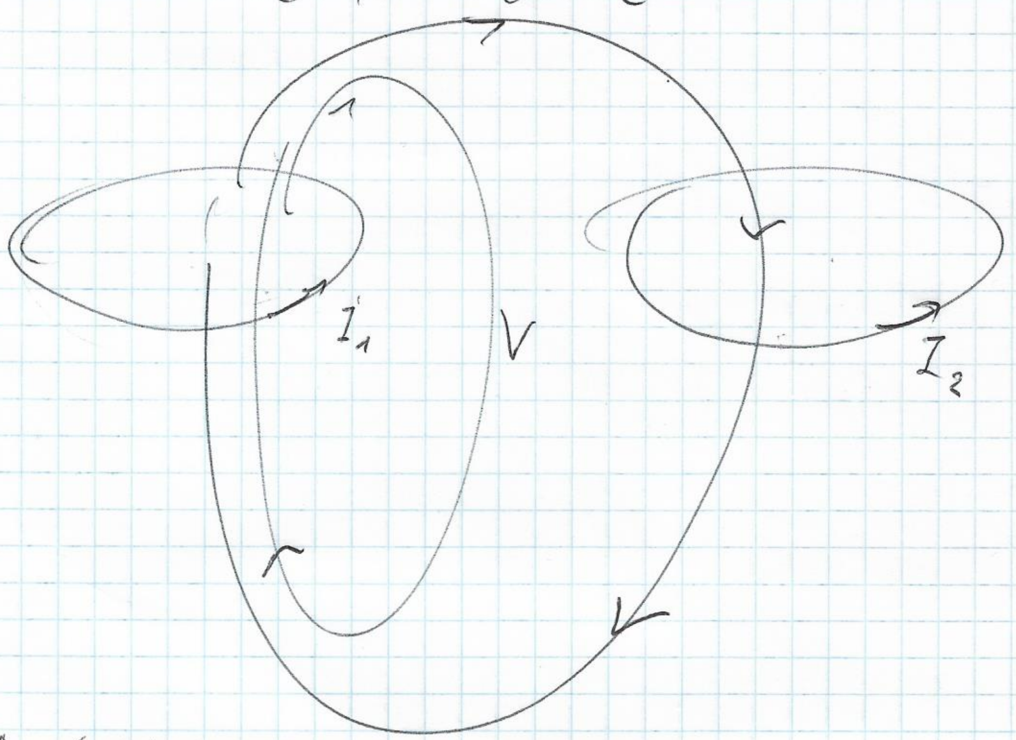


$$\Phi_2(\bar{B}) = \Phi_2(\bar{B}_1) + \Phi_2(\bar{B}_2) =$$

$$= M_{12} I_1 + L_2 I_2$$

$$\Phi_1(\bar{B}) = \Phi_1(\bar{B}_1) + \Phi_1(\bar{B}_2) =$$

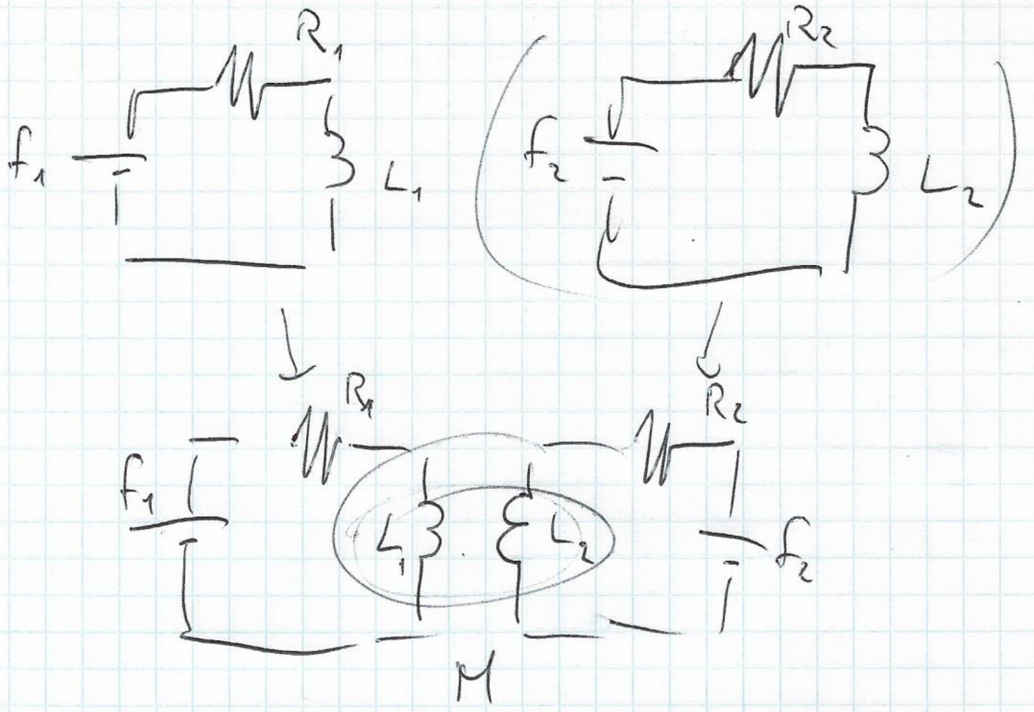
$$= L_1 I_1 + M_{21} I_2$$

$$M_{12} = M_{21} = M$$

$$\Phi_1(\bar{B}_2) = \int_{S_1} \bar{B}_2 \cdot d\bar{S} \qquad = \dots = I_2 \, \underbrace{\oint}_{M_{21}}$$

$$M_{21} = \Phi_1(\bar{B}_2)/I_2$$

$$\Phi_2(\bar{B}_1) \quad =$$

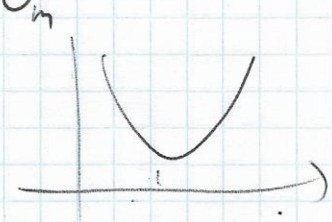


$$\begin{cases} f_1 - L_1 \dfrac{dI_1}{dt} - M \dfrac{dI_2}{dt} = R_1 I_1 & \leftarrow \quad I_1\,dt \ (= dQ_1) \\[2ex] f_2 - L_2 \dfrac{dI_2}{dt} - M \dfrac{dI_1}{dt} = R_2 I_2 & \leftarrow \quad I_2\,dt \ (= dQ_2) \end{cases}$$

$$\underbrace{f_1 I_1\,dt + f_2 I_2\,dt}_{d\mathcal{L}_g} = (R_1 I_1^2 + R_2 I_2^2)\,dt + \underbrace{L_1 I_1\,dI_1 + L_2 I_2\,dI_2 + M(I_1\,dI_2 + I_2\,dI_1)}_{dU_m}$$

$$dU_m = d\Big(\underbrace{\tfrac{1}{2} L_1 I_1^2 + \tfrac{1}{2} L_2 I_2^2 + M I_1 I_2}\Big)$$

$$U_m =$$



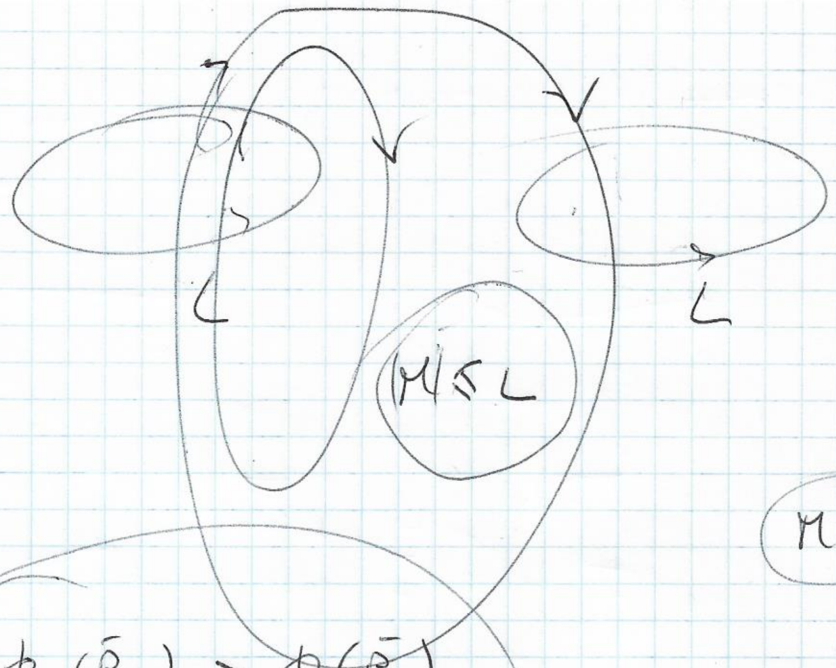
$$U_m = \frac{1}{2} \sum_k I_k \Phi_k$$

Φ_k flusso conc. dal circ. k <u>da tutti i campi</u>

$L_1, \; L_2, \; M$

$x = \dfrac{I_1}{I_2}$

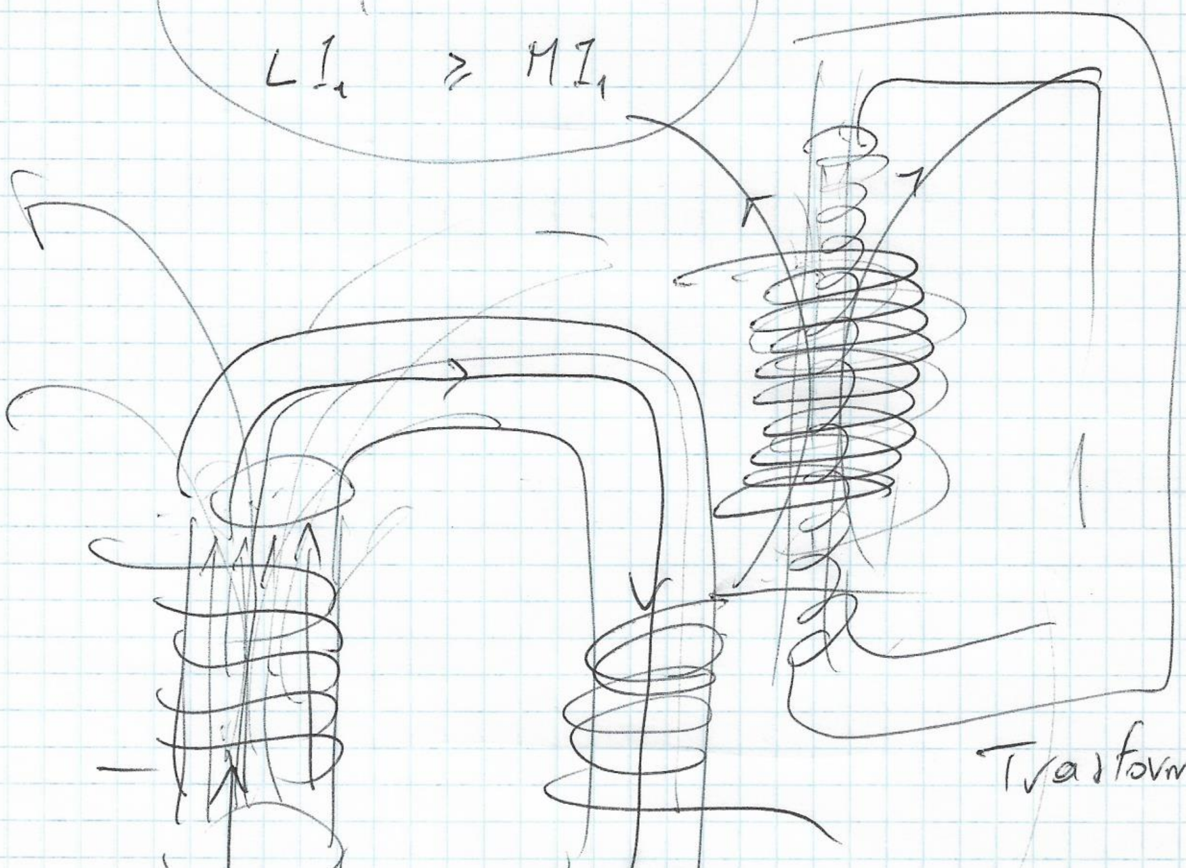
$$\boxed{|M| \leq \sqrt{L_1 L_2}}$$



$M = L$

$$\Phi_1(\bar{B}_1) \geq \Phi_2(\bar{B}_1)$$

$$L I_1 \geq M I_1$$



Trasformatori

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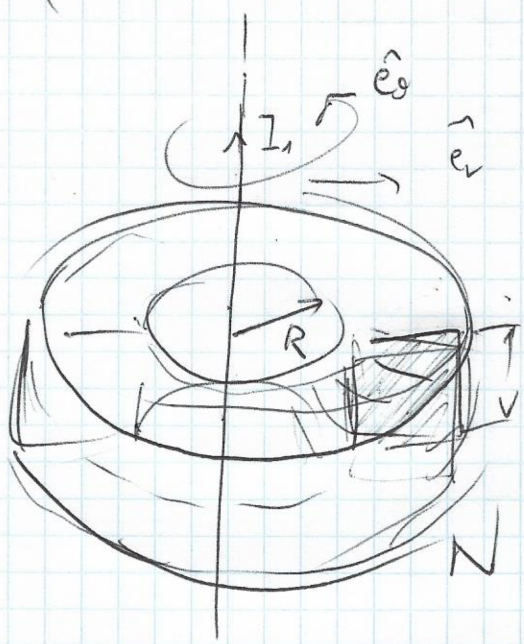
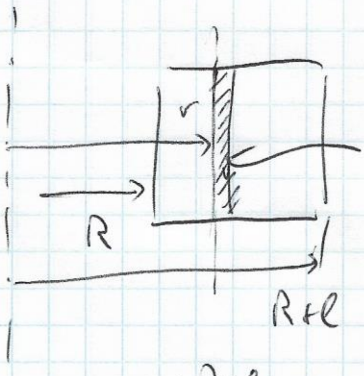


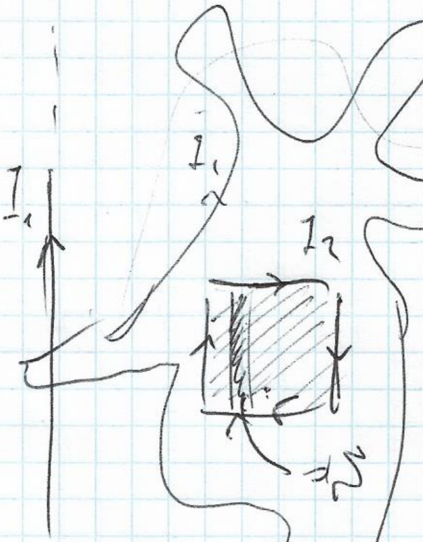
$$\bar{B}_1(r) = \frac{\mu_0 I_1}{2\pi r}\hat{e}_\theta$$



$dS = l\,dr$

$$\Phi_2(\bar{B}_1(I_1)) = N\int_R^{R+l}\frac{\mu_0 I_1}{2\pi r}\,l\,dr = \frac{\mu_0 N I l}{2\pi}\log\left(\frac{R+l}{R}\right)$$

$$M = \frac{\Phi_2(I_1)}{I_1} = \frac{\mu_0 N l}{2\pi}\log\left(\frac{R+l}{R}\right)$$



$$\bar{B}_2(I_2) = \frac{\mu_0 N I_2}{2\pi r}\hat{e}_\theta$$

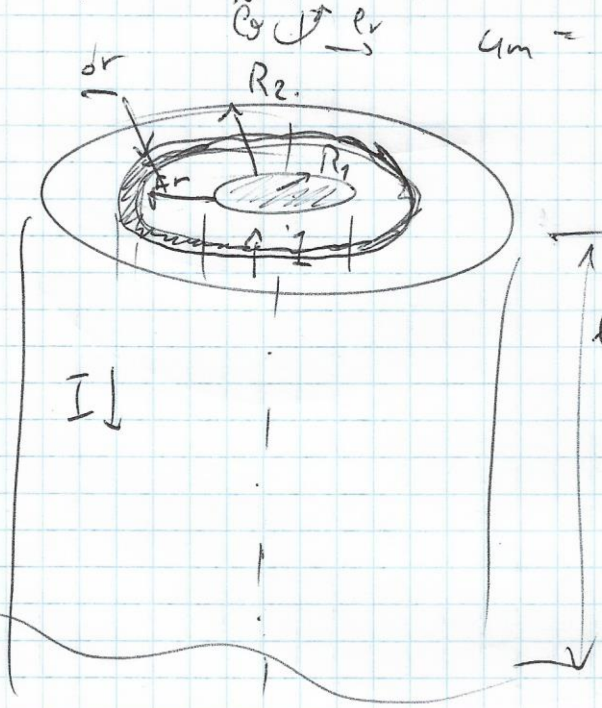
$$\Phi_1(\bar{B}_2(I_2)) = \int_R^{R+l}\frac{\mu_0 N I_2}{2\pi r}\,l\,dr = \frac{\mu_0 N I_2 l}{2\pi}\log\left(\frac{R+l}{R}\right)$$

$$M = \frac{\Phi_1(I_2)}{I_2} = \frac{\mu_0 N l}{2\pi}\log\left(\frac{R+l}{R}\right)$$

L cavo coassiale

$$U_m = \frac{1}{2} L I^2 \quad \& \quad U_m = \int_\tau u_m(\bar{r})\, d\tau$$

$$u_m = \frac{1}{2\mu_0} \bar{B} \cdot \bar{B}$$



$R_1 < r < R_2$

$$\bar{B}_0(r) = \frac{\mu_0 I}{2\pi r} \hat{e}_\theta$$

$$u_m(r) = \frac{B^2(r)}{2\mu_0}$$

$$d\tau = l\, 2\pi r\, dr$$

$$U_m^{ext} = \int_{R_1}^{R_2} \frac{B^2(r)}{2\mu_0}\, l\, 2\pi r\, dr = \int_{R_1}^{R_2} \frac{\mu_0^2 I^2}{4\pi^2 r^2} \frac{2\pi l}{2\mu_0}\, r\, dr = \frac{\mu_0 I^2 l}{4\pi} \log\left(\frac{R_2}{R_1}\right)$$

$$L_u^{ext} = \frac{2 U_m}{l I^2} = \frac{\mu_0}{2\pi} \log\left(\frac{R_2}{R_1}\right)$$

$r < R_1 \qquad \mu \simeq \mu_0 \qquad \bar{B}_0(r) = \frac{\mu_0 I}{2\pi R_1^2} r \hat{e}_\theta$

$$U_m^{int} = \int_0^{R_1} \frac{B^2(r)}{2\mu_0}\, l\, 2\pi r\, dr = \int_0^{R_1} \frac{\mu_0^2 I^2 r^2}{4\pi^2 R_1^2} \frac{1}{2\mu_0}\, l\, 2\pi r\, dr = \frac{\mu_0 I^2 l}{16\pi}$$

$$L_u^{int} = \frac{2 U_m^{int}}{l I^2} = \frac{\mu_0}{8\pi}$$

$R_1 = 1\,mm;\ R_2 = 2.7\,mm$

$$L_u = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log\frac{R_2}{R_1}\right] = 250\, nH$$

Forze su circuiti rigidi a corrente costante

N circuiti j-esimo $f_j \to I_j$ R_j

$$U_m = \frac{1}{2}\sum_{j=1}^{N} I_j \Phi_j \qquad \Phi_j$$

$$\bar{F}^k_{ext} \qquad \delta L_k = \bar{F}^k_{ext}\cdot \delta \bar{x}_k$$

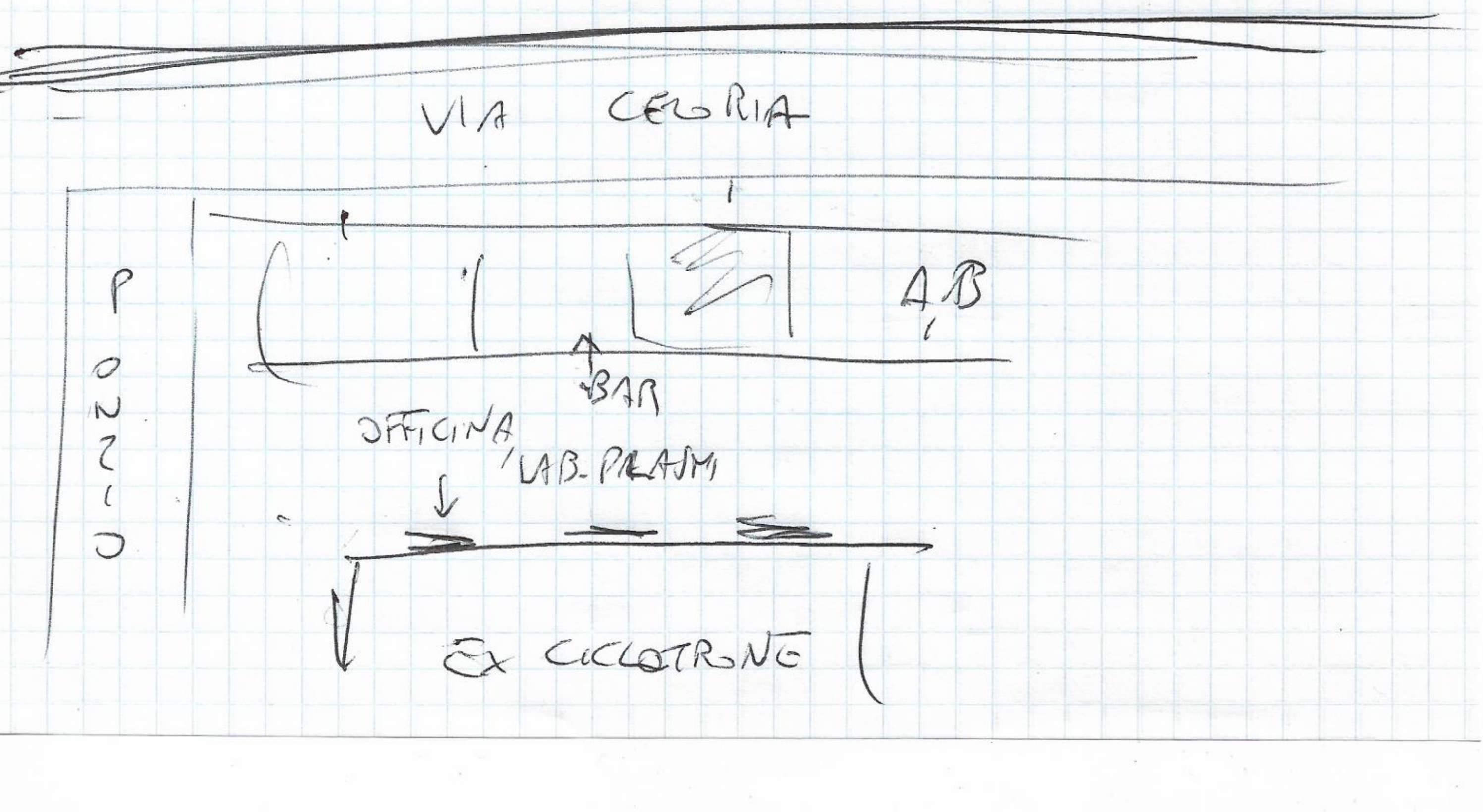
$$\delta L^k + \underbrace{\delta L_g}_{} = dU_m + \underbrace{dU_R}_{\sum_j R_j I_j^2 dt}$$

$$\delta L_g = -dU_g = \sum_j f_j dQ_g = \sum_j f_j I_j dt$$

$$dU_m = d\left(\frac{1}{2}\sum_j I_j \Phi_j\right) \underset{(I_j = cost.)}{=} \frac{1}{2}\sum_j I_j d\Phi_j$$

$$\delta L^k = dU_m - dL_g + dU_R = \frac{1}{2}\sum_j I_j d\Phi_j - \sum_j f_j I_j dt + \sum_j R_j I_j^2 dt$$

$$\forall j \quad f_j = R_j I_j + \frac{d\Phi_j}{dt}$$



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$$\delta L^H = \frac{1}{2}\sum_j I_j d\phi_j - \cancel{\sum_j R_j I_j^2 dt} - \sum_j I_j d\phi_j + \cancel{\sum_j R_j I_j^2 dt} =$$

$$= -\frac{1}{2}\sum_j I_j d\phi_j = -dU_m$$

$$\bar{f}^H = -\bar{F}_H^{ext}$$

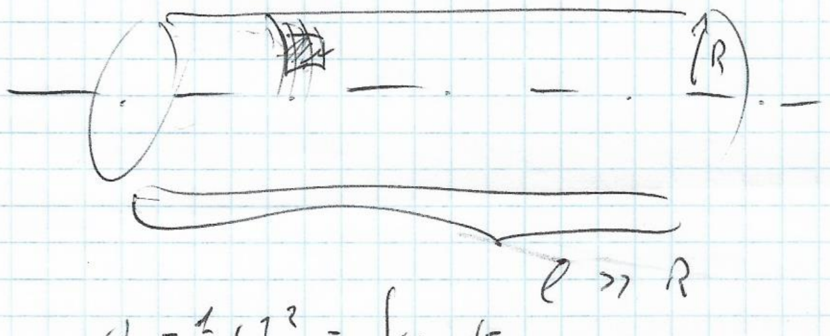
$$\underline{\bar{f}^H \cdot \delta \bar{H}} = -\bar{F}_H^{ext} \cdot \delta \bar{H} = -\delta L^H = \underline{dU_m}$$

$$\bar{f}^H \cdot \delta \bar{H} = dU_m$$

$$\downarrow$$

$$\boxed{\bar{f}^H = \bar{\nabla} U_m \Big|_{I_j = cost.}}$$

(18-11) pressione magn. solenoide



I stazionaria

$$U_m = \frac{1}{2} L I^2 = \int u_m d\tau \qquad B = \mu_0 \frac{N}{l} I$$

$$U_m(r) = u_m \, d\tau = \frac{B^2}{2\mu_0} \pi r^2 l = \frac{1}{2}\mu_0 \frac{N^2 \pi r^2 I^2}{l}$$

$$\bar{f} = \bar{\nabla} \cdot U_m \Big|_{I = cost.}$$

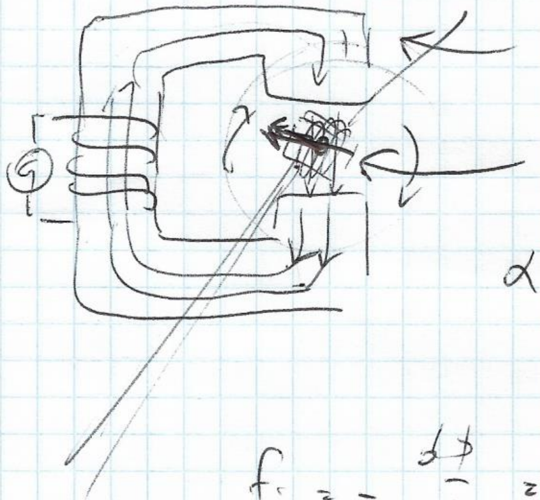
$$f_r = \frac{\partial U_m}{\partial r}\Big|_{I = cost,\ r = R} = \frac{\partial}{\partial r}\left(\frac{1}{2}\mu_0 \frac{N^2 \pi I^2}{l} r^2\right)\Bigg|_{r=R} = \mu_0 \frac{N^2 I^2}{l} \pi R$$

$$p = \frac{dF_{normale}}{dS} = \frac{F_r}{S}$$

$$p = \frac{F_r}{2\pi R l} = \frac{1}{2}\mu_0 \frac{N^2}{l^2} I^2 = \frac{B^2}{2\mu_0} \Rightarrow u_m$$

Macchine elettriche

Generatori di corr. elettrica

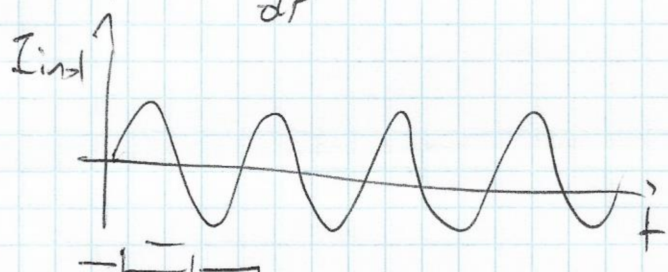


$\Phi(\bar{B}) = NBS\cos\alpha =$
$= NBS\cos(\omega t)$

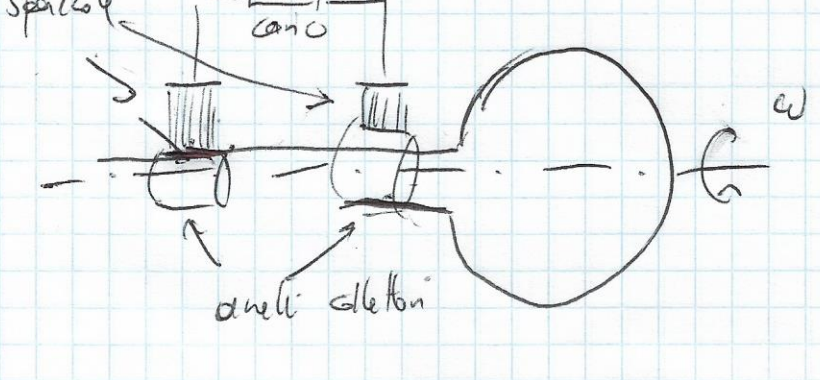
α angolo $\bar{B}$ - spira

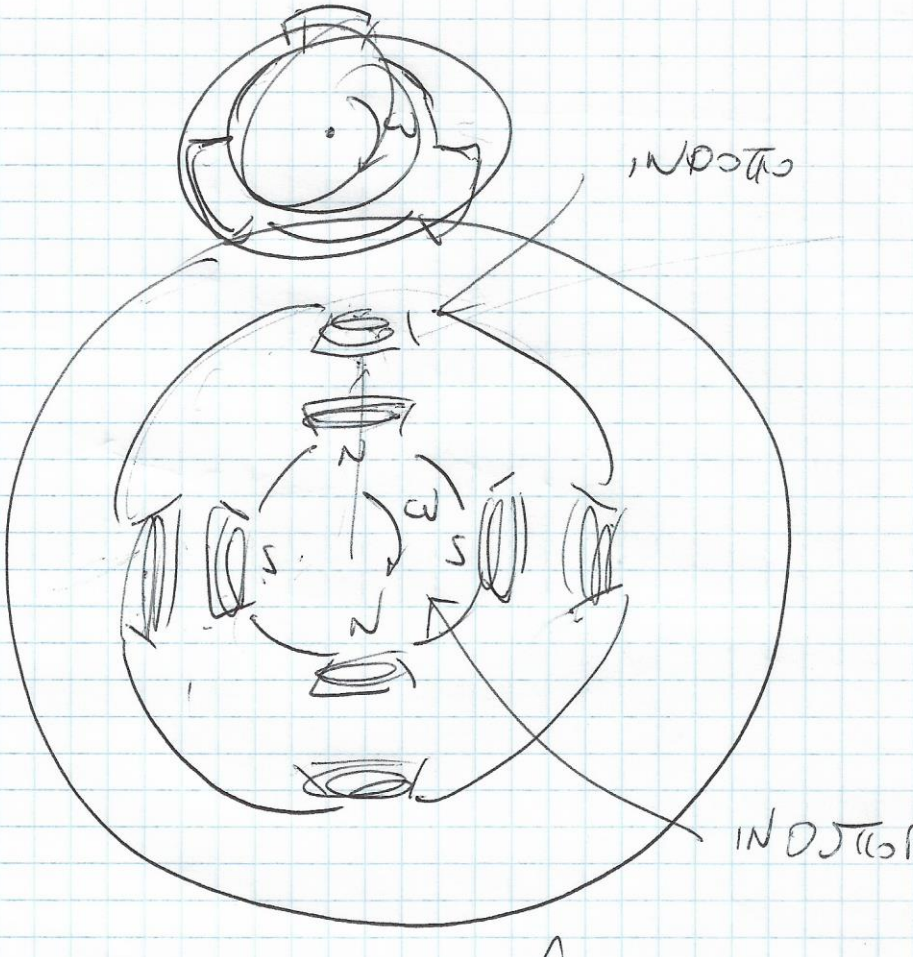
$\alpha = \omega t$

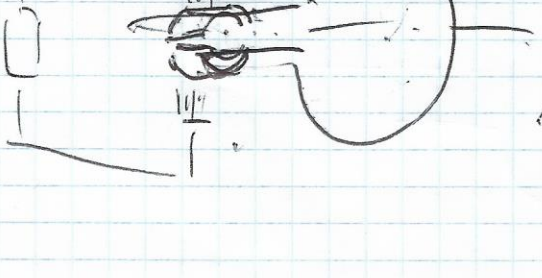
$$f_i = -\frac{d\Phi}{dt} = NBS\omega\sin(\omega t) = F_0\sin(\omega t)$$



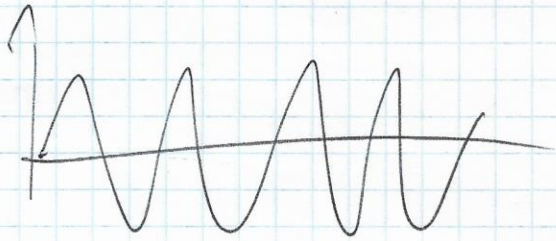
corrente alternata

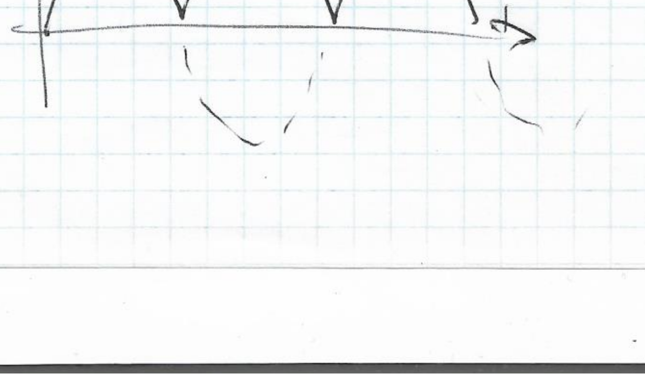






Dinamo





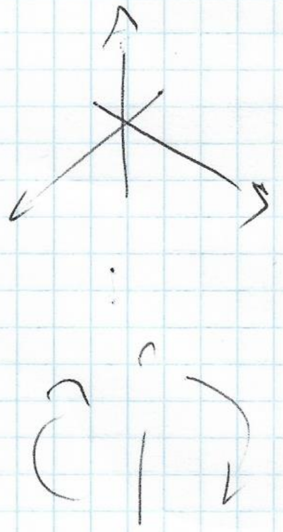
Motori elettrici

corr. continua → STATORE
ROTORE

↳ = dinamo

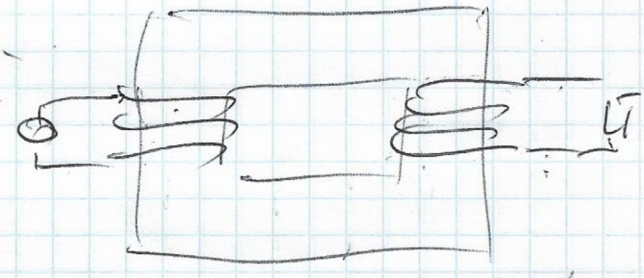
corr. alternata ↗ sincrono (avviamento)

↘ asincrono



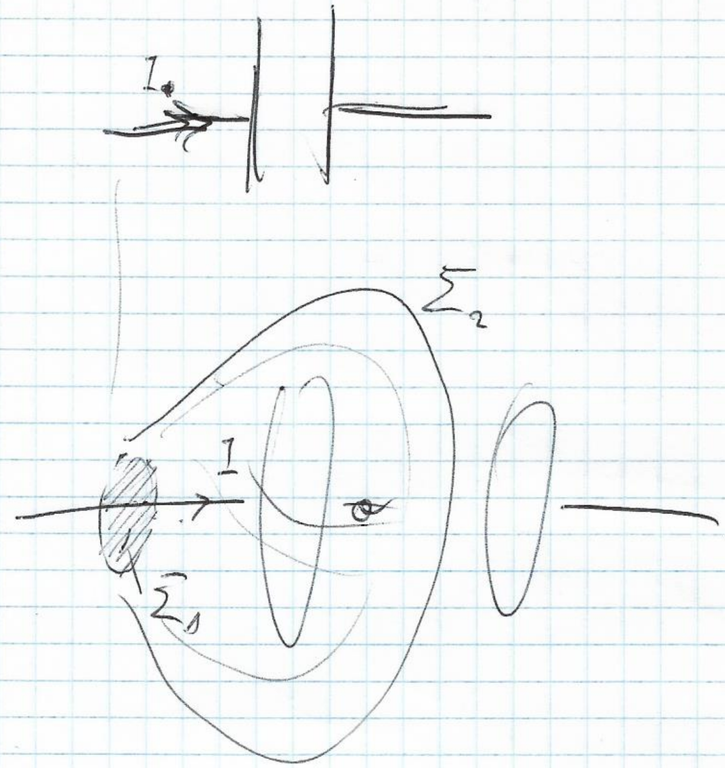
$$\bar{B} = \bar{B}_{01}\cos(\omega t) + \bar{B}_{02}\cos(\omega t + 120°) + \bar{B}_{03}\cos(\omega t + 240°)$$

Trasformatore



innalzatori / abbassatori

t. di isolamento



$$\bar\nabla \times \bar B = \mu_0 \bar J$$

$$\oint_{\ell=\partial S} \bar B \cdot d\bar\ell = \mu_0 \iint_S \bar J \cdot d\bar S$$

Σ_1 $\mu_0 I$

Σ_2 $\emptyset$