ES. #26 corrente di spostamento / IV eq. di Maxwell $\emptyset$ - vettore di Poynting

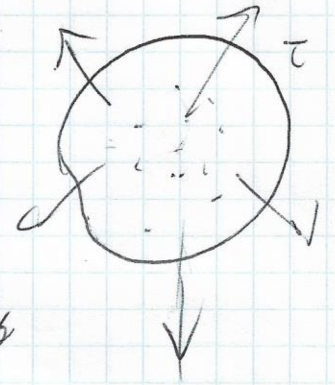
10/V/2021

1

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \emptyset \qquad \vec{\nabla}\cdot\vec{J} = \emptyset$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = \emptyset \qquad \frac{\partial \rho}{\partial t} = \emptyset$$



$$\int_\tau \frac{\partial \rho}{\partial t}\,d\tau + \int_\tau \vec{\nabla}\cdot\vec{J}\,d\tau = \emptyset$$

$$\frac{d}{dt}\int_\tau \rho\,d\tau = \frac{dQ}{dt} + \int_{\partial\tau} \vec{J}\cdot d\vec{S} = \emptyset$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = \frac{\partial}{\partial t}(\varepsilon_0 \vec{\nabla}\cdot\vec{E}) + \vec{\nabla}\cdot\vec{J} = \vec{\nabla}\cdot\underbrace{\left[\vec{J} + \varepsilon_0\frac{\partial \vec{E}}{\partial t}\right]}_{\vec{J}_t} = \emptyset$$

$\vec{J}_t$ dens. corrente generalizzata

$$\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J}_c + \underbrace{\varepsilon_0\frac{\partial \vec{E}}{\partial t}}_{\vec{J}_s,\ \vec{J}_D}\right]$$

$\vec{J}_s, \vec{J}_D$ densità di corr. di spostamento

se staz.

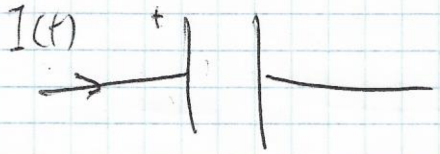
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}_t = \mu_0\vec{J}_c + \mu_0\vec{J}_s = \mu_0\vec{J}_c + \varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$

nella materia

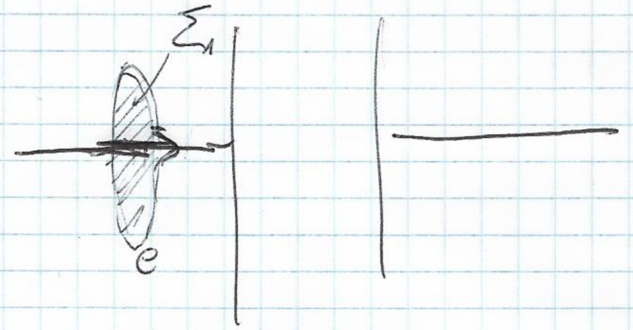
$$\vec{\nabla}\times\vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$$

$$\oint_{C=\partial S} \vec{B}\cdot d\vec{l} = \mu_0\int_S \vec{J}_t\cdot d\vec{S} = \mu_0 I_c + \mu_0\int_S \varepsilon_0\frac{\partial \vec{E}}{\partial t}\cdot d\vec{S}$$



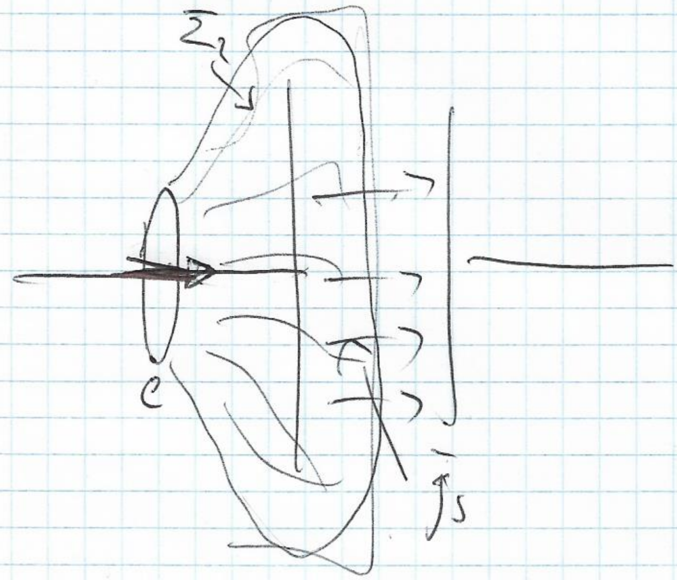
$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

$$\oint_{C=\partial\Sigma} \bar{B} \cdot d\bar{l} = \mu_0 \int_{\Sigma} \bar{J} \cdot d\bar{S}$$



su Σ_1

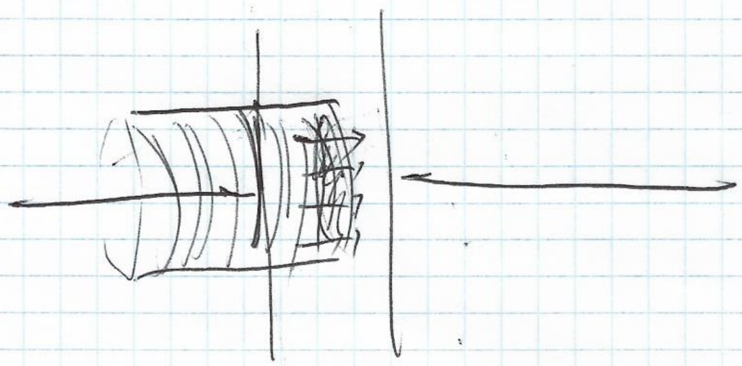
$$\oint_{C=\partial\Sigma_1} \bar{B} \cdot d\bar{l} \neq 0 = \mu_0 I(t)$$

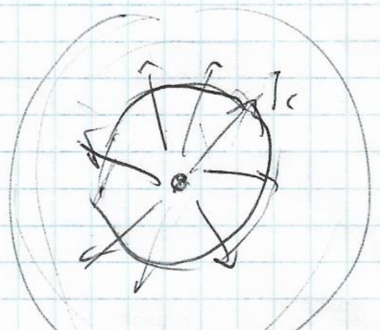


su Σ_2 non intersecante il circuito

$$\oint_{C=\partial\Sigma_2} \bar{B} \cdot d\bar{l} \neq \mu_0 \int_{\Sigma_2} \bar{J} \cdot d\bar{S} = 0$$

$$= \mu_0 I(t)$$

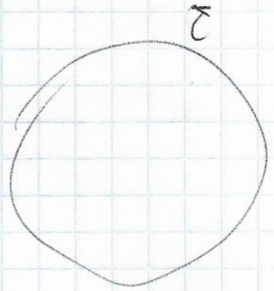




Vettore di Poynting - teorema di conservazione dell' energia elm

$$U_{elm} = \int_\tau \frac{1}{2}\varepsilon_0 \bar{E}\cdot\bar{E}\, d\tau + \int_\tau \frac{1}{2\mu_0}\bar{B}\cdot\bar{B}\cdot d\tau$$

su τ



$$-\frac{dU_{elm}}{dt} = -\int_\tau \varepsilon_0 \bar{E}\cdot\underline{\underline{\frac{\partial\bar{E}}{\partial t}}}\, d\tau - \int \frac{1}{\mu_0}\bar{B}\cdot\underline{\underline{\frac{\partial\bar{B}}{\partial t}}}\, d\tau$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\bar{J} + \mu_0\varepsilon_0\frac{\partial\bar{E}}{\partial t}$$

$$-\frac{dU}{dt} = \int_\tau \bar{E}\cdot\bar{J}\, d\tau - \int \frac{1}{\mu_0}\bar{E}\cdot(\bar{\nabla}\times\bar{B})\, d\tau + \int \frac{1}{\mu_0}\bar{B}\cdot(\bar{\nabla}\times\bar{E})\, d\tau$$

$$\bar{\nabla}\cdot(\bar{E}\times\bar{B}) = \bar{B}\cdot(\bar{\nabla}\times\bar{E}) - \bar{E}\cdot(\bar{\nabla}\times\bar{B})$$

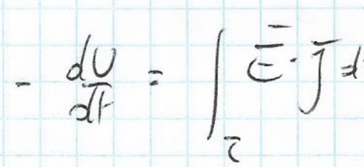
$$-\frac{dU}{dt} = \int_\tau \bar{E}\cdot\bar{J}\, d\tau + \int_\tau \bar{\nabla}\cdot\left(\frac{\bar{E}\times\bar{B}}{\mu_0}\right) d\tau$$

$$\bar{I} = \bar{S} = \frac{\bar{E}\times\bar{B}}{\mu_0}$$

vettore di Poynting

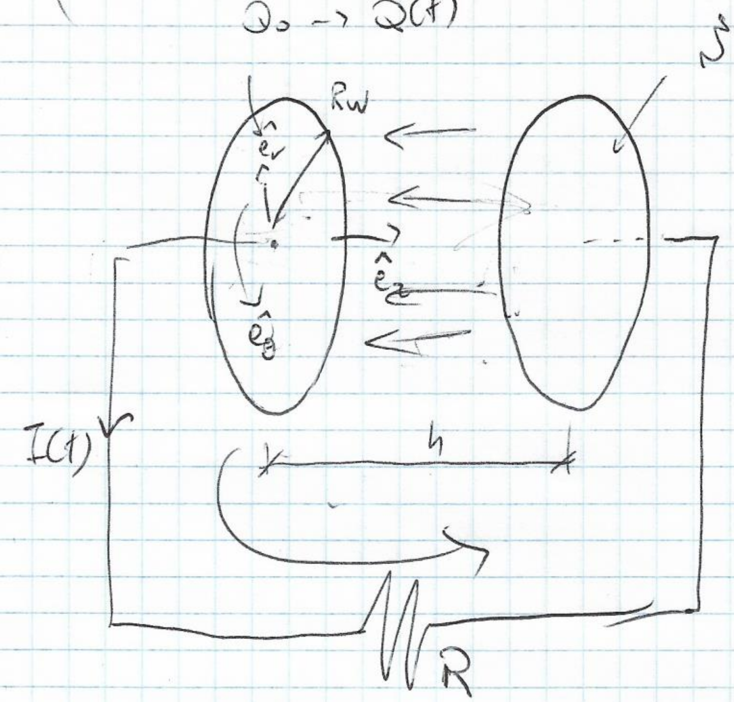
$$\boxed{-\frac{dU_{elm}}{dt} = \int_\tau \bar{E}\cdot\bar{J}\, d\tau + \int_{S=\partial\tau} \bar{I}\cdot d\bar{S}}$$

↳ teor. di Poynting (cons. en. elm.)



$$[I] = \left[\frac{J}{m^2 s}\right]$$

(fig. 1)



S sup. $= \pi R_W^2$

$$Q(t) = Q_0 e^{-t/RC}$$

$$I_c(t) = \frac{dq}{dt} = -\frac{dQ}{dt}$$

$$I_c(t) = \frac{Q_0}{RC} e^{-t/RC}$$

$$\bar{J}_s = \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

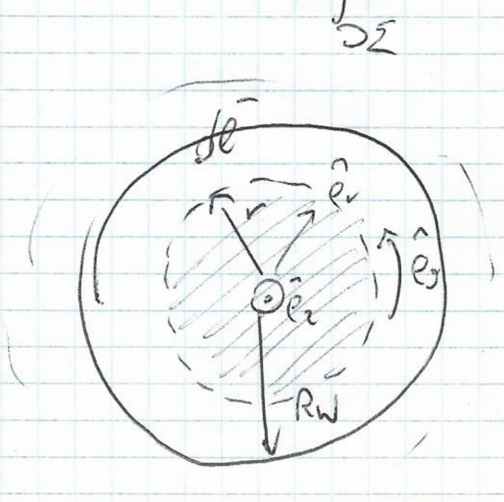
$$\bar{E}(t) = \frac{\sigma(t)}{\varepsilon_0} \hat{e}_z = \frac{Q(t)}{\varepsilon_0 S} \hat{e}_z = \frac{Q_0}{\varepsilon_0 S} e^{-t/RC} \hat{e}_z$$

$$\bar{J}_s = \varepsilon_0 \frac{\partial \bar{E}}{\partial t} = \frac{1}{S} \frac{dQ(t)}{dt} \hat{e}_z = -\frac{1}{S} I_c(t) \hat{e}_z = -\frac{Q_0}{SRC} e^{-t/RC} \hat{e}_z$$

$$I_s(t) = \int \bar{J}_s \cdot d\bar{S} = J_s \cdot S = \frac{Q_0}{RC} e^{-t/RC} = I_c(t)$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \left(\cancel{\bar{J}_c} + \bar{J}_s \right) = \mu_0 \bar{J}_s$$

$$\int_{\partial \Sigma} \bar{B} \cdot d\bar{l} = \mu_0 \int_{\Sigma} \bar{J}_s \cdot d\bar{\Sigma}$$



$B_\theta(r)$

$d\bar{l} = r\,d\theta\, \hat{e}_\theta$

$r < R_W$

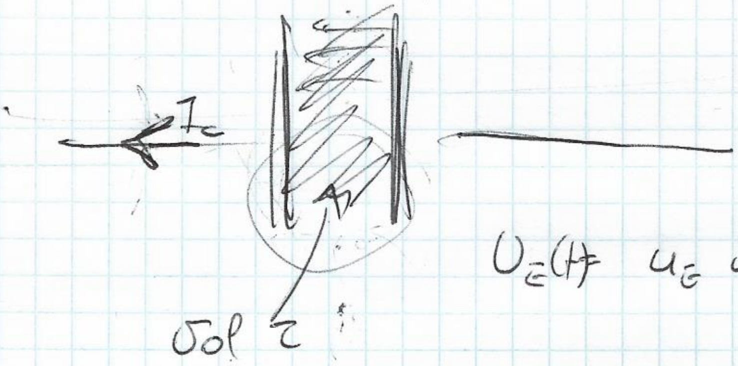
$$B_\theta(r)\, 2\pi r = \mu_0 \pi r^2 J_s$$

$$\bar{B}(r) = \frac{\mu_0}{2} r J_s \hat{e}_\theta =$$

$$= -\frac{\mu_0}{2} \frac{Q}{SRC} r e^{-t/RC} \hat{e}_\theta$$

$r > R_W \qquad B_\vartheta(r)\, 2\pi r = \mu_0 \underbrace{\pi R_W^2 J_s}_{I_s \,=\, I_c}$

$\bar{B}(r) = -\frac{\mu_0 I_c}{2\pi r}\hat{e}_\vartheta$



$U_E(t) = u_E\, \mathrm{Vol} = \frac{1}{2}\varepsilon_0 E^2(t)\, \mathrm{Vol} =$

$= \frac{1}{2}\cancel{\varepsilon_0}\frac{Q_0^2}{\varepsilon_0^{\cancel{2}} S^{\cancel{2}}} e^{-2t/RC}\, \cancel{S} h =$

$C = \frac{\varepsilon_0 S}{h}$

$= \frac{1}{2}\frac{Q_0^2}{C} e^{-2t/RC}$

$\frac{dU_E}{dt} = -\frac{2}{RC}\cdot\frac{1}{2}\frac{Q_0^2}{C} e^{-2t/RC} =$

$= -R\Big(\underbrace{\frac{Q_0}{RC} e^{-t/RC}}_{I_c(t)}\Big)^2 = -R I_c^2(t) = -P_J$

$-\frac{dU_{EM}}{dt} = \int_{S=\partial\tau} \bar{I}\cdot d\bar{S} + \cancel{\int_\tau \bar{E}\cdot\bar{J}_c\, d\tau}$

$\bar{I} = \frac{1}{\mu_0}\bar{E}\times\bar{B} = \frac{1}{\mu_0}\underbrace{\frac{Q_0}{\varepsilon_0 S} e^{-t/RC}}_{E_z(r)} \cdot \underbrace{\frac{\mu_0}{2}\frac{Q_0 r}{SRC} e^{-t/RC}}_{(-)B_\vartheta(r)}\ \underbrace{\hat{e}_z\times(-\hat{e}_\vartheta)}_{\hat{e}_r}$

$P_{out} = \int_{S_{lat}} \bar{I}\cdot d\bar{S} = \rightarrow \frac{Q_0^2 R_W}{2\varepsilon_0 S^2 R} e^{-2t/RC}\cdot 2\pi R_W h = \frac{Q_0^2 h}{\varepsilon_0 S RC} e^{-2t/RC}$

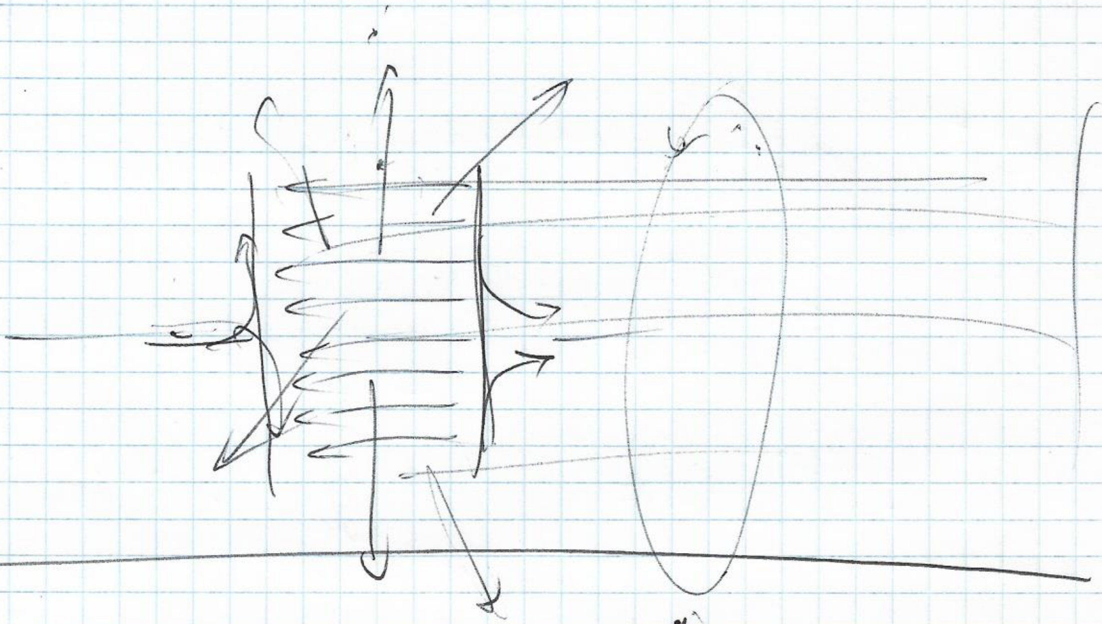
$-\frac{dU_{EM}}{dt} = P_{out} = -\frac{dU_E}{dt}$

(6)

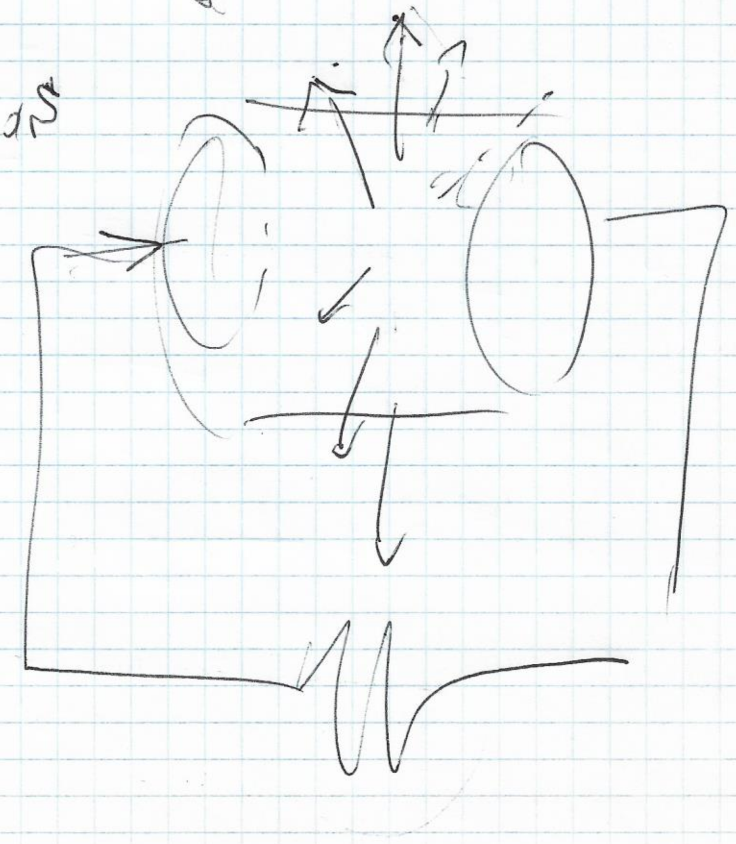
$$\bar\nabla \times \bar B = \mu_0 \bar J + \mu_0 \varepsilon_0 \frac{\partial \bar E}{\partial t}$$

$$\bar\nabla \times \bar E = -\frac{\partial \bar B}{\partial t}$$

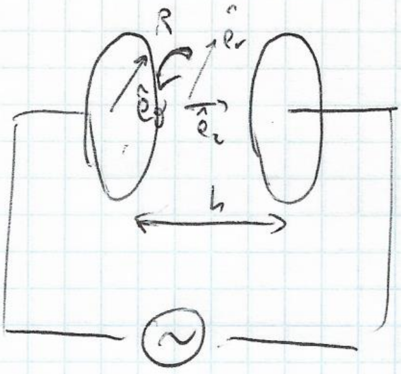
$$\bar J \longrightarrow \bar B' \longrightarrow$$



$$-\frac{dU}{dt} = \int_\Sigma \bar I \cdot d\bar S$$



(19-2)



$$V(t) = V_0 \sin(\omega t)$$

$$\bar{E}(t) = \frac{V(t)}{h}\hat{e}_z = \frac{V_0}{h}\sin(\omega t)\,\hat{e}_z$$

$$U_E = u_E \tau = \frac{1}{2}\varepsilon_0 E^2(t)\,\pi R^2 h = \frac{\varepsilon_0 \pi R^2 V_0^2}{2h}\sin^2(\omega t)$$

$$\oint_{\partial\Sigma} \bar{B}\cdot d\bar{l} = \mu_0 \varepsilon_0 \frac{d}{dt}\int_{\Sigma} \bar{E}\cdot d\bar{S}$$

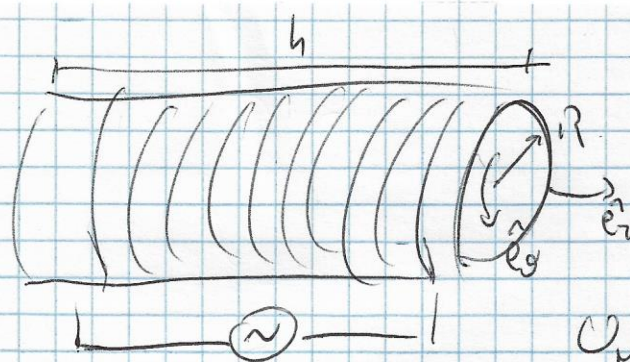
$$B_\varphi(r)\,2\pi r = \mu_0 \varepsilon_0 \underbrace{\frac{V_0}{h}\omega\cos(\omega t)}_{\to\, dE/dt}\pi r^2 \Rightarrow \bar{B}(r,t) = \frac{\mu_0 \varepsilon_0 V_0 \omega}{2h}\, r \cos(\omega t)\,\hat{e}_\theta$$

$$u_m(r,t) = \frac{B^2(r,t)}{2\mu_0} = \frac{\mu_0 \varepsilon_0^2 V_0^2 \omega^2}{8h^2}\, r^2 \cos^2(\omega t)$$

$$U_m(r,t) = \int_0^R u_m(r,t)\, h\, 2\pi r\, dr = \frac{\pi \mu_0 \varepsilon_0^2 V_0^2 \omega^2 R^4}{16 h}\cos^2(\omega t)$$

$$\frac{U_{E\,max}}{U_{M\,max}} = \frac{8}{\mu_0 \varepsilon_0 \omega^2 R^2} \qquad \omega^2 \qquad = 4.6\cdot 10^{+17}$$

$\omega = 2\pi\cdot 50$ rad/s $\qquad R = 10$ mm



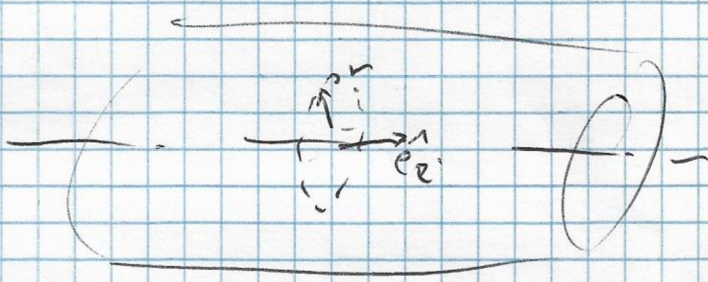
$$I(t) = I_0 \sin(\omega t)$$

$$\vec{B}(t) = \mu_0 n I_0 \sin(\omega t)\,\hat{e}_z$$

$$U_M = \frac{B^2}{2\mu_0}\,\pi R^2 h = \frac{\pi \mu_0 n^2 I_0^2 h}{2}\sin^2(\omega t)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightsquigarrow \vec{E}_\vartheta(r,t)$$

$$\oint_{\ell=\partial\Sigma} \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_\Sigma \vec{B}\cdot d\vec{S}$$



$$E_\vartheta(r,t)\,2\pi r = -\pi r^2 \frac{\partial B_z}{\partial t} =$$

$$= -\pi r^2 n I_0 \omega \cos(\omega t)$$

$$\vec{E}(r,t) = -\frac{\mu_0}{2} r\, n I_0\, \omega \cos(\omega t)\,\hat{e}_\vartheta$$

$$U_E = \int_{\tau_{sol}} u_E\, d\tau = \int_0^R \frac{1}{2}\varepsilon_0 E^2(r,t)\, h\, 2\pi r\, dr = \frac{\pi \varepsilon_0 \mu_0^2 n^2 I_0^2 \omega^2 h R^4}{16}\cos^2(\omega t)$$

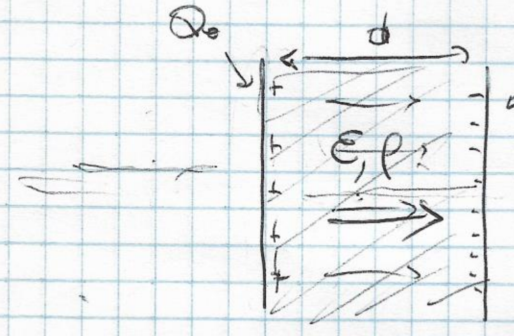
$$\frac{U_{E,max}}{U_{M,max}} = \frac{\varepsilon_0 \mu_0 \omega^2 R^2}{8} \simeq 2.2\cdot 10^{-18}$$

$\omega = 2\pi\cdot 50$ rad/s

$R = 10$ mm

Condensatore in perdita

$\varepsilon = k\varepsilon_0 = \varepsilon_r \varepsilon_0$; ρ resistività finita



$t = 0 \quad Q(t) = 0$, isolati

$I_c(t)$

$\varepsilon(t) \rightarrow I_s(t)$

$$I_c(t) = -\frac{dQ(t)}{dt}$$

$$R = \frac{\rho d}{A} \rightsquigarrow I_c(t) = \frac{V(t)}{R} = \frac{Q(t)}{RC}$$

$$-\frac{dQ}{dt} = \frac{Q}{RC} \rightsquigarrow Q(t) = Q_0 e^{-t/RC}$$

$$I_c(t) = -dQ/dt = \frac{Q_0}{RC} e^{-t/RC}$$

$$\bar{J}_c = \frac{I_c(t)}{A}\hat{e}_z = \frac{Q_0}{RCA} e^{-t/RC}\hat{e}_z$$

$$\bar{J}_s = \frac{\partial \bar{D}}{\partial t} = \varepsilon \frac{\partial \bar{E}}{\partial t} =$$

$$= \varepsilon \frac{d}{dt}\frac{Q(t)}{\varepsilon A}\hat{e}_z = \frac{1}{A}\frac{dQ}{dt}\hat{e}_z =$$

$$= -\frac{1}{A}\frac{Q_0}{RC} e^{-t/RC}\hat{e}_z$$

$$\bar{J}_s = -\bar{J}_c \qquad I_s = -I_c$$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 \int \bar{J}_T \cdot d\bar{S} = 0$$

$$\hookrightarrow \bar{J}_T = \bar{J}_c + \bar{J}_s$$