ES. #27 Vettore di Poynting - magnetismo nella materia

17/V/2021

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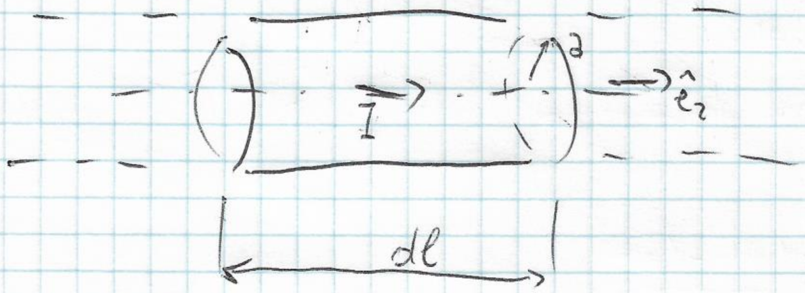

I staz.

a raggio

$\sigma = 1/\rho$

$$-\frac{dU_{em}}{dt} = \int_{\partial\tau} \bar{I}\cdot d\bar{S} + \int_{\tau} \bar{E}\cdot\bar{J}\, d\tau \qquad \bar{E} = ?$$

$$\bar{I} = \frac{\bar{E}\times\bar{B}}{\mu_0} \qquad \bar{J} = \sigma\bar{E} \quad (V = RI)$$

$$\bar{E} = \frac{1}{\sigma}\bar{J} = \frac{1}{\sigma}\frac{I}{\pi a^2}\hat{e}_z \qquad \bar{E}(r\leq a, z)$$

$\boxed{r\leq a}$ $\bar{B}(r) = \frac{\mu_0 I}{2\pi a^2} r\,\hat{e}_\theta$

$\boxed{r > a}$ $\bar{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{e}_\theta$

$$\bar{I}(r=a) = \frac{1}{\mu_0}\bar{E}(a)\times\bar{B}(a) = \frac{1}{\mu_0}\frac{I}{\sigma\pi a^2}\frac{\mu_0 I}{2\pi a}\underbrace{\hat{e}_z\times\hat{e}_\theta}_{-\hat{e}_r} = -\frac{I^2}{2\pi^2 a^3\sigma}\hat{e}_r$$

$$P_p = \int_{\partial\tau = S_{lat}} \bar{I}\cdot d\bar{S} = \bar{I}\cdot\underbrace{2\pi a\, dl}_{S_{lat}}(-\hat{e}_r)$$

$$P_p = -\frac{1}{\sigma}\frac{I^2}{\pi a^2}\, dl = -I^2 dR \qquad \left(\rho\frac{dl}{\pi a^2} \Rightarrow dR\right)$$

$$-\frac{dU_{em}}{dt} = \int_{\partial\tau}\bar{I}\cdot d\bar{S} + \int_{\tau}\bar{E}\cdot\bar{J}\, d\tau = \phi$$

$$P_p = -\int \bar{E}\cdot\bar{J}\, d\tau$$

$$\bar E = \frac{1}{\sigma}\bar J \qquad \int \bar E\cdot\bar J\, d\tau = \frac{J^2}{\sigma}\pi a^2\, dl =$$

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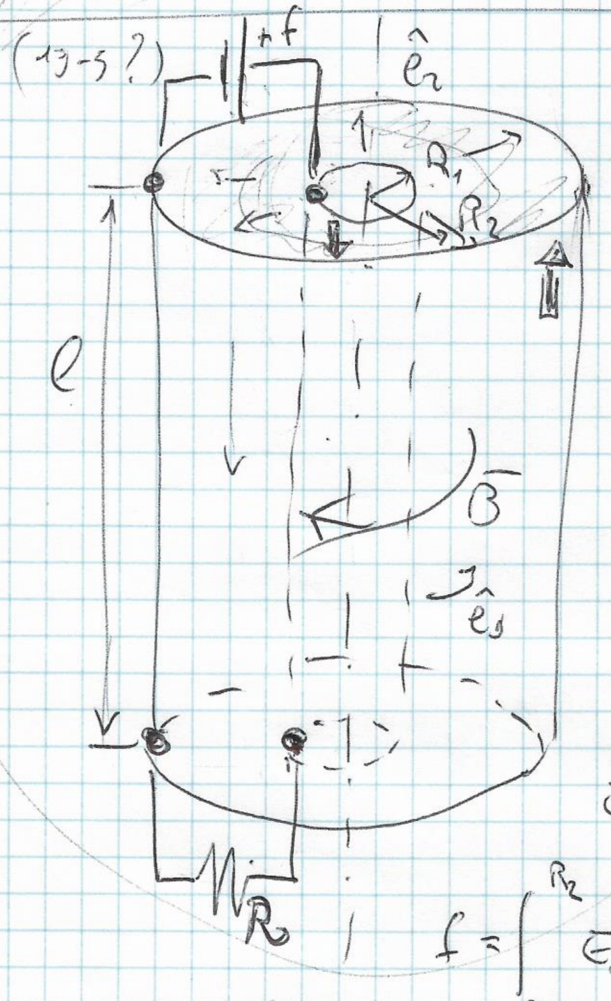
$$= \frac{I^2}{\sigma(\pi a^2)^2}\pi a^2\, dl = I^2\,\frac{1}{\sigma}\,\frac{dl}{\pi a^2} = I^2\, dR = dP_J$$

$$J = I/\pi a^2$$



$$\bar B(r) = -\frac{\mu_0 I}{2\pi r}\hat e_\varphi = -\frac{\mu_0 f}{2\pi R r}\hat e_\varphi$$

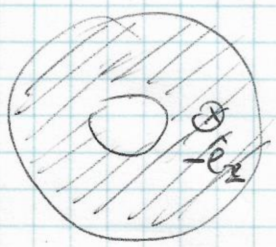
$$I = f/R$$

$$\int_S \bar E\cdot d\bar S = \frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$S \to R_1 < r < R_2$

$$E_r(r)\, 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\bar E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat e_r$$



$$f = \int_{R_1}^{R_2} E_r(r)\, dr = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{R_2}{R_1}\right)$$

$$\lambda = \frac{2\pi\varepsilon_0 f}{\log(R_2/R_1)} \Rightarrow \bar E(r) = \frac{f}{\log(R_2/R_1)}\,\frac{1}{r}\,\hat e_r$$

$$\hat e_r \times (-\hat e_\varphi) = -\hat e_z$$

$$\bar I = \frac{1}{\mu_0}\bar E\times\bar B = -\frac{1}{\mu_0}E(r)B(r)\,\hat e_z = -\frac{f^2}{2\pi R\log(R_2/R_1)}\,\frac{1}{r^2}\,\hat e_z$$

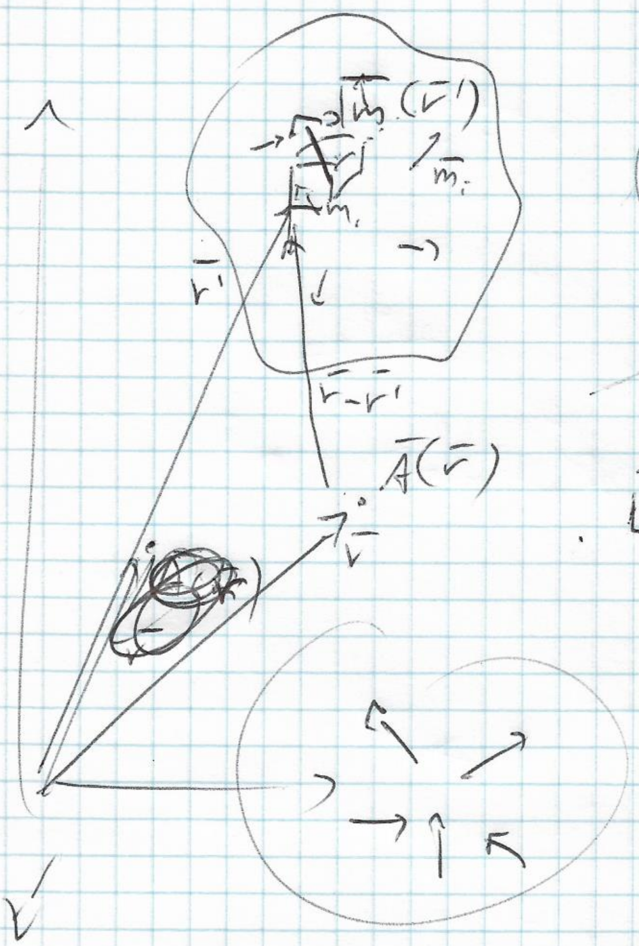
$$P = \int \bar I\cdot d\bar S = \int_{R_1}^{R_2} \underset{\text{Poynting}}{I(r)}\, 2\pi r\, dr = \frac{f^2}{R\log\left(\frac{R_2}{R_1}\right)}\log\left(\frac{R_2}{R_1}\right) = \frac{f^2}{R} = P_J$$

Magn. nella materia

$\bar M$ (densità di) magnetizzazione

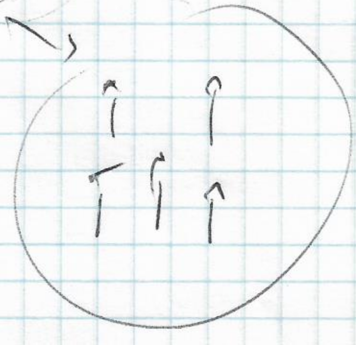
$$\bar M = \frac{\sum \bar m_i}{\Delta\tau} = \lim_{\Delta\tau\to 0}\frac{\sum \bar m_i}{\Delta\tau}$$

$$\frac{N\bar m}{\Delta\tau}$$



il mezzo è continuo

$$\sum \bar m_i = N\bar m$$



$$\bar A(\bar r) = \frac{\mu_0}{4\pi}\frac{\overbrace{\bar m}^{\bar m(\bar r')}\times(\bar r-\bar r')}{|\bar r-\bar r'|^3}$$

$$\bar M = \frac{\sum \bar m_i}{\Delta\tau} \longrightarrow d\bar m(\bar r') = \bar M(\bar r')\,d\tau'$$

$$\bar A(\bar r) = \frac{\mu_0}{4\pi}\int_{\tau'}\frac{\bar M(\bar r')\times(\bar r-\bar r')}{|\bar r-\bar r'|^3}d\tau' = \dots =$$

$$= \frac{\mu_0}{4\pi}\int_{\tau'}\frac{\overbrace{\bar\nabla\times\bar M(\bar r')}^{\bar J_m}}{|\bar r-\bar r'|}d\tau' + \frac{\mu_0}{4\pi}\int_{\partial\tau'}\frac{\overbrace{\bar M(\bar r')\times\hat e_n(\bar r')}^{\bar K_m}}{|\bar r-\bar r'|}dS'$$

$$\bar A(\bar r) = \frac{\mu_0}{4\pi}\int_{\tau'}\frac{\bar J_m(\bar r')}{|\bar r-\bar r'|}d\tau' + \frac{\mu_0}{4\pi}\int_{\partial\tau'}\frac{\bar K_m(\bar r')\,dS}{|\bar r-\bar r'|}$$

$$\begin{cases} \bar{J}_m = \bar{\nabla} \times \bar{M} & [A/m^2] \\ \bar{K}_m = \bar{M} \times \hat{e}_n & [A/m] \end{cases}$$

$$\bar{\nabla} \times \bar{B} = \mu_0 (\bar{J}_c + \underbrace{\bar{J}_m}_{\bar{\nabla} \times \bar{M}})$$

$$\bar{\nabla} \times \left(\frac{\bar{B} - \mu_0 \bar{M}}{\mu_0} \right) = \bar{J}_c \qquad \bar{H} \doteq \frac{\bar{B}}{\mu_0} - \bar{M} \quad \text{campo magnetico}$$

$$\bar{\nabla} \times \bar{H} = \bar{J}_c \qquad \sim \left(\text{vacuum } \bar{\nabla} \times \bar{B} = \mu_0 \bar{J}_c \right)$$

$$\oint_{\partial S} \bar{H} \cdot d\bar{l} = \int_S \bar{J}_c \cdot d\bar{S}$$

$$\bar{\nabla} \cdot \bar{B} = \phi$$

$$\bar{H}, \ \bar{B}, \ \bar{M} \qquad (\bar{E}, \bar{D}, \bar{P})$$

$$\cancel{\bar{M}(\bar{B})} \qquad \bar{M}(\bar{H})$$

rel. costitutiva $\bar{M} = \chi_m \bar{H}$ $\quad$ χ_m suscettività magnetica

$(\bar{P} = \varepsilon_0 \chi_e \bar{E})$

$\chi_m < \phi$ DIAMAGNETICI

$|\chi_m| \sim 10^{-5} - 10^{-8}$

$\chi_m > \phi$ PARAMAGNETICI

$\chi_m = f(H)$ non lineari

$\gg 1$ FERROMAGNETICI

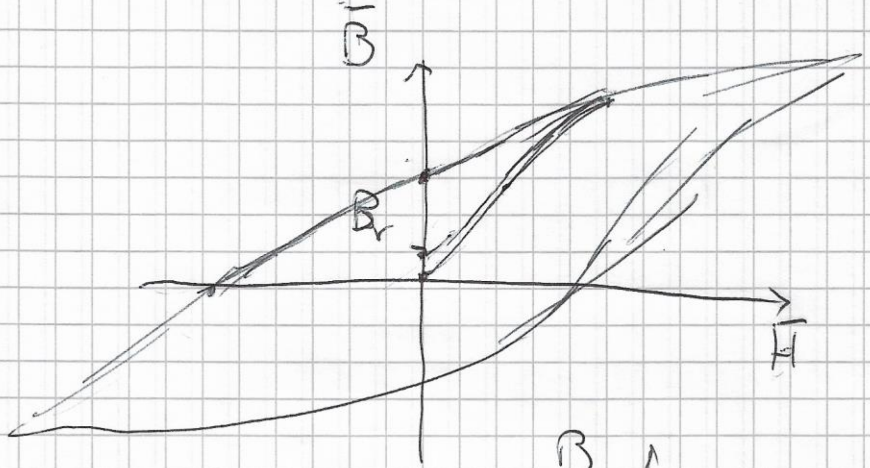
$$\bar{B} = \mu_0 (\bar{H} + \bar{M}) = \mu_0 (1 + \chi_m) \bar{H} = \mu_0 \mu_r \bar{H}$$

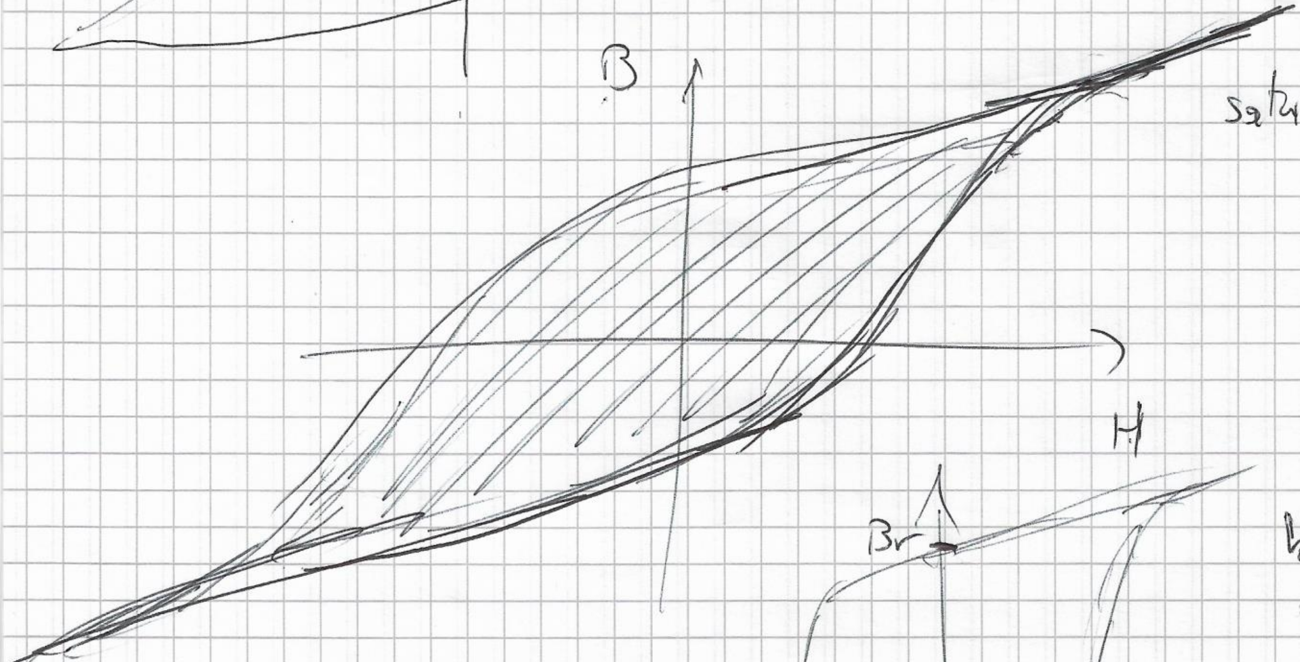
$\mu_r \doteq 1 + \chi_m$ perm. magn. relativa

$\bar{M} = \bar{M}(\bar{H})$

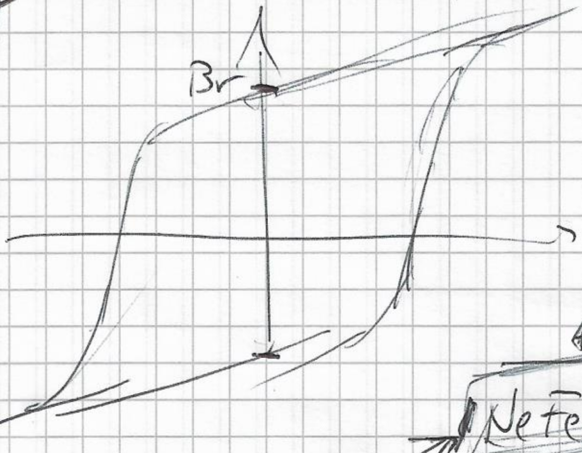
soft / hard

magneti permanenti

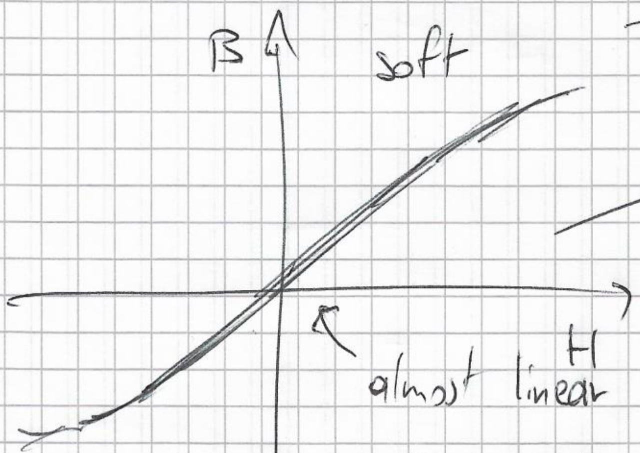




saturazione



hard



soft

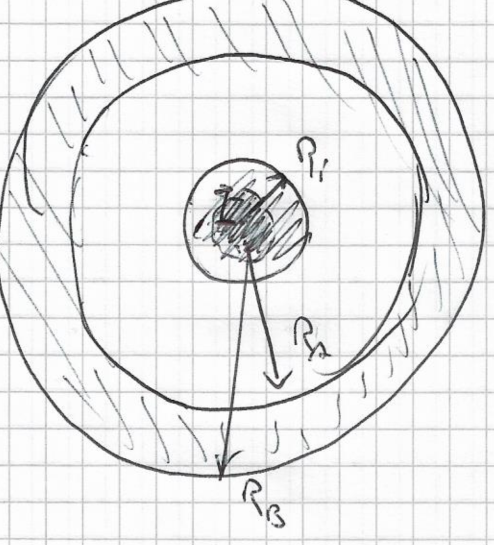
almost linear

elettromagneti

NdFeB

AlNiC II IV

Sm V



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μ_{r1}, μ_{r2} lineari, omog., iso.

$$\oint \bar{H}\cdot d\bar{l} = I_{conc.}$$

$\boxed{r < R_i}$ $\quad H_\vartheta(r)\, 2\pi r = \frac{r^2}{R_i^2} I$

$$\bar{B} = \mu_0 \mu_r \bar{H} \rightarrow \begin{cases} \bar{H}(r) = \dfrac{I}{2\pi R_i^2}\, r\, \hat{e}_\vartheta \\ \bar{B}(r) = \dfrac{\mu_0 \mu_{r1} I r}{2\pi R_i^2}\, \hat{e}_\vartheta \\ \bar{M}(r) = \chi_{m1} \bar{H} = \dfrac{\chi_{m1} \bar{H}}{2\pi R_i^2}\, r\, \hat{e}_\vartheta \end{cases}$$

$\boxed{R_i < r < R_A}$ $\quad 2\pi r\, H_\vartheta(r) = I$

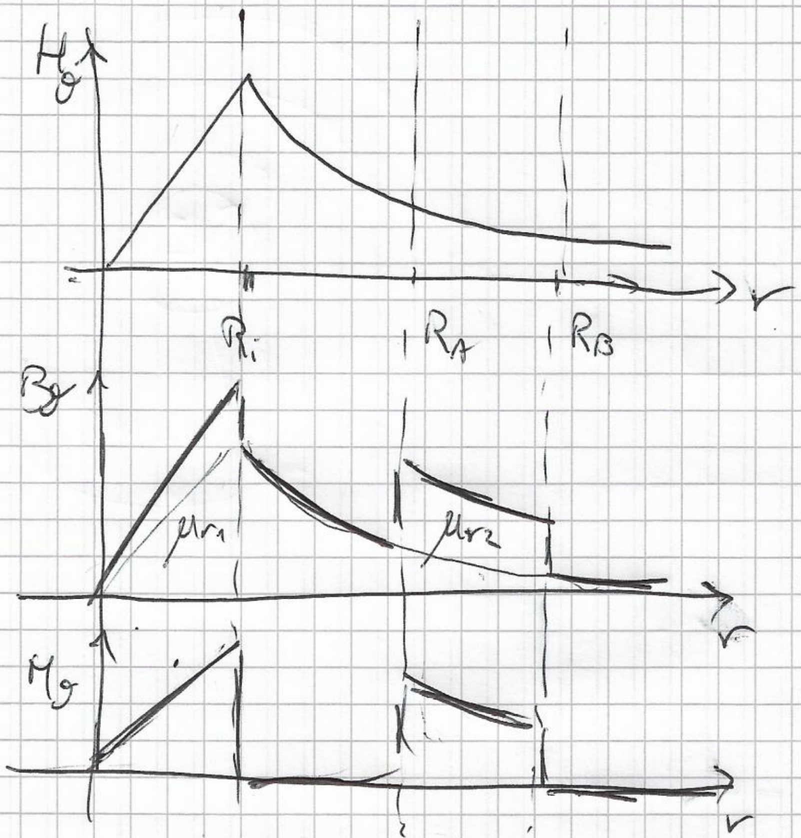
$$\begin{cases} \bar{H}(r) = \dfrac{I}{2\pi r}\, \hat{e}_\vartheta \\ \bar{B}(r) = \dfrac{\mu_0 I}{2\pi r}\, \hat{e}_\vartheta \\ \bar{M}(r) = \phi \end{cases}$$

$\boxed{R_A < r < R_B}$ $\quad 2\pi\, H_\vartheta(r) = I$

$$\begin{cases} \bar{H}(r) = \dfrac{I}{2\pi r}\, \hat{e}_\vartheta \\ \bar{B}(r) = \dfrac{\mu_0 \mu_{r2} I}{2\pi r}\, \hat{e}_\vartheta \\ \bar{M}(r) = \cancel{\phi}\, \chi_{m2} \bar{H} = \chi_{m2} I/2\pi r\, \hat{e}_\vartheta \end{cases}$$

$\boxed{r > R_B}$ $\quad 2\pi\, H_\vartheta(r) = I$

$$\begin{cases} \bar{H}(r) = I/2\pi r\, \hat{e}_\vartheta \\ \bar{B}(r) = \mu_0 I/2\pi r\, \hat{e}_\vartheta \\ \bar{M}(r) = \phi \end{cases}$$



$\boxed{r<R_i}$ $\quad \bar J_m = \bar\nabla\times\bar M = \frac{\chi_m I}{2\pi R_i^2}\cdot\frac{1}{r}\left[\frac{\partial}{\partial r}(\overbrace{r\cdot r}^{M_\vartheta}) - \frac{\partial}{\partial\vartheta}\phi\right]\hat e_z \Rightarrow$

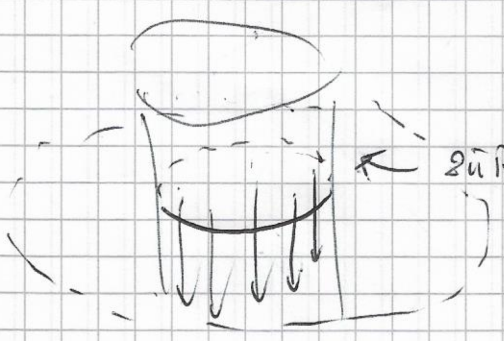
$$\boxed{\bar J_m = \frac{\chi_{m1} I}{\pi R_i^2}\hat e_z = \chi_{m1}\bar J_c}$$

$\bar\nabla\times\bar M = \chi_{m1}\bar\nabla\times\bar H = \chi_{m1}\bar J_c$ $\quad\otimes$

$I_{mv} = J_m \pi R_i^2 = \chi_m I$

$\bar H_m = \bar M(R_i)\times\hat e_n = \bar M(R_i)\times\hat e_r = -M(R_i)\hat e_z$?

$\underbrace{\hat e_\vartheta\times\hat e_r}_{-\hat e_z} \qquad = -\frac{\chi_{m1} I}{2\pi R_i}\hat e_z$



$I_{ms} = -\chi_{m1} I$

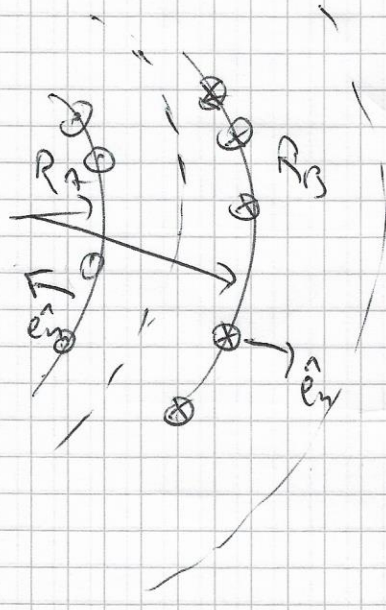
$\underline{I_{mv} + I_{ms} = \emptyset}$

Cond. 2

$$\bar{J}_m = \vec{\nabla} \times \bar{M} = \cancel{\frac{\chi_{m2} I}{2\pi} \frac{1}{r} \left[\frac{\partial}{\partial r}\left(r \cdot \frac{1}{r}\right)\phi - \frac{\partial}{\partial \phi}\phi \right]_z}$$

$$\bar{J}_m = \phi$$

$$\bar{K}_m(R_A) = \underbrace{\bar{M}(R_A)}_{\hat{e}_\theta} \times (-\hat{e}_r) = M_\theta(R_A)\,\hat{e}_z = \frac{\chi_{m2} I}{2\pi R_A}\hat{e}_z$$



$$I_{ms}(R_A) = K_m(R_A)\, 2\pi R_A = \chi_{m2} I$$

$$\bar{K}_m(R_B) = \bar{M}(R_B) \times \hat{e}_r = -M_\theta(R_B)\,\hat{e}_z = -\frac{\chi_{m2}}{2\pi R_B} I\,\hat{e}_z$$

$$I_{ms}(R_B) = K_m(R_B)\, 2\pi R_B = -\chi_{m2} I$$

$$\Delta B = \mu_0 K$$