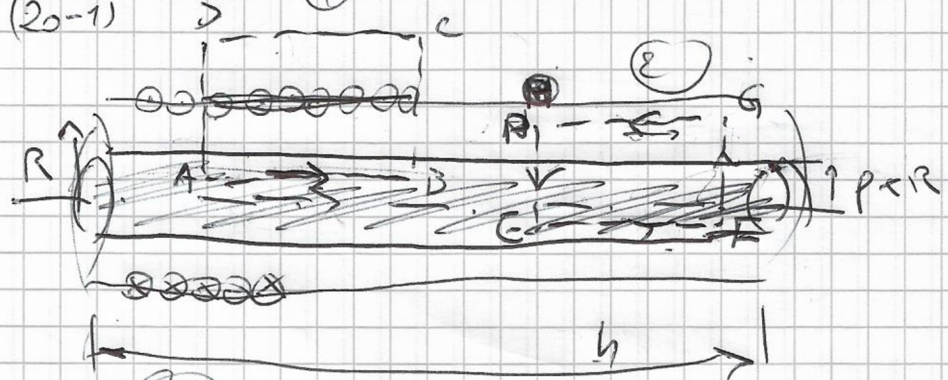
ES. #28 Magnetismo nella materia 24/V/2021

(1)

(20-1)



$h \gg R$

n, I

$\bar{H}, \bar{B}, \bar{M}$

$\bar{J}_m \quad \bar{K}_m$

(1) $\bar{H}$ – corr. conduzione

$$\oint \bar{H}\cdot d\bar{l} = \int_A^B \bar{H}_1\cdot d\bar{l} + \underbrace{\cancel{\int_C^D \bar{H}_0\cdot d\bar{l}}}_{=\emptyset} = nlI$$

$l = \overline{CD}$

$$H_1 l = nlI \Rightarrow H_1 = nI$$

(2)

$$\oint \bar{H}\cdot d\bar{l} = \int_E^F \bar{H}\cdot d\bar{l} + \int_G^P \bar{H}\cdot d\bar{l} = H_1 l - H_2 l = \emptyset$$

$$H_1 = H_2$$

$[H] = [A/m]$

$([As/m])$

mat. lineare omog. - iso - $\mu_r = \chi_m + 1$

$$\begin{cases} \bar{B}_1 = \mu_0 \mu_r nI\hat{e}_z & r < \rho \\ \bar{B}_2 = \mu_0 nI\hat{e}_z & \rho < r < R \\ \bar{B}_3 = \emptyset & r > R \end{cases}$$

$\bar{M} = \chi_m \bar{H} = \chi_m nI\hat{e}_z$

$\bar{M} = \emptyset$

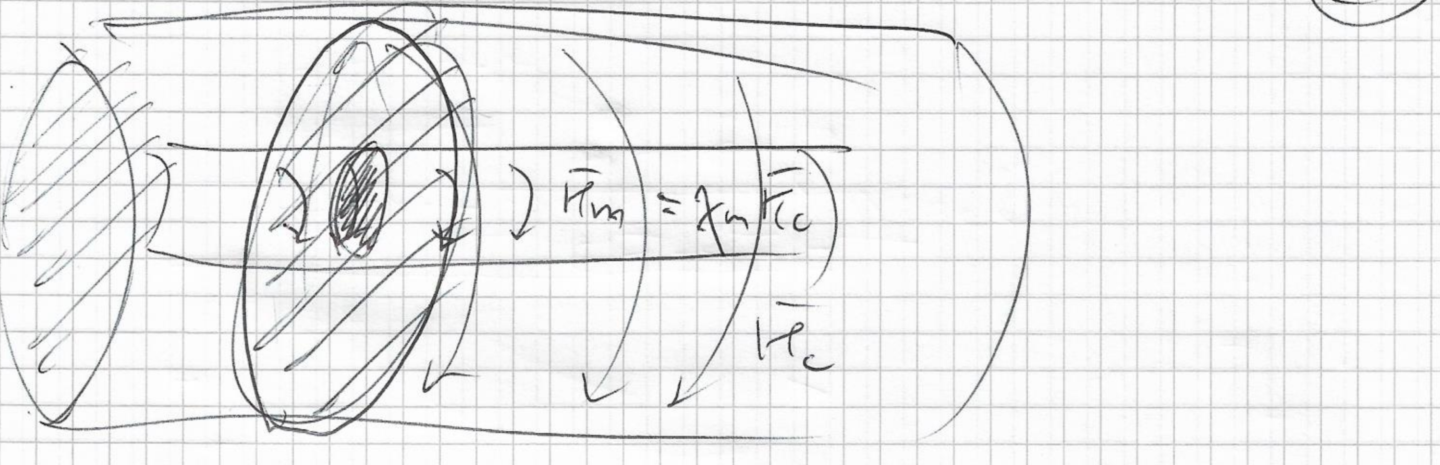
$M = \emptyset$

$$\bar{J}_m = \bar{\nabla}\times\bar{M} = \emptyset$$

$$\bar{K}_m = \bar{M}\times\hat{e}_n = \bar{M}\times\hat{e}_r = \chi_m nI\hat{e}_\theta = \chi_m \bar{K}_c$$

$\hat{e}_z\times\hat{e}_r = \hat{e}_\theta$

$\bar{K}_{cond} = nI\hat{e}_\theta$



$$\bar B_1 = \mu_0 \mu_r n I \hat e_z = \mu_0 (1+\chi_m) n I \hat e_z = \mu_0 \bar H_c + \mu_0 \bar H_m =$$

$$= \mu_0 \bar H + \mu_0 \bar M$$

$$\bar B = \mu_0 (\bar H + \bar M)$$

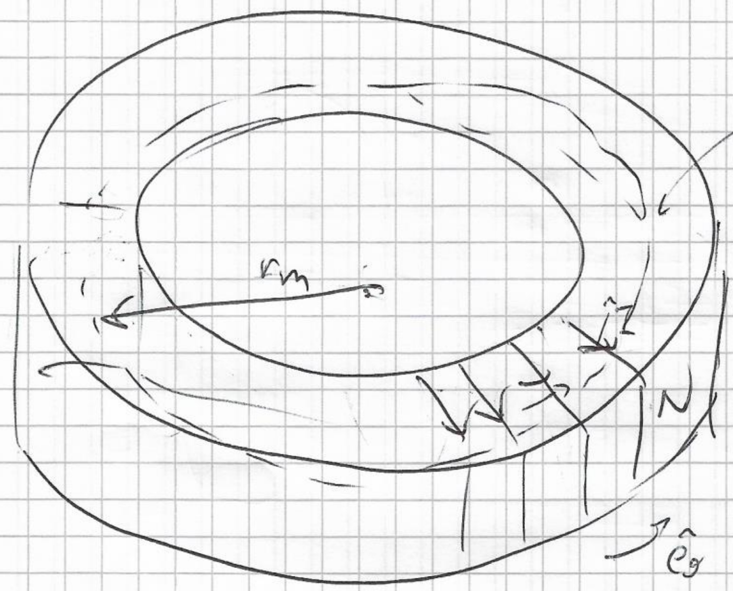
$$\Phi(\bar B) = L I \qquad N = nh$$

$$\Phi = N \Phi_s = N B_1 \pi \rho^2 + N B_2 \pi (R^2 - \rho^2) =$$

$$= \pi \mu_0 n^2 h I \left[\mu_r \rho^2 + (R^2 - \rho^2) \right]$$

$$L = \frac{\Phi}{I} = \mu_0 \pi n^2 h \left[\mu_r \rho^2 + (R^2 - \rho^2) \right]$$

(20-3)



mat. magneticamente non trasparente

1 a) Spessore trascurabile

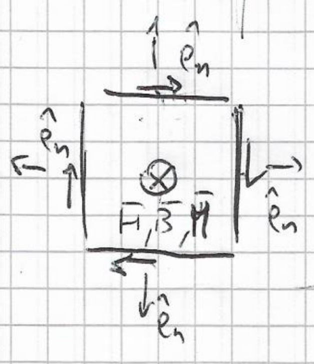
r_m

b) mat. lineare

$$\oint \bar{H} \cdot d\bar{l} = \underbrace{2\pi r_m}_{l} H = lH = NI$$

$$\bar{H} = \frac{N}{l} I \hat{e}_\theta \; ; \quad \bar{B} = \mu_0 \mu_r \frac{N}{l} I \hat{e}_\theta$$

$$\bar{M} = \chi_m \bar{H} = \chi_m \frac{N}{l} I \hat{e}_\theta$$

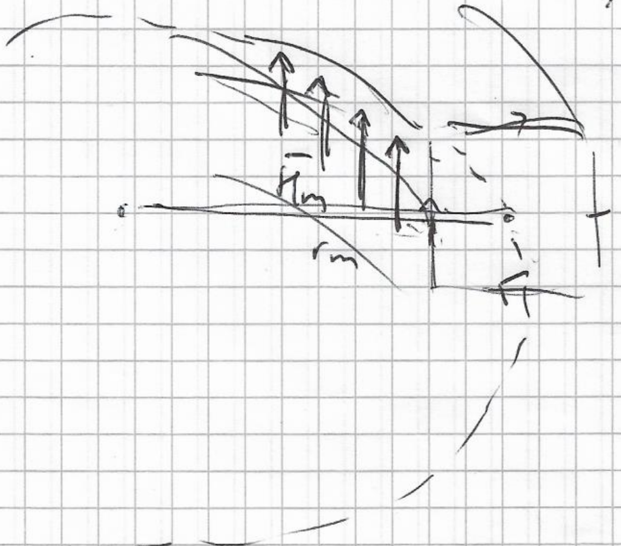


$\bar{\nabla} \times \bar{M} = \phi$ per approx $\Rightarrow$ $J_m = \phi$

$$\bar{K}_m = \bar{M} \times \hat{e}_n = \chi_m \frac{N}{l} I \; \hat{e}_\theta \times \hat{e}_n \text{ (locale)}$$

$$\oint \bar{B} \cdot d\bar{l} = \underline{\underline{lB}} = \mu_0 (NI + \underset{2\pi r_m}{l} K_m) \overset{m}{=} \mu_0 (1+\chi_m) NI =$$

$$= \underline{\underline{\mu_0 \mu_r NI}}$$



② $R_1 - R_2$, mat. lineare μ_r

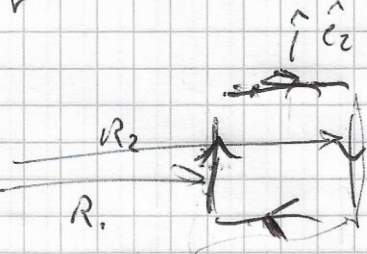
$R_1 < r < R_2 \qquad 2\pi r H_\theta = NI$

$$\bar{H} = \frac{NI}{2\pi r}\hat{e}_\theta \; ; \quad \bar{B} = \mu_0 \mu_r \frac{NI}{2\pi r}\hat{e}_\theta \; ; \quad \bar{M} = \chi_m \frac{NI}{2\pi r}\hat{e}_\theta$$

$$\bar{J}_m = \bar{\nabla}\times\bar{M} = \frac{1}{r}\left(\cancel{\frac{\partial M_z}{\partial\theta}} - \cancel{\frac{\partial M_\theta}{\partial z}}\right)\hat{e}_r + \left(\cancel{\frac{\partial M_r}{\partial z}} - \cancel{\frac{\partial M_z}{\partial r}}\right)\hat{e}_\theta +$$

$$+ \frac{1}{r}\Big(\frac{\partial}{\partial r}\underbrace{(r M_\theta)}_{\propto \frac{1}{r}} - \cancel{\frac{\partial M_r}{\partial\theta}}\Big)\hat{e}_z = \emptyset$$

$$\nabla_{cyl} \times \frac{1}{r}\hat{e}_\theta = \phi$$



$$\bar{K}_m = \bar{M}\times\hat{e}_n = \chi_m \frac{NI}{2\pi r}\hat{e}_\theta \times \hat{e}_n$$

$\bar{K}_m(R_1) = \chi_m \dfrac{NI}{2\pi R_1}\hat{e}_z \qquad I_{ms}(R_1) = \chi_m NI$

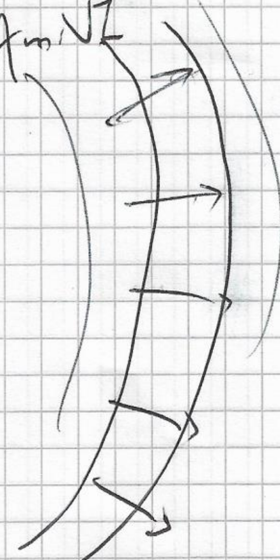
$(-\hat{e}_r)$

$\bar{K}_m(R_2) = -\chi_m \dfrac{NI}{2\pi R_2}\hat{e}_z \qquad I_{ms}(R_2) = K_m(R_2)\, 2\pi R_2 = -\chi_m NI$

$(+\hat{e}_r)$

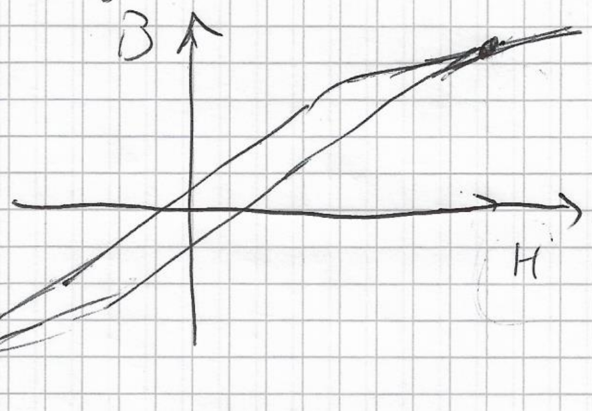
$\bar{K}_m^{\,up/down}(r) = \pm \chi_m \dfrac{NI}{2\pi r}\hat{e}_r \quad \times 2\pi r \quad I_{ms}^{\,up/down} = \pm\chi_m NI$

$(\hat{e}_z , -\hat{e}_z)$



③ Ferro - a saturazione (complet. magnetizzato)

$\bar{H}, \ \bar{B} = f(\bar{H})$



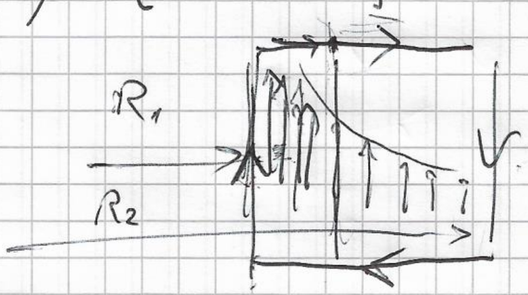
$\bar{M} = \chi_m \bar{H}$

$\bar{M} = M_s \, \hat{e}_\vartheta$

$$\bar{B} = \mu_0(\bar{H} + \bar{M}) = \mu_0 \bar{H} + \mu_0 \bar{M} = \mu_0 H_\vartheta \hat{e}_\vartheta + \mu_0 M_s \hat{e}_\vartheta$$

$\bar{H}(r) = \dfrac{NI}{2\pi r} \hat{e}_\vartheta \qquad \bar{B} = \mu_0 \left[\dfrac{NI}{2\pi r} + M_s \right] \hat{e}_\vartheta$

$\bar{J}_m = \bar{\nabla} \times \bar{M} = \dfrac{M_s}{r} \hat{e}_z$



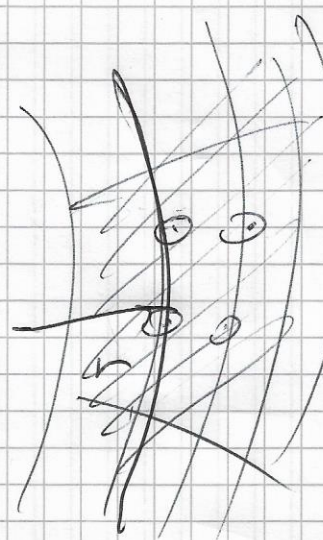
$\bar{K}_m = M_s \, \hat{e}_\vartheta \times \hat{e}_n$

$$I_{mv} = \int_0^{2\pi} \int_{R_1}^{R_2} \frac{M_s}{r} \, r \, dr \, d\vartheta = \int_{R_1}^{R_2} \frac{M_s}{\cancel{r}} \, 2\pi \cancel{r} \, dr = 2\pi M_s (R_2 - R_1)$$

$\uparrow \ I_{ms}(R_1) = 2\pi R_1 K_m(R_1) = 2\pi R_1 M_s$

$\qquad\qquad\qquad\qquad\qquad\qquad \ \ M_s$

$\downarrow \ I_{ms}(R_2) = 2\pi R_2 K(R_2) = -2\pi R_2 M_s$



lat. $\quad 2\pi r M_s$

verso l'alto $\quad 2\pi (r - R_1) M_s \quad$ (vol.)

$\qquad + 2\pi R_1 M_s \quad$ (sup.)

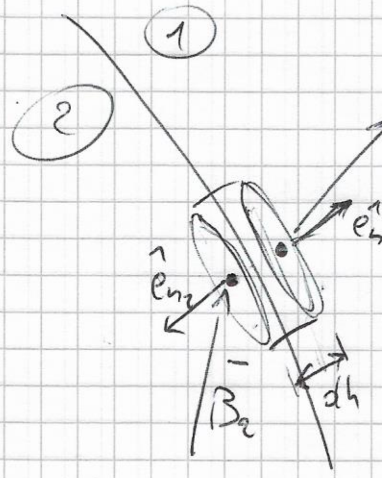
$\qquad = 2\pi r M_s$

Cond. di raccordo per $\bar B$ $\bar H$

no corr. cond.

$$\oint_S \bar B \cdot d\vec S = 0$$

$$\oint \bar H \cdot d\bar l = 0$$

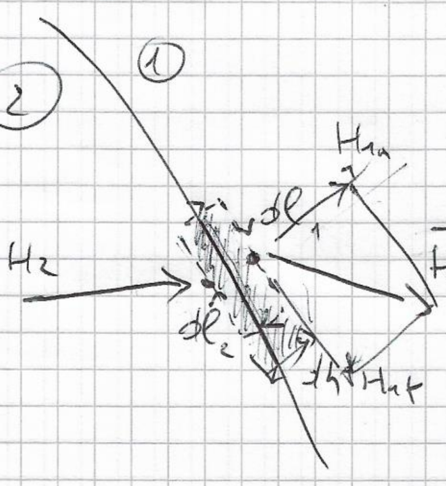


$$d\vec S = d\vec S_1 = d\vec S_2 \qquad \sqrt{dS_{1,2}} \gg dh$$

$$\Phi_{tot} = \Phi_{b1} + \Phi_{b2} + \cancel{\Phi_{lat}} = \bar B_1 \cdot d\vec S_1 + \bar B_2 \cdot d\vec S_2 =$$

$$= (B_{n1} - B_{n2})\, dS_2 = 0$$

$$\boxed{B_{n1} = B_{n2}}$$



$$dl = dl_1 = dl_2 \qquad dh \ll dl$$

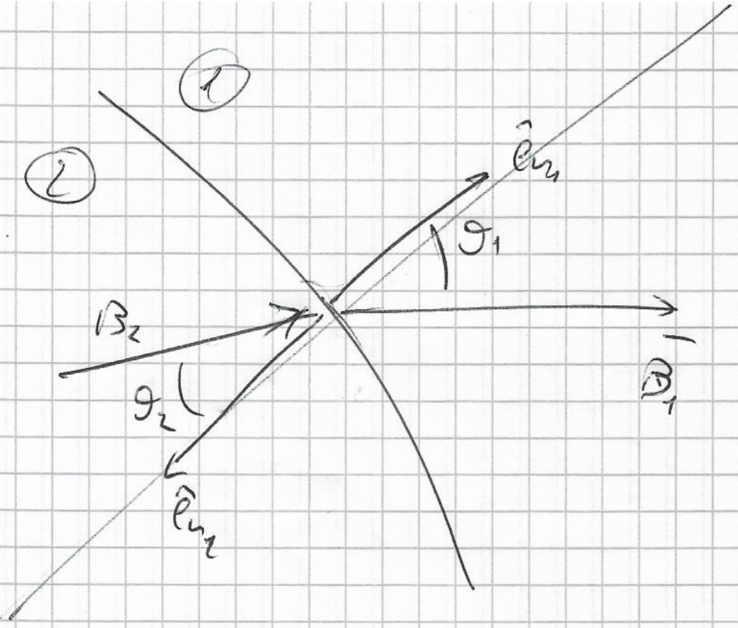
$$\oint \bar H \cdot d\bar l = \bar H_1 \cdot d\bar l_1 + \bar H_2 \cdot d\bar l_2 =$$

$$= H_{1t}\,dl - H_{2t}\,dl = 0$$

$$\boxed{H_{1t} = H_{2t}}$$

$$\bar B = \mu \bar H \qquad \boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

$$\left[(H_{1t} - H_{2t})\cancel{dl} = J_{1\perp}\,\cancel{dl}\,dh/2 + J_{2\perp}\,\cancel{dl}\,dh/2 + K_\perp\,\cancel{dl} \qquad H_{1t} - H_{2t} = K_\perp \right]$$



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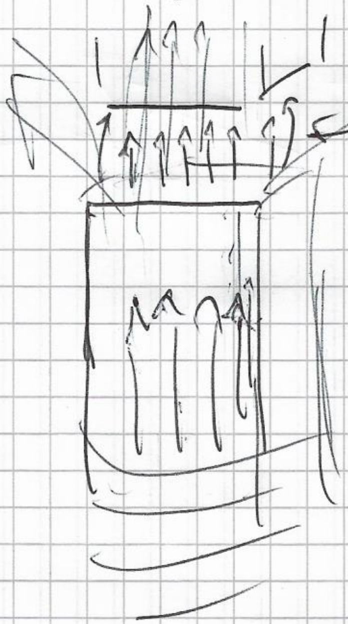
$$\frac{B_{t_1}}{B_{t_2}} = \frac{B_{t_1}/B_{n_1}}{B_{t_2}/B_{n_2}} = \boxed{\frac{tg\,\vartheta_1}{tg\,\vartheta_2} = \frac{\mu_1}{\mu_2}}$$

legge di rifrazione delle linee di $\vec{B}$

① ferro ; ② vuoto $(\mu_{r_2} \cong 1)$
$(\mu_{r_1} \gg 1)$

$$\frac{tg\,\vartheta_1}{tg\,\vartheta_2} = \frac{\mu_1}{\cancel{\mu}} \gg 1 \qquad tg\,\vartheta_1 \gg tg\,\vartheta_2 \qquad \vartheta_2 \to \emptyset$$

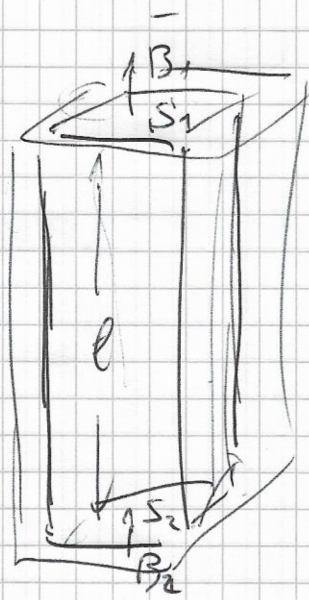
$\vec{B}$ esce +o- $\perp$ alla sup. del ferro-



traferro / interferro

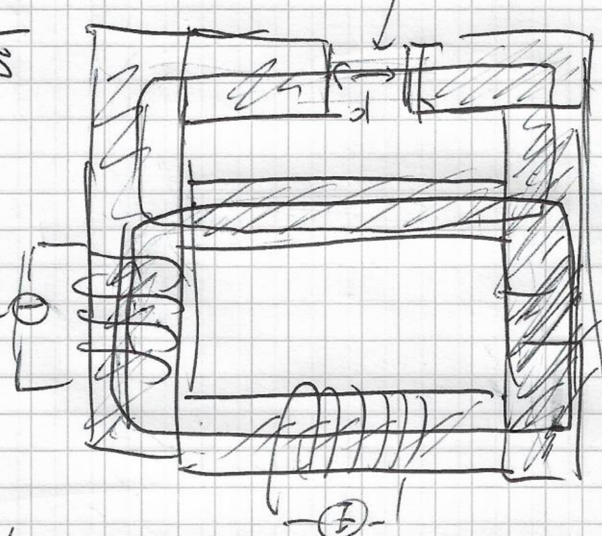
no effetti di bordo

Circuiti magnetici – legge di Hopkinson



$d < \sqrt{S}$

$l \gg \sqrt{S}$



$$\oint \bar{B} \cdot d\bar{S} = 0$$

$$\oint \bar{H} \cdot d\bar{l} = \cancel{0}\; NI$$

$$\Phi_{lat} = 0 \quad \rightarrow \quad \Phi = B_1 S_1 - B_2 S_2 = 0$$

$$B_1 S_1 = B_2 S_2 \qquad \Phi_1 = \Phi_2 \qquad \boxed{\Phi = \text{cost.}}$$

omo. iso. $\quad \bar{B} = \mu \bar{H} \qquad\qquad B = \Phi / S$

$$NI = \oint \bar{H} \cdot d\bar{l} = \oint \frac{\bar{B}}{\mu} \cdot d\bar{l} = \oint \frac{\Phi}{\mu S} \, dl = \Phi \oint \frac{dl}{\mu S}$$

$\mathcal{F}$ forza magnetomotrice $\quad NI = \boxed{\Phi} \cdot \oint \frac{dl}{\mu S} \rightarrow \mathcal{R}$ riluttanza

$$f = RI = \oint \frac{dl}{\sigma S} \cdot \boxed{I}$$

$$\mathcal{R} = \frac{l}{\mu S} = \sum \mathcal{R}_i = \sum_i \frac{l_i}{\mu_i S_i} \qquad \boxed{\mathcal{F} = \mathcal{R} \Phi} \quad \text{legge di Hopkinson}$$

$$f = RI$$