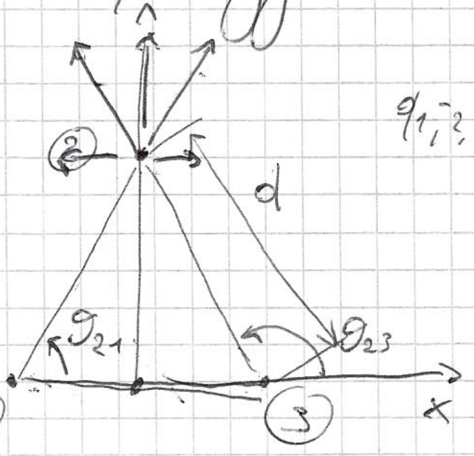
ES. #3 Campo elettrostatico; legge di Gauss

14/X/2020

(1-3)



$q_{1,2,3} = q = 1\mu C = 10^{-6}\,C$

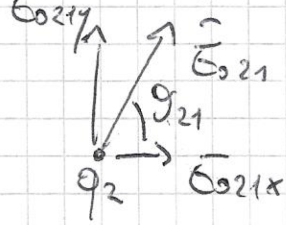
$d = 10\,cm$

$m = 1g$

$\bar{E}_0 = \bar{E}_{021} + \bar{E}_{023}$

principio di sovrapposizione

$\bar{F}_C = q_2 \bar{E}_0 = q\bar{E}_0$



$\bar{E}_{021x} = \bar{E}_{021}\cos\vartheta_{21}$

$\bar{E}_{021y} = \bar{E}_{021}\sin\vartheta_{21}$

$\bar{E}_{021} = ?$

$$\bar{E}_{021} = \frac{1}{4\pi\varepsilon_0}\,\underbrace{\frac{q_{(1)}}{|\bar{r}_2-\bar{r}_1|^3}\,(\bar{r}_2-\bar{r}_1)}_{d^3}$$

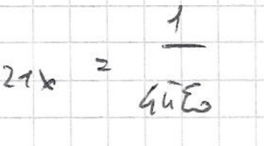
$$\bar{E}_{021x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\cos\vartheta_{21} = \frac{1}{8\pi\varepsilon_0}\frac{q}{d^2} = 4.50\cdot 10^5\ N/C$$

opp. V/m

$$\bar{E}_{021y} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\sin\vartheta_{21} = \frac{\sqrt{3}}{8\pi\varepsilon_0}\frac{q}{d^2} = 7.79\cdot 10^5\ V/m$$

$\bar{E}_{021z} = \phi \qquad z_1 = z_2 = z_3$ (sistema planare)

$\bar{E}_{023}$



$$\bar{E}_{023x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\cos\vartheta_{23} = -\frac{1}{8\pi\varepsilon_0}\frac{q}{d^2} = -\bar{E}_{021x}$$

$$\Rightarrow \bar{E}_{0x}^{(2)} = \phi$$

$$E_{023y} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\sin\vartheta_{23} = +\frac{\sqrt{3}}{8\pi\varepsilon_0}\frac{q}{d^2} = \bar{E}_{021x}$$

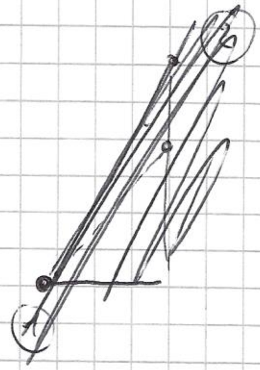
$$\Rightarrow \bar{E}_{0y} = 2\bar{E}_{021y} = 1.56\cdot 10^6\ V/m \Rightarrow \bar{F}_C = q\bar{E}_{0y}\,\hat{e}_y = 1.56\,N\,\hat{e}_y$$

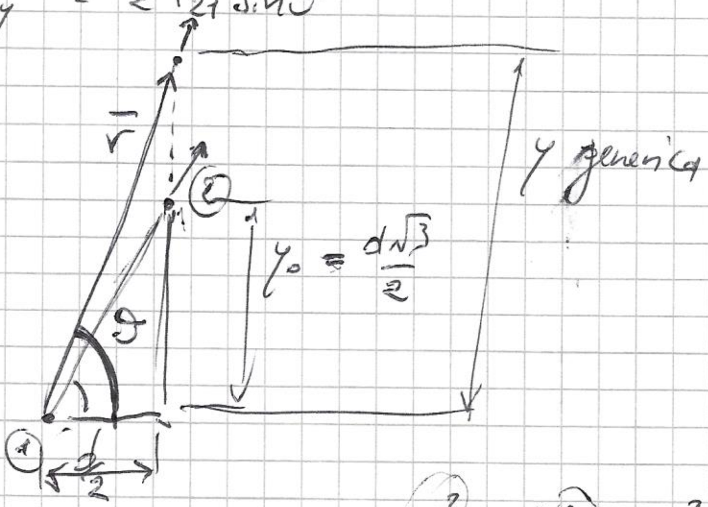
Velocità limite $\qquad \overline{F_C} \sim \frac{1}{r^2}$

$\overline{F_C} \to$ Lavoro

$$\mathcal{L} = \int_\gamma \overline{F} \cdot d\overline{l} = \int_{y_0}^{+\infty} F_y \, dy$$

$$F_y = 2\overline{F}_{21,y} = 2\overline{F}_{21} \sin\vartheta$$





$$F_y = 2F_{21y} = 2F_{21}\sin\vartheta = 2\frac{q^2}{4\pi\varepsilon_0}\frac{\sin\vartheta}{r^2} = \frac{q^2}{2\pi\varepsilon_0}\frac{r\sin\vartheta}{r^3} =$$

$r\sin\vartheta \to y$

$$= \frac{q^2}{2\pi\varepsilon_0}\frac{y}{(y^2+d^2/4)^{3/2}}$$

$$\mathcal{L} = \int_{y_0}^{+\infty} \frac{q^2}{2\pi\varepsilon_0}\underbrace{\frac{y}{(y^2+d^2/4)^{3/2}}}_{\text{der. di } -\frac{1}{(y^2+d^2/4)^{1/2}}}\,dy =$$

$$= -\frac{q^2}{2\pi\varepsilon_0}\left[\frac{1}{(y^2+d^2/4)^{1/2}}\right]_{y_0}^{+\infty} = \frac{q^2}{2\pi\varepsilon_0}\frac{1}{(y_0^2+d^2/4)^{1/2}}$$

$y_0 = \frac{d\sqrt{3}}{2}$

$$\mathcal{L} = \frac{q^2}{2\pi\varepsilon_0 d} = 0.18\text{ J} \quad \underline{\text{Joule}}$$

En. pot. $\longrightarrow$ en. cin. $\quad \mathcal{L} = E_{k\infty} = \frac{1}{2}mv_\infty^2$

$$v_\infty = 18.97 \text{ m/s}$$

Distribuzioni continue di carica

$\bar{E}_0(P) = ?$ linee, superfici, volumi

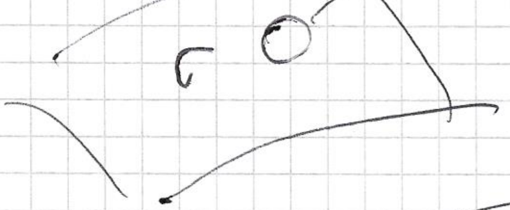
$\lambda \; [q/L]$

$dl \to dq = \lambda\, dl \; (\lambda(\bar r)\, d\bar l(\bar r))$

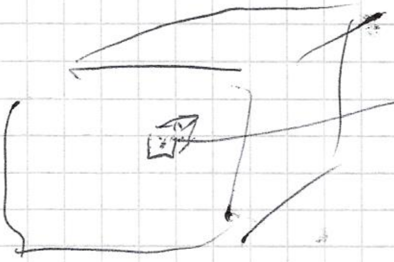
$q \to \bar E_0 \quad \rightsquigarrow \quad dl \to dq \to d\bar E_0$

$\to$ integrale $\int d\bar E_0$

~~dl → dq =~~

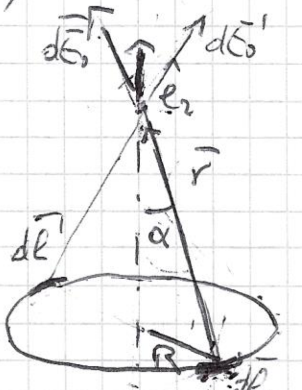


$dS \to dq = \sigma\, dS$



$d\tau \to dq = \rho\, d\tau$

(1-4) λ uniforme $\to$ C raggio R (vuoto)

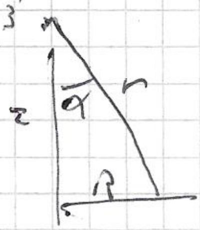


$dq = \lambda\, dl \rightsquigarrow d\bar E_0$

$$d\bar E_0 = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^3}\bar r$$

$$dE_{0z} = dE_0 \cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r^2}\cos\alpha$$

$$E_{0z} = \int dE_{0z} = \frac{1}{4\pi\varepsilon_0}\frac{\cos\alpha}{r^2}\int_C dl = \frac{\lambda\cos\alpha}{4\pi\varepsilon_0 r^2}\, 2\pi R = \frac{\lambda R}{2\varepsilon_0}\frac{r\cos\alpha}{r^3}$$

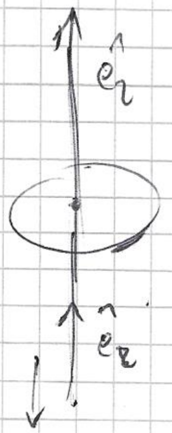


$$E_{0z}(z) = \frac{\lambda R}{2\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}$$

$$\bar{E}_0(z) = \frac{\lambda R}{2\varepsilon_0} \frac{z}{\left(z^2+R^2\right)^{3/2}} \hat{e}_z$$

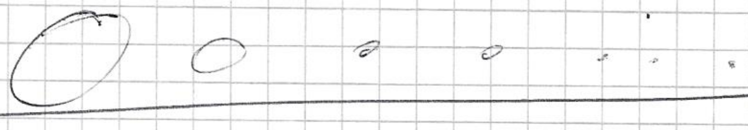
Nota: a grande distanza, $z \gg R$ $\quad \lim_{z \to +\infty} (z^2+R^2) = z^3$

$$\bar{E}_0(z) = \mathrm{sgn}(z) \frac{\lambda R}{2\varepsilon_0} \frac{1}{z^2} \hat{e}_z = \mathrm{sgn}(z) \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, 2\pi R}{z^2} \hat{e}_z = \frac{Q}{4\pi\varepsilon_0} \frac{1}{z^2} \hat{e}_z$$



$$\left(\mathrm{sgn}(z) = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \end{cases} \right)$$

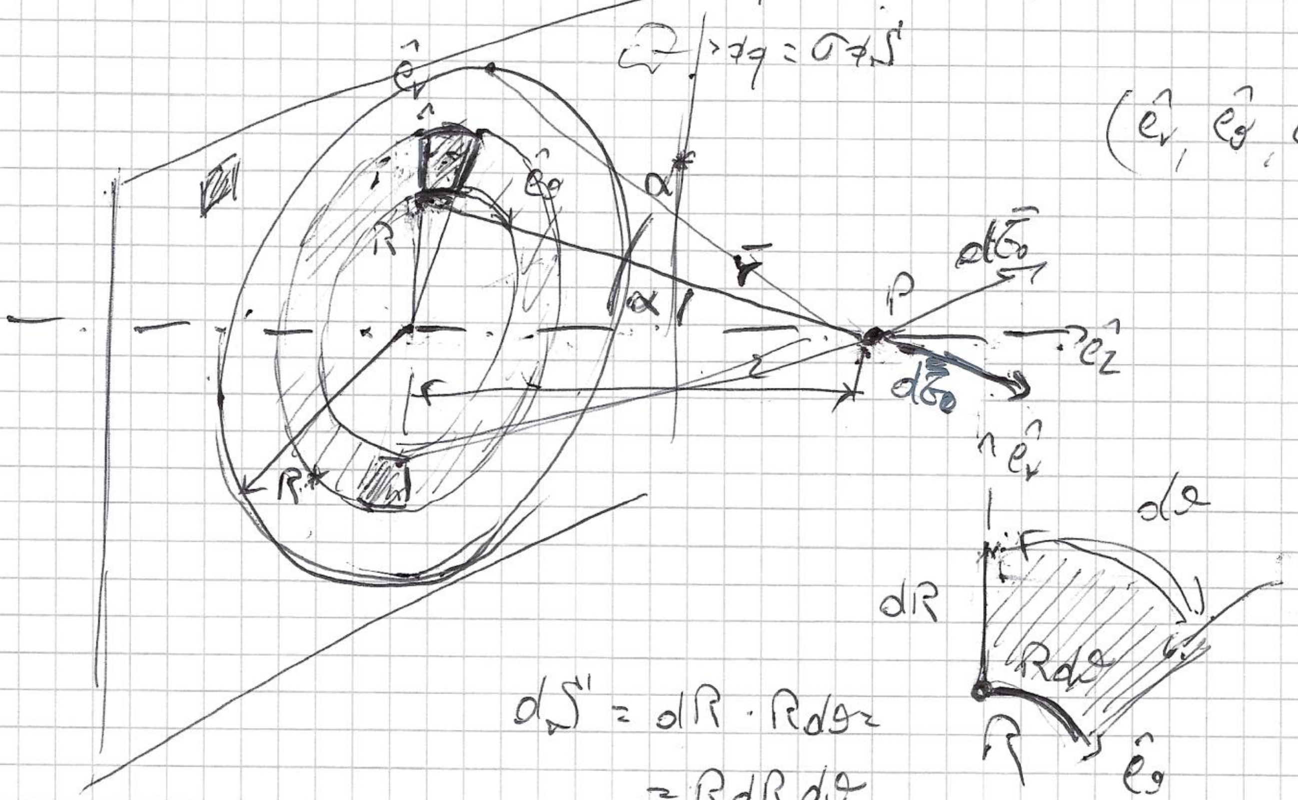
$\lambda\, 2\pi R \doteq Q$



(1-5) $\quad \sigma$ uniforme sup. circolare R^*

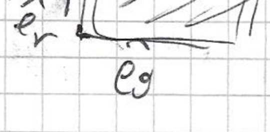
$dq = \sigma\, d_sS'$

$(\hat{e}_r, \hat{e}_\theta, \hat{e}_z)$



$$d_sS' = dR \cdot R\,d\theta = R\,dR\,d\theta$$

$$dq' = \sigma\, d_sS' = \sigma R\,dR\,d\theta$$



$$d_sS = 2\pi R\,dR \to dq = \sigma\, 2\pi R\,dR$$

$$\boxed{dE_{0z} = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma\cdot 2\pi R\,dR}{r^2}\cos\alpha}$$

$$z = r\cos\alpha \qquad R = r\sin\alpha = z\,\mathrm{tg}\,\alpha$$

$$\frac{dR}{d\alpha} = z\,\frac{1}{\cos^2\alpha} \Rightarrow dR \rightarrow \frac{z\,d\alpha}{\cos^2\alpha}$$

$$E_{0z} = \int dE_{0z} = \int_{\phi}^{\alpha^*} \frac{\sigma}{2\varepsilon_0}\,\underbrace{z\,\mathrm{tg}\,\alpha}_{R}\,\underbrace{\frac{\cos^2\alpha}{z^2}}_{1/r^2}\,\cos\alpha\,\underbrace{\frac{z\,d\alpha}{\cos^2\alpha}}_{dR} =$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{\phi}^{\alpha^*}\sin\alpha\,d\alpha = \frac{\sigma}{2\varepsilon_0}\Big[-\cos\alpha\Big]_{\phi}^{\alpha^*} = \frac{\sigma}{2\varepsilon_0}\left[1-\cos\alpha^*\right]$$

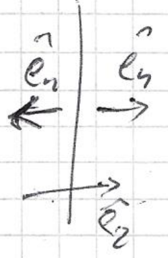
$$\bar{E}_0(z) = \mathrm{sgn}(z)\frac{\sigma}{2\varepsilon_0}\left[1-\cos\alpha^*\right]\hat{e}_z = \mathrm{sgn}(z)\frac{\sigma}{2\varepsilon_0}\left[1-\frac{|z|}{(z^2+R^{*2})^{1/2}}\right]\hat{e}_z$$

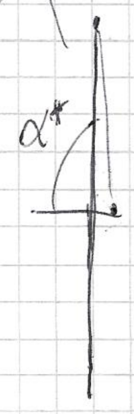
$$\underbrace{\bar{E}_0(z)}_{z>\phi} = -\bar{E}_0(-z)$$

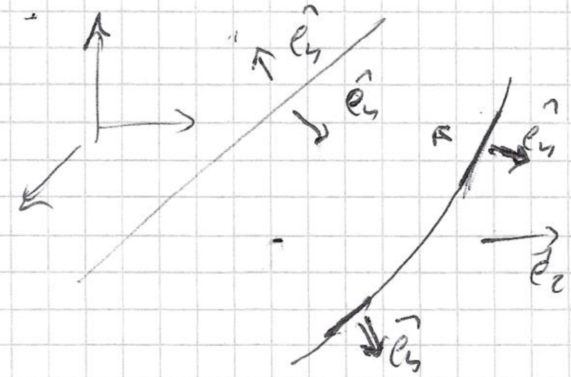
Nota: $R^* \to \infty$ $\quad(\alpha^* \Rightarrow \pi/2\ ;\ \cos\alpha^* \Rightarrow \phi)$

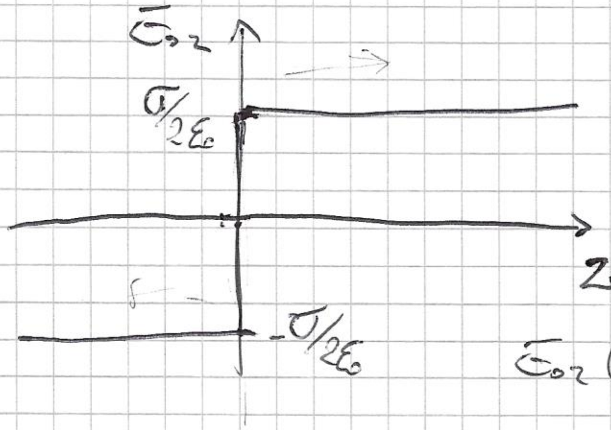
= piano indefinitamente esteso di σ

$$\bar{E}_0(z) = \mathrm{sgn}(z)\frac{\sigma}{2\varepsilon_0}\hat{e}_z \qquad \bar{E}_0 = \frac{\sigma}{2\varepsilon_0}\hat{e}_n$$



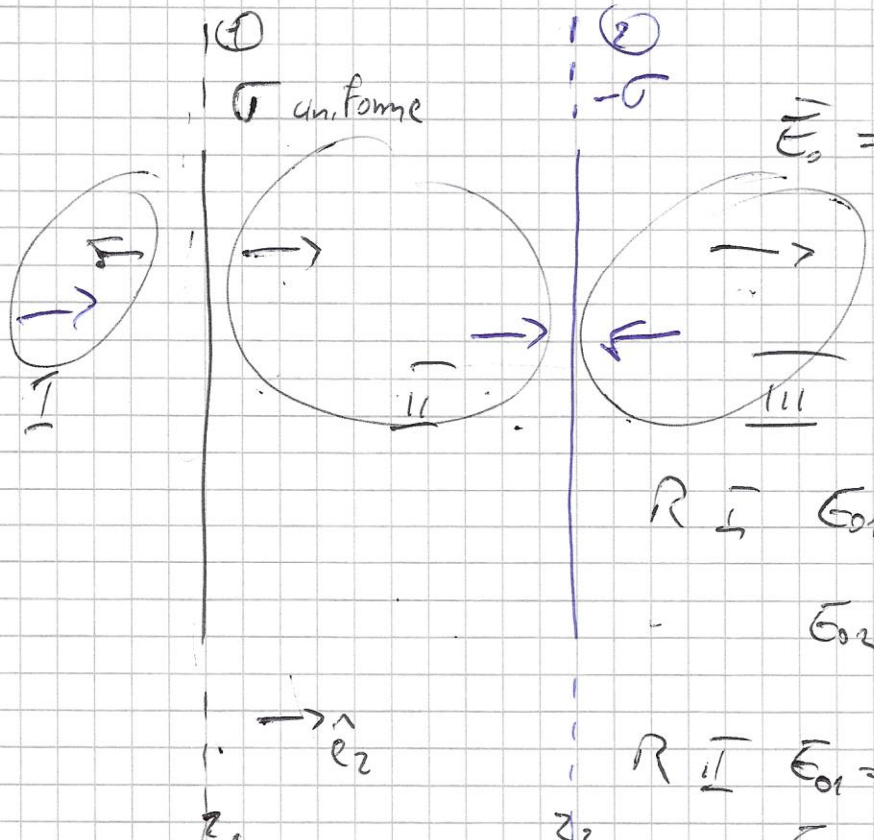






$$E_{0z}(0^+) - E_{0z}(0^-) = \sigma/\varepsilon_0$$

(1-6)

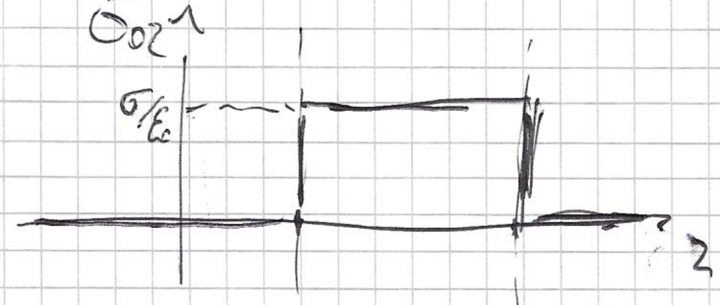


$$\vec{E}_0 = \vec{E}_{01} + \vec{E}_{02}$$

R I $\left.\begin{array}{l} E_{01} = -\sigma/2\varepsilon_0 \\ E_{02} = \sigma/2\varepsilon_0 \end{array}\right\} = \phi$

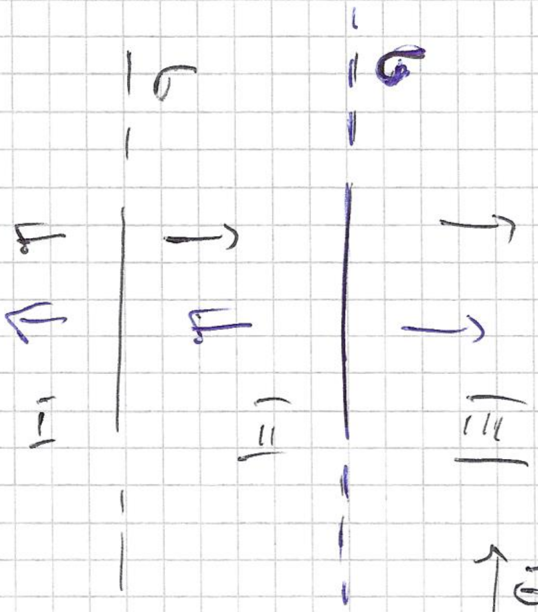
R II $\left.\begin{array}{l} E_{01} = \sigma/2\varepsilon \\ E_{02} = \sigma/2\varepsilon_0 \end{array}\right\} \dfrac{\sigma}{\varepsilon_0}$

R III $\left.\begin{array}{l} E_{01} = \sigma/2\varepsilon_0 \\ E_{02} = -\sigma/2\varepsilon_0 \end{array}\right\} \phi$



$$E_{0z}(z_1^+) - E_{0z}(z_1^-) = \sigma/\varepsilon_0$$

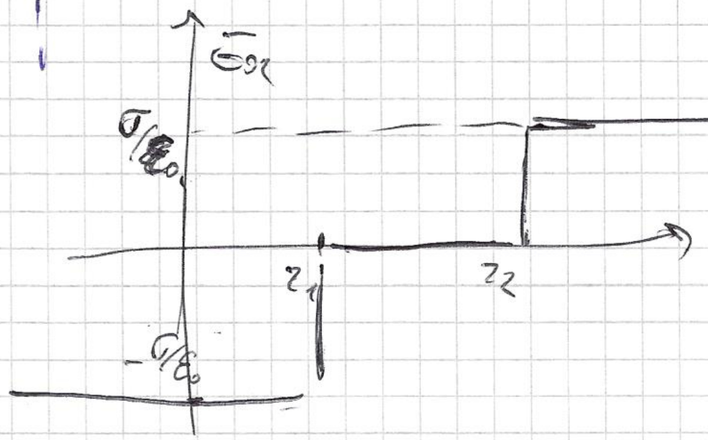
$$E_{0z}(z_2^+) - E_{0z}(z_2^-) = -\sigma/\varepsilon_0$$



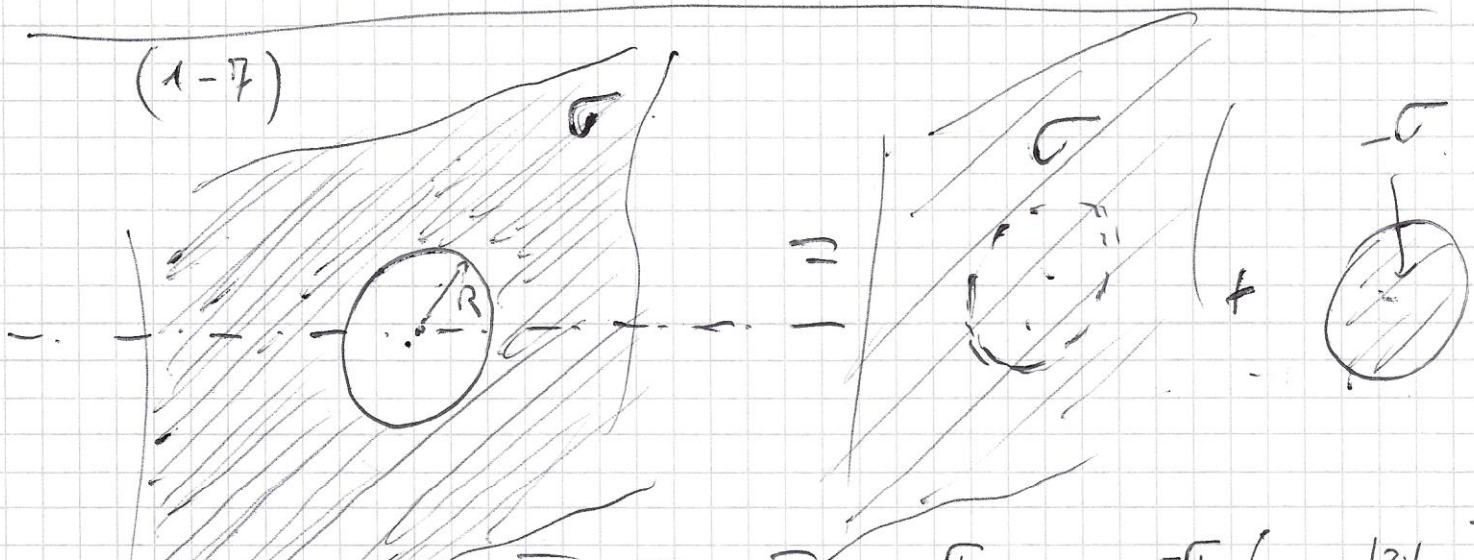
I $\quad E_{0z} = -\sigma/\varepsilon_0$

II $\quad E_{0z} = 0$

III $\quad E_{0z} = \sigma/\varepsilon_0$



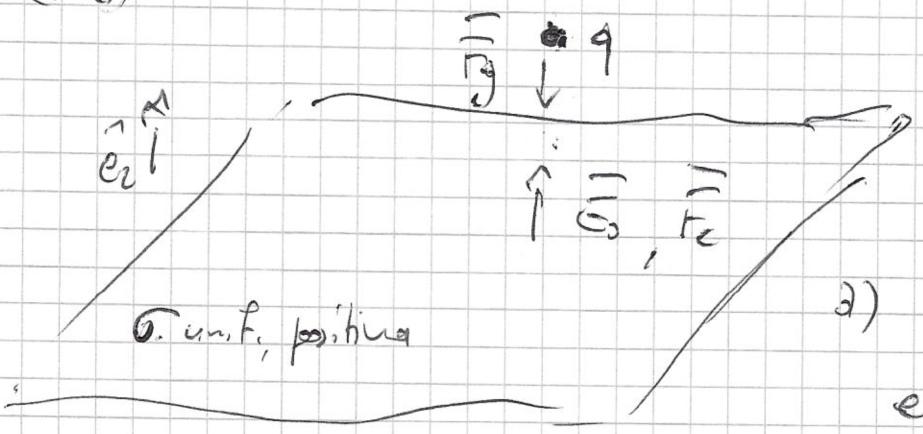
(1-7)



$$\bar{E}_0 = \bar{E}_{0p} + \bar{E}_{0f} = \frac{\sigma}{2\varepsilon_0}\hat{e}_z + \frac{-\sigma}{2\varepsilon_0}\left[1 - \frac{|z|}{(z^2+R^2)^{1/2}}\right]\hat{e}_z$$

$$\bar{E}_0 = \frac{\sigma}{2\varepsilon_0}\frac{z}{[z^2+R^2]^{1/2}}\hat{e}_z$$

(1-8)



a) $H^+ = p^+$

e $\quad m_p$

$F_g = F_c$ $\quad \boxed{mg = q E_{0z}}$

$\boxed{mg = q\, \sigma / 2\varepsilon_0}$

$E_{0z} = \sigma / 2\varepsilon_0$

$$E_{0z} = \frac{mg}{q} = \frac{1.67 \cdot 10^{-27}\ kg}{1.6 \cdot 10^{-19}\ C} \cdot 9.81 = 1.02 \cdot 10^{-7}\ V/m$$

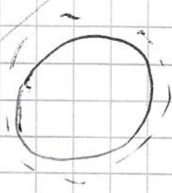
$$\sigma = 2\varepsilon_0 E_{0z} = 1.81 \cdot 10^{-18}\ C/m^2$$

↳ not much dense

b) "dust" polveri micrometriche

$R = 1\,\mu m$ $\qquad \rho = 1.05\ g/cm^3$

$q = 1000\, e$



$$m = \rho\, \frac{4\pi}{3} R^3 = 4.40 \cdot 10^{-15}\ kg \gg m_p$$

$$E_{0z} = \frac{4.40 \cdot 10^{-15}}{10^3 \cdot 1.6 \cdot 10^{-19}} \cdot 9.81 = 270\ V/m$$

$$\sigma = 2\varepsilon_0 E_z = 4.77 \cdot 10^{-9}\ C/m^2$$