ES. #30 Onde elettromagnetiche

07/VI/2021

(1)

$$\begin{cases} \bar{\nabla}\cdot\bar{E} = 0 & \bar{\nabla}\cdot\bar{B} = 0 \\ \bar{\nabla}\times\bar{E} = -\partial\bar{B}/\partial t & \bar{\nabla}\times\bar{B} = \varepsilon_0\mu_0\,\partial\bar{E}/\partial t \end{cases}$$

$\bar{\nabla}\times$ ...

$$\begin{cases} \nabla^2\bar{E} = \varepsilon_0\mu_0 \dfrac{\partial^2\bar{E}}{\partial t^2} \\ \nabla^2\bar{B} = \varepsilon_0\mu_0 \dfrac{\partial^2\bar{E}}{\partial t^2} \end{cases} \quad \bar{f}$$

$$\nabla^2\bar{f} = \frac{1}{v^2}\frac{\partial^2\bar{f}}{\partial t^2}$$

$$\varepsilon_0\mu_0 = \frac{1}{v^2} = \frac{1}{c^2}$$

Onde piane → fronte d'onda piano

$$\bar{E} = \bar{E}_0\, e^{i(\bar{k}\cdot\bar{r}-\omega t)} \qquad \bar{k} \qquad \hat{k} = \frac{\bar{k}}{|\bar{k}|}$$

$$\bar{B} = \bar{B}_0\, e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

onde elm = trasversali

Propagazione lungo $\hat{e}_x$ $\quad \bar{k} = k\hat{e}_x$ $\qquad \bar{E}, \bar{B} \perp \hat{k}$

fronti d'onda $x = ct$,

$$\bar{E}_0 = E_{0y}\hat{e}_y = E_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_y$$

$$\left(\text{se } \bar{E}_0 = E_{0x}\, e^{i(\ldots)}\,\hat{e}_x \Rightarrow \nabla\cdot\bar{E} \neq 0 \right)$$

$$\bar{B}_0 = B_{0z}\hat{e}_z$$

$$\begin{cases} \bar{E}(x,t) = E_{0y}\, e^{i(kx-\omega t)}\,\hat{e}_y \\ \bar{B}(x,t) = B_{0z}\, e^{i(kx-\omega t)}\,\hat{e}_z \end{cases}$$

$B_{0z} \to E_{0y}/c$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar{E} = 0$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$k^2 \cancel{E} = \frac{\omega^2}{c^2} \cancel{E}$$

$$\boxed{k^2 = \frac{\omega^2}{c^2}}$$ rel. di dispersione (banale)

$$k = \frac{2\pi}{\lambda}$$

$$k = \pm \frac{\omega}{c}$$

Maxwell?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \left[\frac{\partial E_y}{\partial x} - \frac{\partial \cancel{E_x}}{\partial y} \right] \hat{e}_z = ik E_{0y} e^{i(kx-\omega t)} \hat{e}_z$$

$$-\frac{\partial \vec{B}}{\partial t} = + i\omega B_{0z} e^{i(kx-\omega t)} \hat{e}_z$$

$$k E_{0y} = \omega B_{0z}$$

$$\boxed{\frac{E_{0y}}{B_{0z}} = \frac{\omega}{k} = c}$$

E_{0z} e $\rightarrow$ B_{0y}

$$\boxed{\frac{E_{0z}}{B_{0y}} = -c}$$

$$\vec{E} = \vec{B} \times \vec{c}$$

$$\vec{E} = \vec{B} \times \vec{v}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\vec{B} = \frac{\vec{v} \times \vec{E}}{v^2} = \frac{1}{v} \hat{k} \times \vec{E}$$

$$v = \frac{1}{\sqrt{\varepsilon \mu}}$$

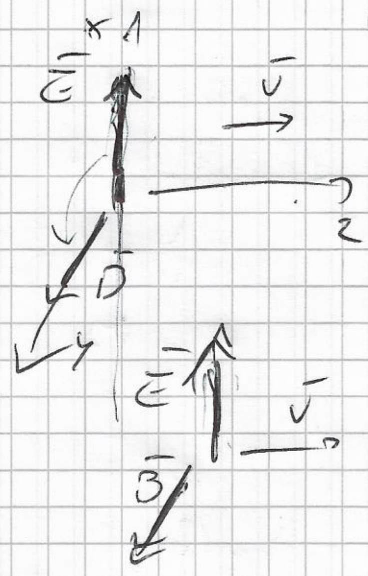
$$\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$$

Polarizzazione

Lineare $\bar{E}(z,t) = E_{0x} e^{i(kz-\omega t)} \hat{e}_x$

$\bar{B}(z,t) = B_{0y} e^{i(kz-\omega t)} \hat{e}_y$

$\downarrow E_{0x}/c$



$\bar{E}_1 = E_{01} \cos(kz - \omega t + \varphi_1) \hat{e}_x$

$+$

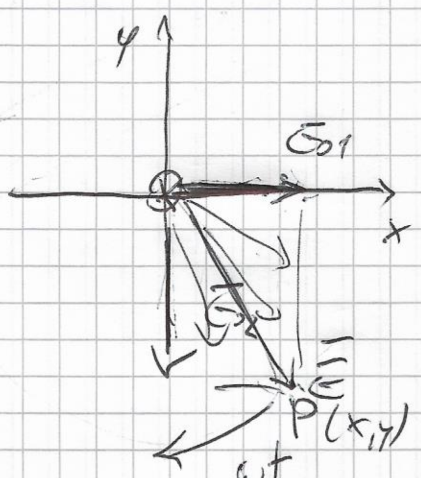
$\bar{E}_2 = E_{02} \cos(kz - \omega t + \varphi_2) \hat{e}_y$

Es.: $\varphi_1 = 0$ $\leadsto$ $\bar{E}_1(z,t) = E_{01} \cos(kz-\omega t) \hat{e}_x$

$+$

$\varphi_2 = -\pi/2$ $\quad \bar{E}_2(z,t) = E_{02} \sin(kz-\omega t) \hat{e}_y$

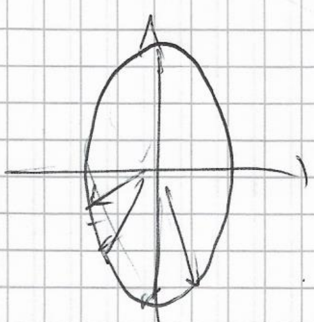
$z = 0$



$\bar{E}_1 = E_{01} \cos(\omega t) \hat{e}_x$

$\bar{E}_2 = -E_{02} \sin(\omega t) \hat{e}_y$

$P \begin{cases} x = E_{01} \cos(\omega t) \\ y = -E_{02} \sin(\omega t) \end{cases}$



$\frac{x^2}{E_{01}^2} + \frac{y^2}{E_{02}^2} = \cos^2(\omega t) + \sin^2(\omega t) = 1$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$ ellisse

Pol. ellittica

$E_{01} = E_{02}$ Pol. circolare

Poynting $\bar{S} = \frac{\bar{E} \times \bar{B}}{\mu}$ // $\hat{n}$ $v = \frac{1}{\sqrt{\varepsilon\mu}}$

$$\bar{S} = \frac{\bar{E} \times \bar{B}}{\mu} = \frac{1}{v\mu} \bar{E} \cdot \bar{E} \, \hat{n} = \sqrt{\frac{\varepsilon}{\mu}} \, \bar{E}^2 \hat{n}$$

$$\bar{B} = \frac{v \times \bar{E}}{v^2} = \frac{\hat{n} \times \bar{E}}{v}$$

$Z \doteq \sqrt{\frac{\mu}{\varepsilon}}$ $[\Omega]$ impedenza caratteristica $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$

Intensità = v. Poynting $\langle I \rangle$

I intensità media $I = \langle \bar{S} \rangle = \frac{1}{T} \int_T S(t)\,dt$

Pressione di radiazione $\bar{p} = \frac{\bar{S}}{v}$

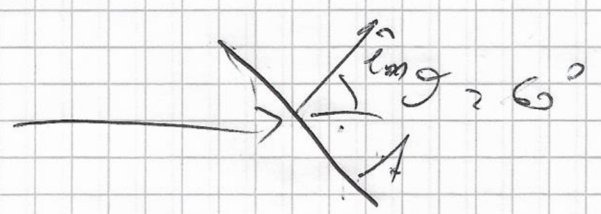
$\frac{q.d.m}{\text{unit Sup}} = \int \bar{p}\,dt$ $\langle q.d.m \rangle$ = media sul periodo

$$\langle \bar{p} \rangle = \frac{\langle \bar{S} \rangle}{v}$$

$$\langle q.d.m. \rangle = \frac{1}{v} I \, T \; (\Delta t)$$

(21-1) $\quad \hat{e}_x \quad$ vuoto $\quad \bar{k} = k\,\hat{e}_x$

$A\hat{e}_n$



$$\bar{B} = B_0\left[2\cos(kx-\omega t)\,\hat{e}_y + 3\sin(kx-\omega t)\,\hat{e}_z\right] \qquad B_0 = 10^{-8}\,T$$

$$\bar{E} = \bar{B}\times\bar{c} = E_0\left[3\sin(kx-\omega t)\,\hat{e}_y - 2\cos(kx-\omega t)\,\hat{e}_z\right]$$

$$E_0 = B_0 c = 3\ V/m$$

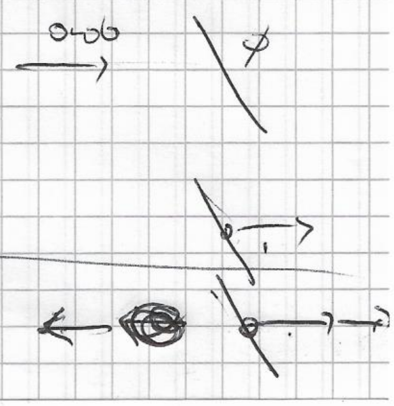
$$\langle W\rangle = \langle\bar{S}\rangle\cdot\bar{A} = \langle S\rangle A\,(\cos 60°) = \frac{1}{2}\langle S\rangle A =$$

$$= \frac{1}{2}A\,\langle \bar{E}\times\bar{B}/\mu_0\rangle = \frac{1}{2\mu_0}A\,\langle E^2/c\rangle = \frac{A}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,\langle E^2\rangle$$

$$\langle E^2\rangle = \frac{1}{T}\int_0^T E^2\,dt = \frac{1}{T}\int_0^T E_0^2\left[9\sin^2(\dots) + 4\cos^2(\dots)\right]dt =$$

$$= \frac{9}{2} + \frac{4}{2} = \frac{13}{2} \qquad\qquad \frac{1}{T}\int_T \sin^2 dt = \frac{1}{2}$$

$$\langle W\rangle \quad \frac{AE_0^2}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,\frac{13}{2} = 0.70\ W \qquad\qquad A = 9\ m^2$$



$$\langle p\rangle = \langle S\rangle/c = 2.32\cdot 10^{-2}\ N/m^2$$

$$\frac{q.d.m}{u\,dt} = \langle p\rangle A = 20.88\ N$$

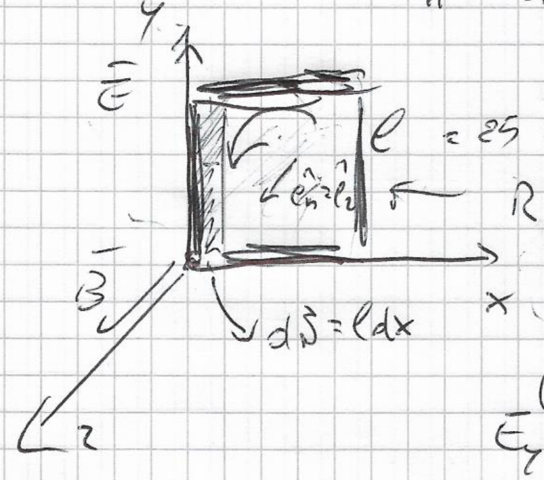
riflessione $\rightarrow \quad 2\langle p\rangle A = 41.76\ N$

(21-2)

$\bar{E}_{0y} = 100\ mV/m \qquad \nu = 300\ MHz$

$\hat{H} = \hat{e}_x \qquad$ vuoto

$\bar{E}_y(x=\phi, t=\phi) = \bar{E}_{0y}$



$\ell = 25\ cm = \lambda/4$

$R = 30\ \Omega$

$i_{ind}(t)$

$$\bar{E}_y^{(x,t)} = \bar{E}_{0y} \cos(kx - \omega t)$$

$$B_z(x,t) = \bar{E}_{0y}/c \, \cos(kx - \omega t)$$

$$\Phi(t) = \int_\phi^\ell B_z(x,t)\, \ell dx = \frac{\bar{E}_{0y}\ell}{c} \int_\phi^\ell \cos(kx - \omega t) = \frac{\bar{E}_{0y}\ell}{ck} \Big[\sin(kx - \omega t) \Big]_\phi^\ell =$$

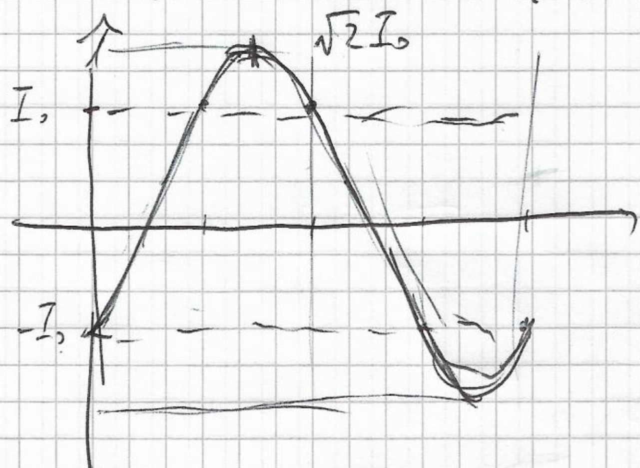
$ck \to \omega$

$$= \frac{\bar{E}_{0y}\ell}{\omega} \Big[\sin(\overset{\pi/2}{k\ell} - \omega t) - \sin(\phi - \omega t) \Big] = \frac{\bar{E}_{0y}\ell}{\omega} \Big[\cos(\omega t) + \sin(\omega t) \Big]$$

$$k\ell = k\frac{\lambda}{4} = k\frac{1}{4}\frac{2\pi}{k} = \frac{\pi}{2}$$

$$f_{ind} = -\frac{d\Phi}{dt} \Rightarrow i_{ind} = -\frac{1}{R}\frac{d\Phi}{dt} = \frac{\bar{E}_{0y}\ell}{R}\Big[\sin(\omega t) - \cos(\omega t)\Big]$$

$\frac{\bar{E}_{0y}\ell}{R} \to I_0$



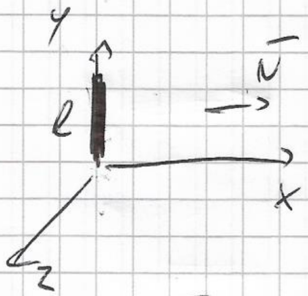
$$i_{ind}(max) = i_{ind}\left(\omega t = \frac{3\pi}{4}\right)$$

$$I_{max} = \sqrt{2} I_0 = 1.17\ mA$$

(21-3)

mezzo $K(\varepsilon_r) = 4$ ⑦

$\mu_r = 1$



$I = 1.19 \cdot 10^{-2}\ W/m^2$

$\ell = 20\ cm$

$$\bar{E} = E_0 \left[\cos(Kx - \omega t)\, \hat{e}_y + \sin(Kx - \omega t)\, \hat{e}_z \right]$$

$$\bar{B} = \frac{\bar{v} \times \bar{E}}{v^2} = \frac{E_0}{v} \left[-\sin(\dots)\, \hat{e}_y + \cos(\dots)\, \hat{e}_z \right]$$

$$\bar{S} = \sqrt{\frac{\varepsilon}{\mu}}\, E^2\, \hat{e}_x$$

$$I = \langle S \rangle = \sqrt{\frac{\varepsilon}{\mu}} \frac{1}{T} \int_0^T \bar{E} \cdot \bar{E}\, dt = \sqrt{\frac{\varepsilon}{\mu}} E_0^2 \underbrace{\frac{1}{T} \int_0^T (\cos^2 + \sin^2)\, dt}_{1}$$

$$I = \sqrt{\frac{\varepsilon}{\mu}} E_0^2 = \frac{1}{Z} E_0^2$$

$$E_0 = \left(I \sqrt{\mu/\varepsilon} \right)^{\frac{1}{2}} = 1.5\ V/m$$

$$\left[v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{2} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{c}{2} \right] B_0 = E_0 / v = 10^{-8}\ T$$

$$fem = \int \bar{E} \cdot d\bar{\ell} = \int_0^\ell E_y\, dy = \int_0^\ell E_0 \cos(\underset{x=0}{Kx} - \omega t)\, dy = E_0 \int_0^\ell \cos(\omega t)\, dy \Rightarrow$$

$$\Rightarrow fem = E_0\, \ell \cos(\omega t)$$

$\hookrightarrow 0.3\ V$

(21-4)

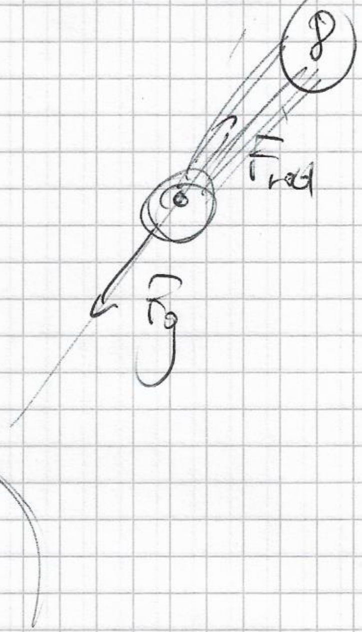
a_0

$P = 3.9 \cdot 10^{26}\ W$

$G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2$

$M_\odot = 1.99 \cdot 10^{30}\ kg$

$\rho = 3\ g/cm^3$



$p = \frac{S}{c}$ $\qquad S(r) = \frac{P}{4\pi r^2}$

$p = P/4\pi r^2 c$ $\qquad F_{rad} = pA = p\pi a_0^2$

$F_g = G\frac{M_\odot m}{r^2}$ $\qquad m = \frac{4\pi}{3} a_0^3 \rho$

$$F_{rad} = F_g$$

$$\frac{P}{4\pi r^2 c}\pi a_0^2 = G\frac{M}{r^2}\frac{4\pi}{3}a_0^3\rho$$

$a_0 = \frac{3P}{16\pi c\rho GM}$ $\qquad a_0 = 0.19\ \mu m$

$F_{rad} > F_g \iff a_0 < 0.19\ \mu m$

(21-5)

9

$$V'(\bar{r},t) = \phi \quad ; \quad \bar{A}'(\bar{r},t) = -\frac{qt}{4\pi\varepsilon_0 r^2}\hat{e}_r$$

$$\bar{E} = -\bar{\nabla}V' - \frac{\partial \bar{A}'}{\partial t} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{e}_r$$

$$\bar{B} = \bar{\nabla}\times\bar{A}' = \frac{qt}{4\pi\varepsilon_0}\bar{\nabla}\times(\hat{e}_r/r^2) = \phi$$

$$V(\bar{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad ; \quad \bar{A}(\bar{r},t) = \phi \qquad \lambda$$

$\lambda(\bar{r},t) \qquad (V,\bar{A}) \to (V',\bar{A}')$

$$V' = V - \frac{\partial\lambda}{\partial t} \quad ; \quad \bar{A}' = \bar{A} + \bar{\nabla}\lambda$$

$$\phi = \frac{q}{4\pi\varepsilon_0 r} - \frac{\partial\lambda}{\partial t} \sim \lambda(\bar{r},t) = \frac{qt}{4\pi\varepsilon_0 r} + f(\bar{r})$$

$$-\frac{qt}{4\pi\varepsilon_0 r^2}\hat{e}_r = \phi + \overbrace{\bar{\nabla}\frac{qt}{4\pi\varepsilon_0}}^{\bar{\nabla}\lambda} + \bar{\nabla}f(\bar{r}) = -\frac{qt}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{e}_r + \bar{\nabla}f$$

$$\phi = \bar{\nabla}f(\bar{r}) \Rightarrow f(\bar{r}) = \phi$$

$$\boxed{\lambda(\bar{r},t) = \frac{qt}{4\pi\varepsilon_0 r}}$$