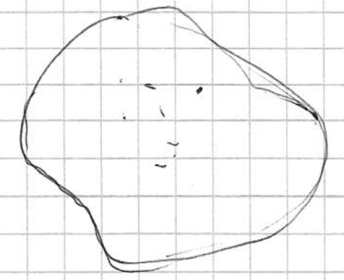
ES. #4 Legge di Gauss 21/X/2020

$$\Phi(\bar{E}_0) = \int_{S_{chiusa}} \bar{E}_0 \cdot d\bar{S} = \frac{1}{\varepsilon_0} Q_{int}$$



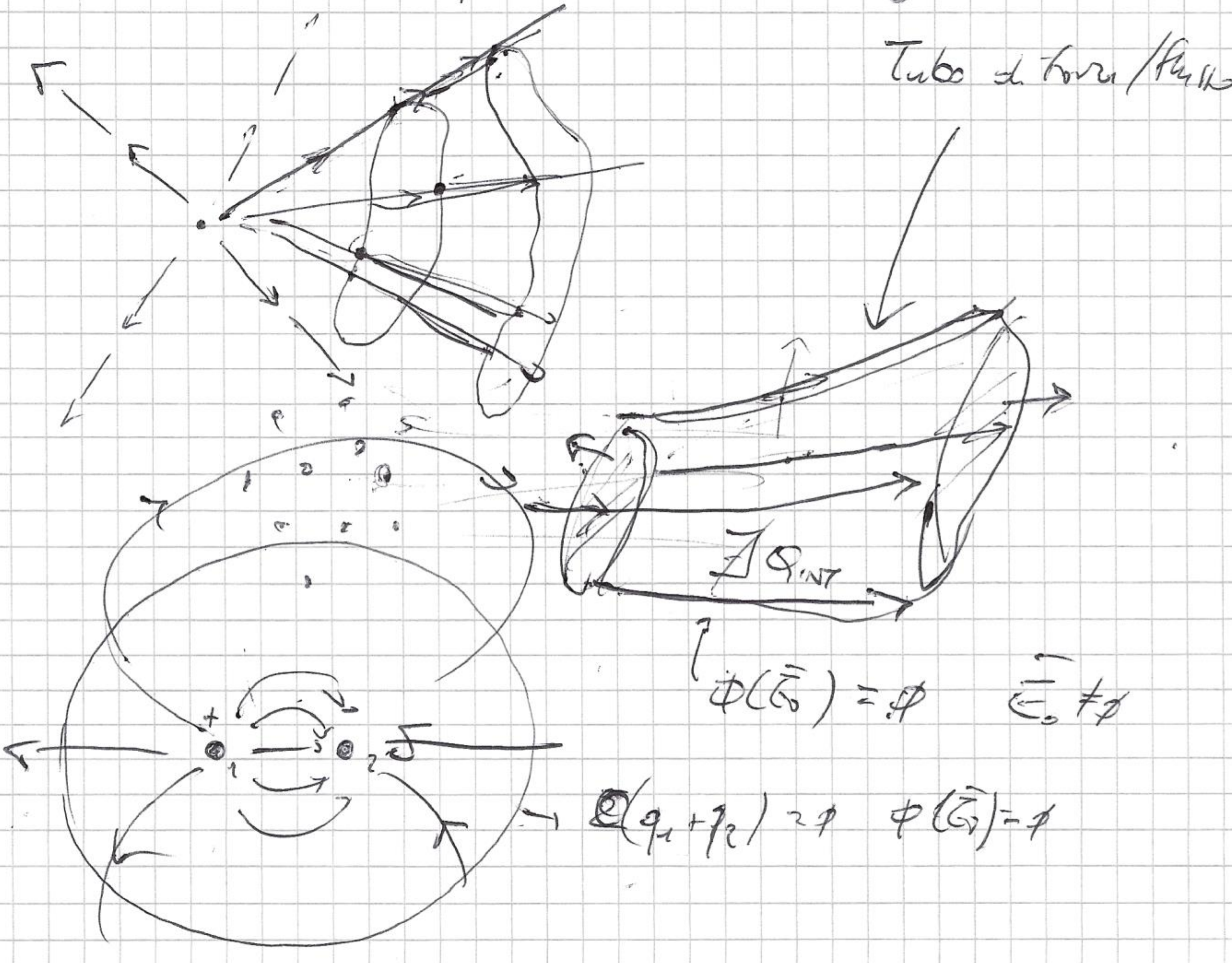
$$Q_{int} = \sum_i q_i + q_S + q_V$$

$$q_S \to \int_A \sigma(\bar{r})\, da \qquad q_V \to \int_V \rho(\bar{r})\, d\tau$$

$$\Phi(\bar{E}_0) = 0 \quad \cancel{\Rightarrow} \quad \bar{E}_0 = 0$$

Linea di forza (campo) : $\forall$ pt $\in$ linea, $\bar{g} \parallel \bar{E}_0$

Tubo di forza/flusso

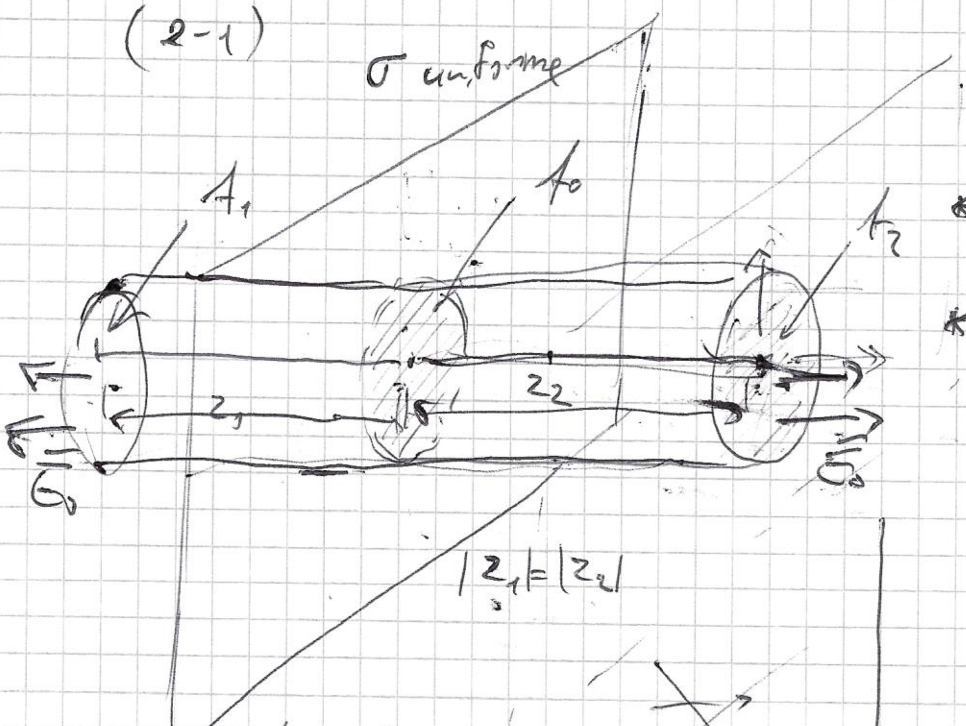


$\nexists Q_{INT}$

$\Phi(\bar{E}_0) = 0 \quad \bar{E}_0 \neq 0$

$Q(q_1+q_2) = 0 \quad \Phi(\bar{E}_0) = 0$

(2-1)



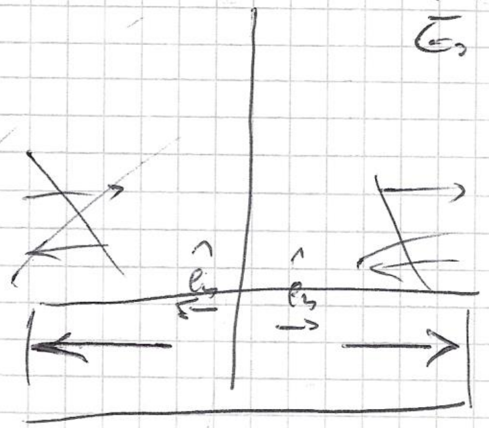
$A_0 = A_1 = A_2 = A$

* $\bar{E}_0 \; f(z)$

* $\bar{E}_0 = E_{0z}\,\hat{e}_z$

$E_{0n}\,\hat{e}_n$

$\bar{E}_0 = E_{0n}(d)\,\hat{e}_n$



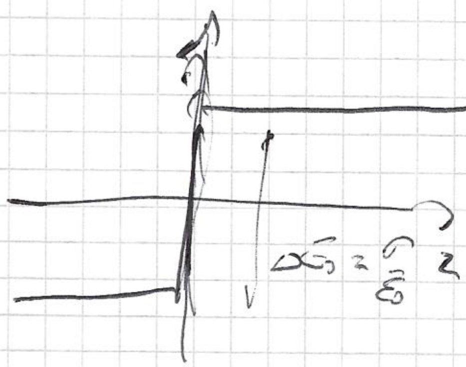
$$\Phi(\bar{E}_0) = \int_{S_{cyl}} \bar{E}\cdot d\bar{S} = \int_{A_1} \bar{E}_0(z_1)\cdot d\bar{S}_1 + \int_{A_2} \bar{E}_0(z_2)\cdot d\bar{S}_2 =$$

$$= E_0(z_1)A_1 + E_0(z_2)A_2 =$$

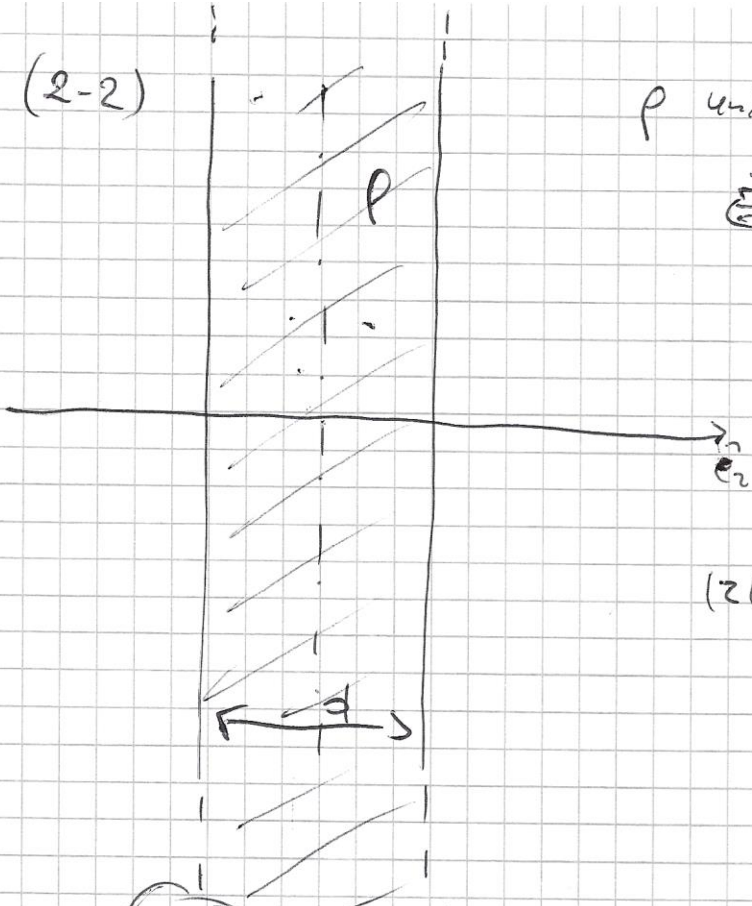
$d = |z_1| = |z_2|$ $\qquad = \boxed{2E_0(d)A}$

$$\Phi(\bar{E}_0) = \frac{1}{\varepsilon_0} Q_{INT} = \boxed{\frac{1}{\varepsilon_0}\sigma A}$$

$$\boxed{\bar{E}_0 = \frac{\sigma}{2\varepsilon_0}\,\hat{e}_n}$$



(2-2)

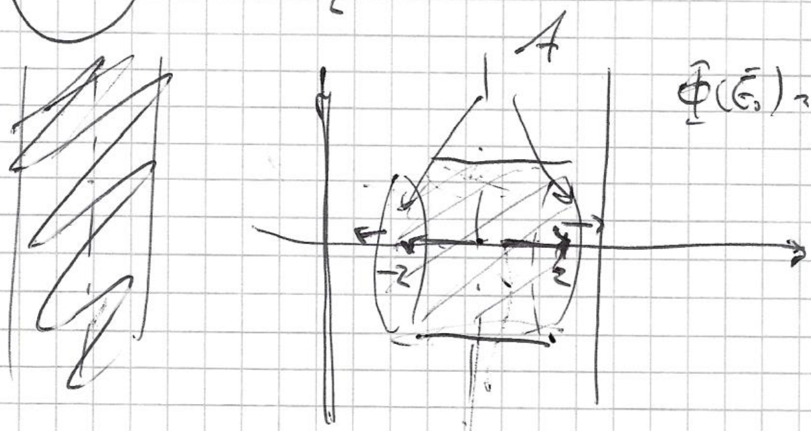


ρ uniforme

$\bar{E}_0 = ?$ $\qquad$ $E_0(d)\,\hat{e}_n$

$|z| < \frac{d}{2}$

IN $\quad |z| < \frac{d}{2}$

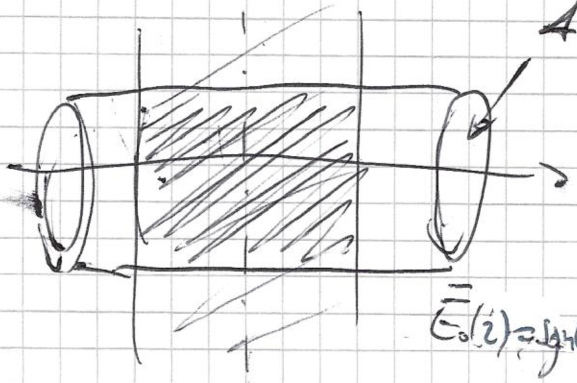


$$\Phi(\bar{E}_0) = \int_{\Sigma_{cyl}} \bar{E}_0 \cdot d\bar{\Sigma} = E_{0n}(|z|)\,2A = \frac{Q_{INT}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\rho A \cdot 2|z|$$

$$\boxed{\bar{E}_0(z) = \frac{\rho z}{\varepsilon_0}\hat{e}_z}$$

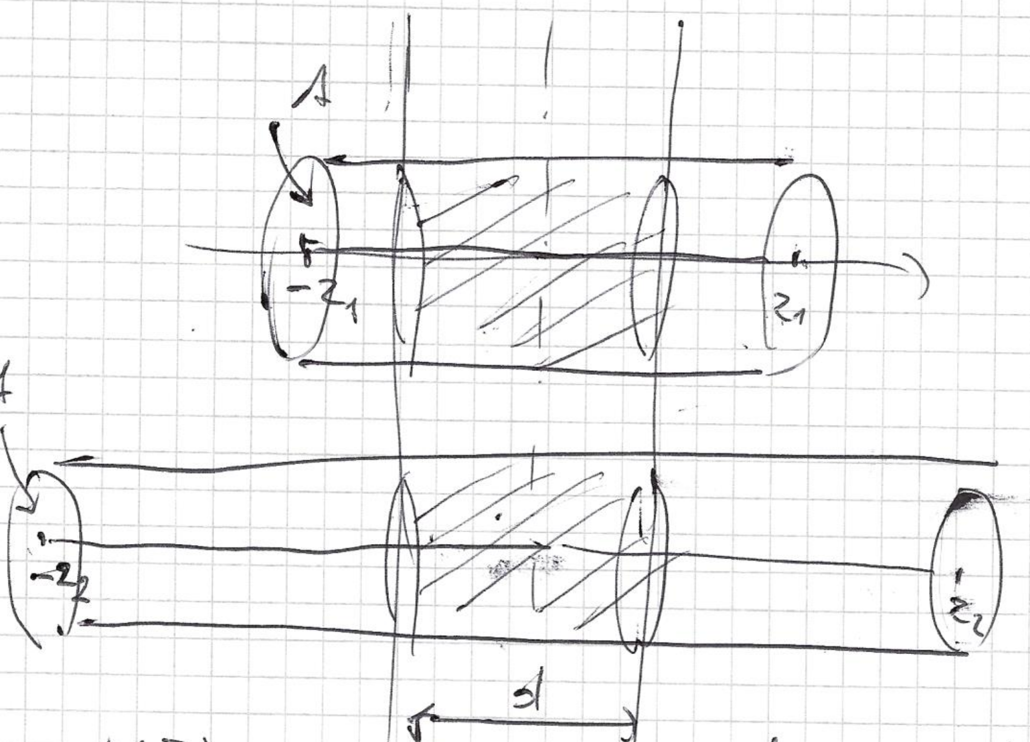
OUT

$|z| \geq \frac{d}{2}$



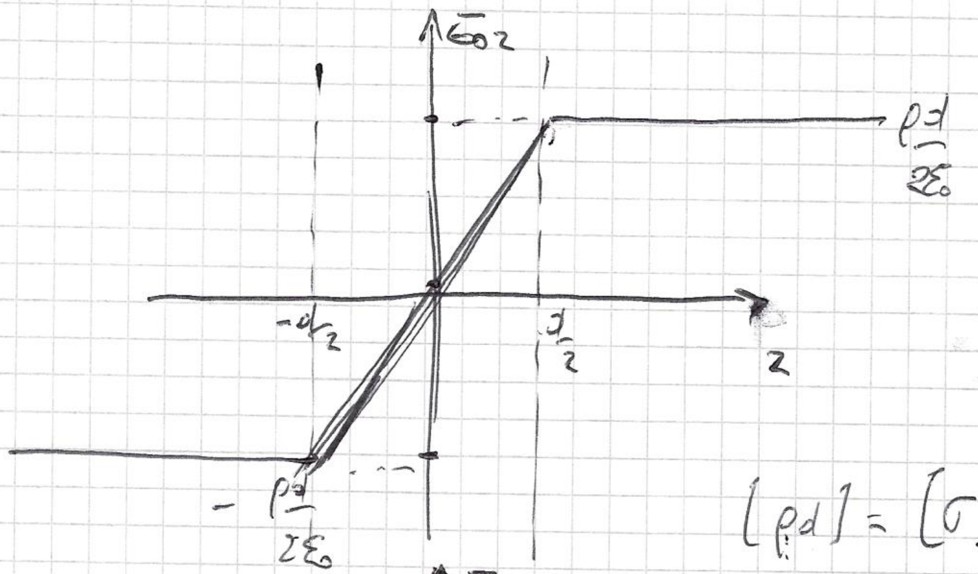
$$\Phi(\bar{E}_0) = E_0(|z|)\,2A = \frac{Q_{INT}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\underbrace{\rho d A}_{\text{vol cyl}}$$

$$\bar{E}_0(z) = \mathrm{sgn}(z)\frac{\rho d}{2\varepsilon_0}\hat{e}_z$$

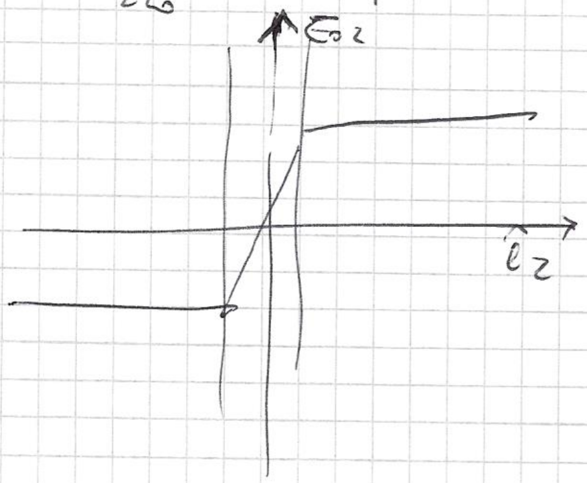


$$\Phi(\vec{E}_0) = 2 A E_0 (|z_i|) = \frac{1}{\varepsilon_0} Q_{INT} \; \underbrace{\rho A d}$$

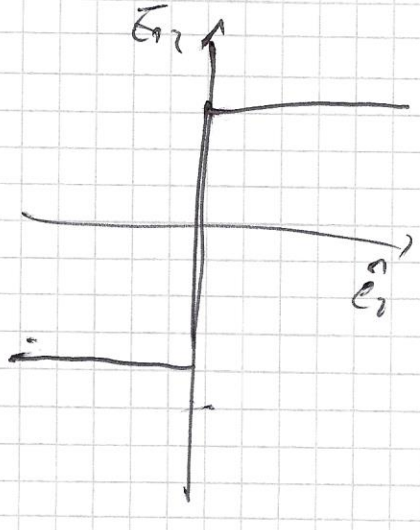
vol. cyl altezza d



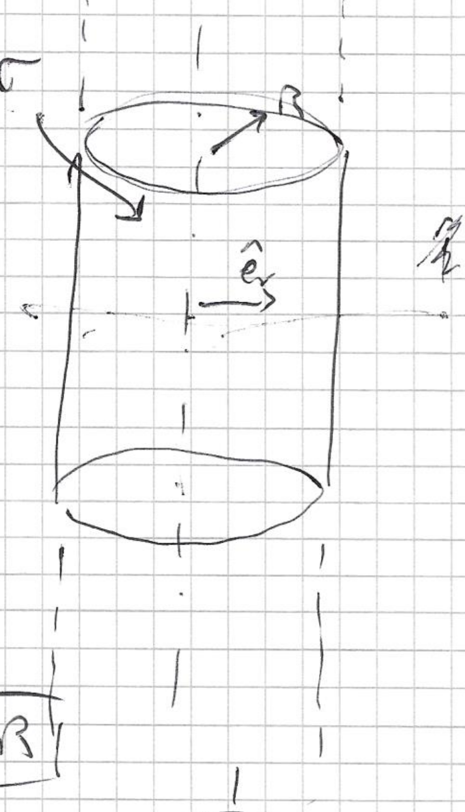
$$[\rho d] = [\sigma]$$



$\rightsquigarrow$



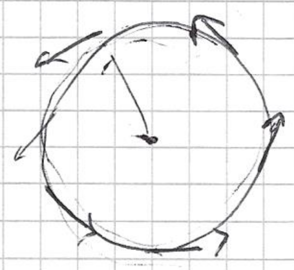
(2-3) σ uniforme vacuum



$(r, \cancel{\theta}, \cancel{z})$

$\bar{\bar{E}}_0(r)$

$E_{0r}(r), E_{0\theta}(r), E_{0z}(r)$



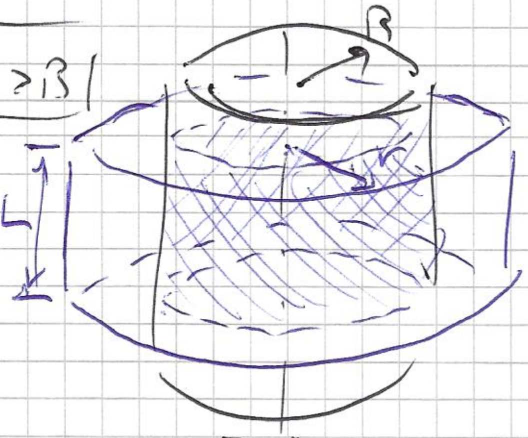
$\oint \bar{E}_0 \cdot d\bar{l} \neq \emptyset$

$\boxed{r<R}$

$$\Phi(\bar{E}_0) = \int \bar{E}_0 \cdot d\bar{S} = \boxed{E_{0r}(r)\,\underbrace{2\pi r L}_{S_{lat}}} = \frac{Q_{INT}}{\varepsilon_0} = \emptyset$$

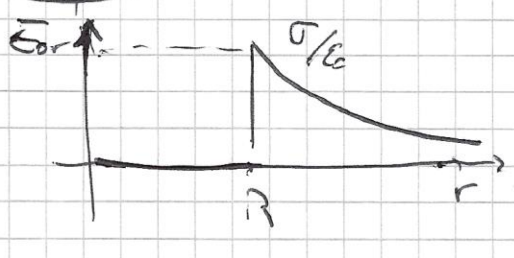
$$E_{0r}(r) = \emptyset$$

$\boxed{r>R}$



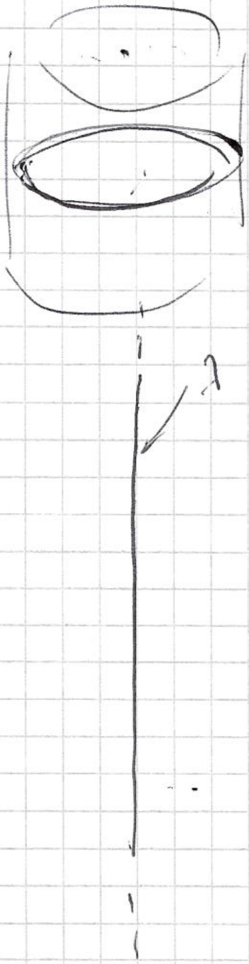
$$\Phi(\bar{E}_0) = E_{0r}(r)\, 2\pi r L = \frac{Q_{INT}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\sigma\, 2\pi R L$$

$$\bar{\bar{E}}_0(r) = \frac{R\sigma}{\varepsilon_0}\frac{1}{r}\hat{e}_r$$



$$\bar{E}_0(r) = \frac{R\sigma}{\varepsilon_0}\frac{1}{r}\hat{e}_r$$

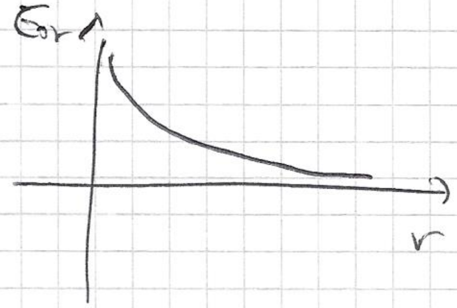
$$[R\sigma] = [\lambda]$$

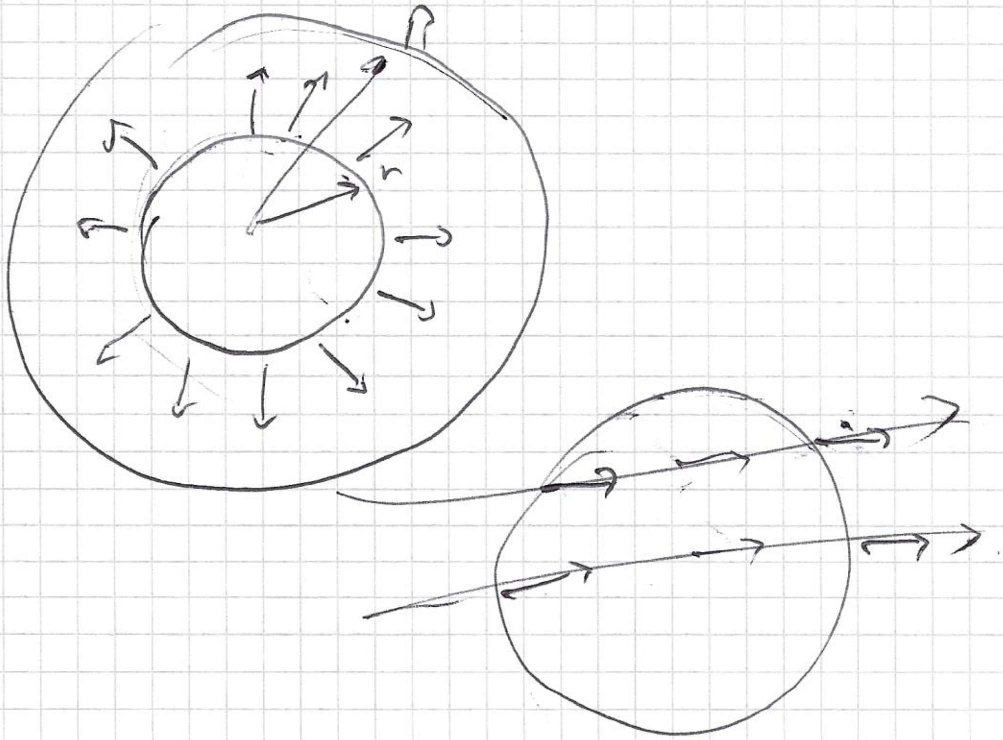


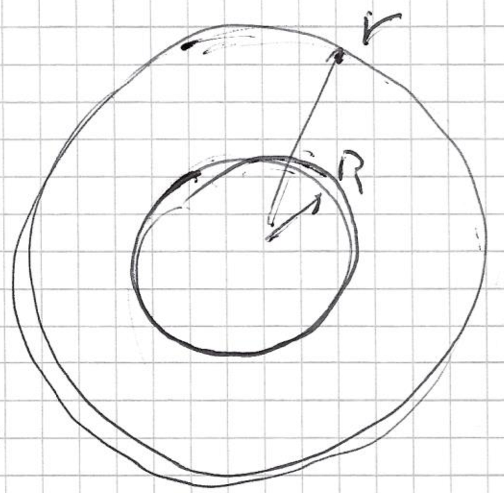
$$\lambda \doteq 2\pi R\sigma$$

$$\boxed{\bar{E}_0(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{e}_r}$$

campo di filo ∞mente esteso, rettilineo



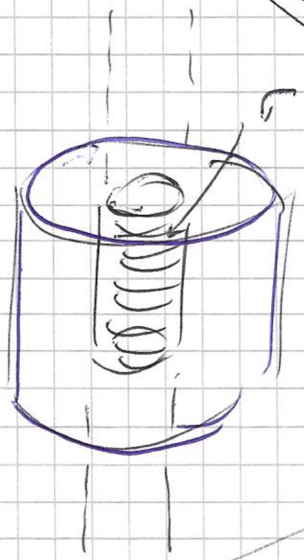




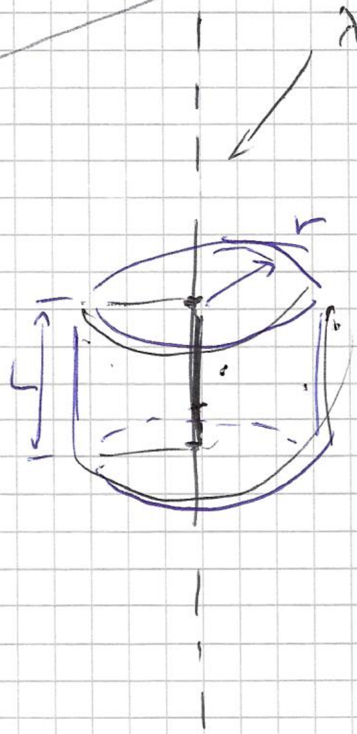
$$\int \vec{E}_0(r)\cdot d\vec{S} =$$

$$= E_{0r}(r)\, \cancel{2\pi} r \cancel{L}$$

$$= \frac{1}{\varepsilon}\, \sigma\, \cancel{2\pi} R \cancel{L}$$



$$E_{0r}(r) = \frac{\sigma R}{\varepsilon_0}\, \frac{1}{r}$$



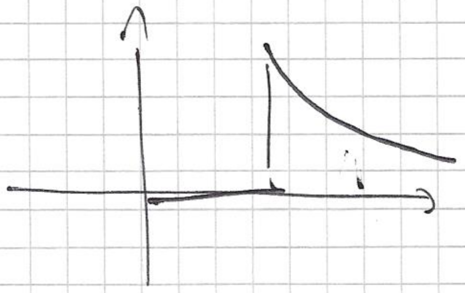
$$\vec{E}_{0r}(r)$$

$$\Phi(\vec{E}_0) = E_{0r}(r)\, 2\pi r \cancel{L} =$$

$$= \frac{1}{\varepsilon_0} Q_{INT}\ ?$$

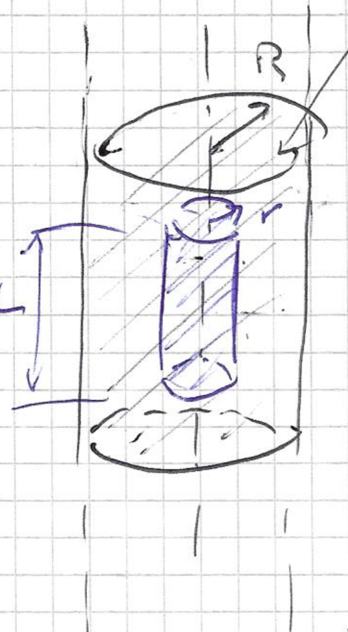
$$= \frac{1}{\varepsilon}\, \lambda \cancel{L}$$

$$E_{0r} = \frac{\lambda}{2\pi\varepsilon_0}\, \frac{1}{r}$$



$$\bar{E}_{0r} = \frac{R\sigma}{\varepsilon}\, \frac{1}{r}$$

(2-4) ~~λ~~ ~~σ~~ ρ uniforme , vacuum



$\bar{E}_0 = E_{0r}(r)\,\hat{e}_r$

$\boxed{r<R}$

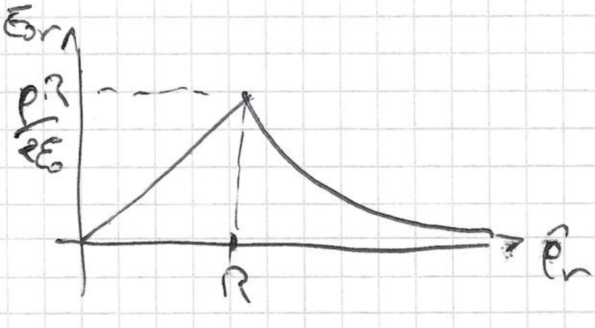
$$\Phi(\bar{E}_0) = \int_{S_{cyl}} \bar{E}_0 \cdot d\bar{S} = E_{0r}(r)\, 2\pi r L = \frac{Q_{INT}}{\varepsilon_0} = \rho\, \frac{\pi r^2 L}{\varepsilon_0}$$

$$\rightsquigarrow \bar{E}_0(r) = \frac{\rho r}{2\varepsilon_0}\,\hat{e}_r$$

$\boxed{r>R}$

$$\Phi(\bar{E}_0) = E_{0r}(r)\, 2\pi r L = \frac{1}{\varepsilon_0}\,\rho \pi R^2 L$$

$$\bar{E}_0(r) = \frac{\rho R^2}{2\varepsilon_0}\,\frac{1}{r}\,\hat{e}_r$$

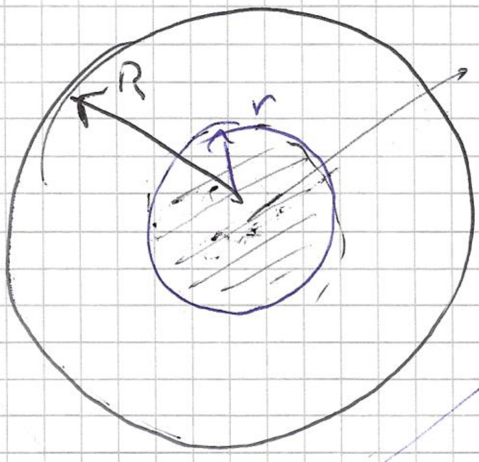


$$\lambda \doteq \rho \pi R^2$$

$$\rightsquigarrow \bar{E}_0(r) = \frac{1}{2\pi\varepsilon_0}\,\frac{\lambda}{r}\,\hat{e}_r$$

(2-5) ρ uniforme vacuum R

(r, θ, φ)



$\boxed{r<R}$ $\bar{E}_0 = E_0(r)\,\hat{e}_r$

$$\Phi(\bar{E}_0) = \int_{S_{sph}} \bar{E}_0 \cdot d\bar{S} = E_{0r}(r)\,4\pi r^2 = \frac{Q_r}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{4\pi}{3} r^3 \rho$$

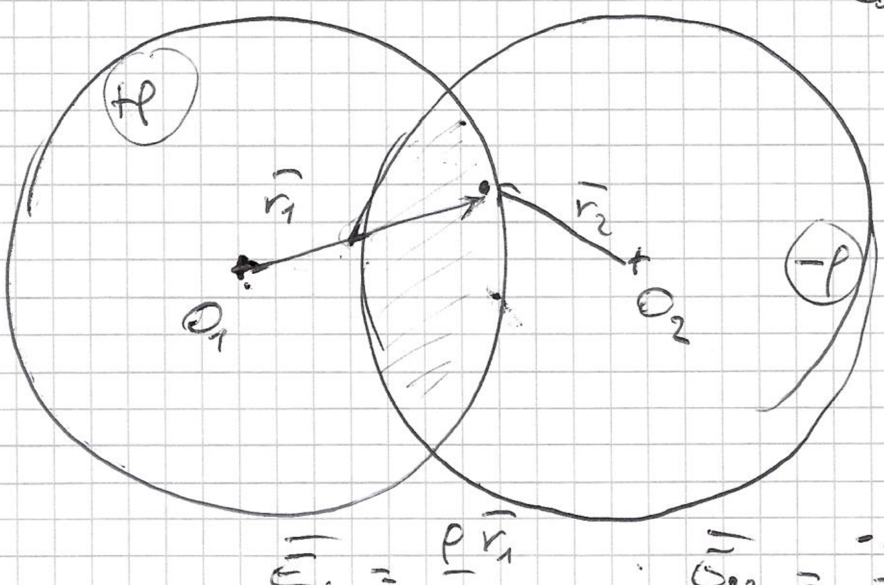
$$\boxed{\bar{E}_0(r) = \frac{\rho}{3\varepsilon_0} r\hat{e}_r}$$

$\boxed{r>R}$ $\Phi(\bar{E}_0) = E_{0r}(r)\,4\pi r^2 = \dfrac{Q_{tot}}{\varepsilon_0} = \dfrac{1}{\varepsilon_0}\underbrace{\dfrac{4\pi}{3}R^3\rho}_{\to Q_{tot}}$.

$$\bar{E}_0(r) = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{e}_r = \frac{1}{4\pi\varepsilon_0}\frac{Q_{tot}}{r^2}\hat{e}_r$$

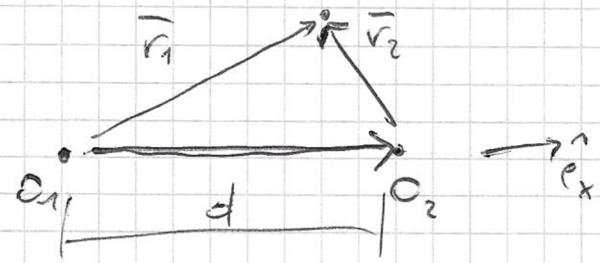
$+\rho$ $-\rho$

$\bar{E}_0 = \bar{E}_{01} + \bar{E}_{02}$

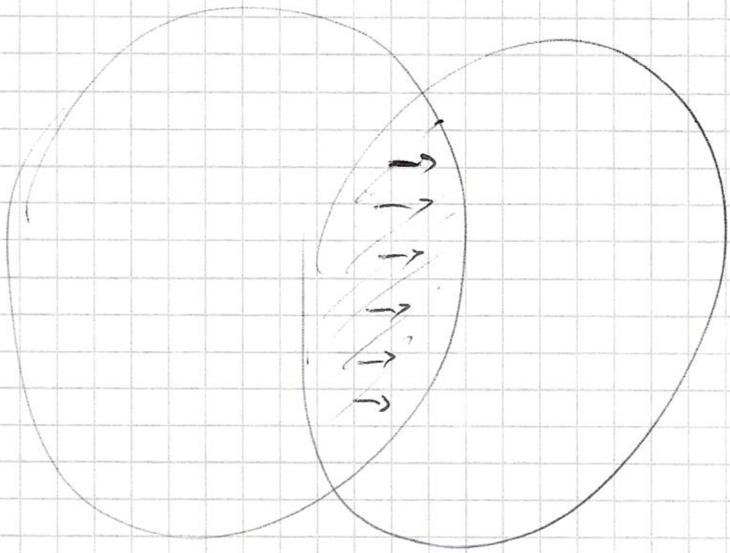


$$\bar{E}_{01} = \frac{\rho\,\bar{r}_1}{3\varepsilon_0} \quad ; \quad \bar{E}_{02} = \frac{-\rho\,\bar{r}_2}{3\varepsilon_0}$$

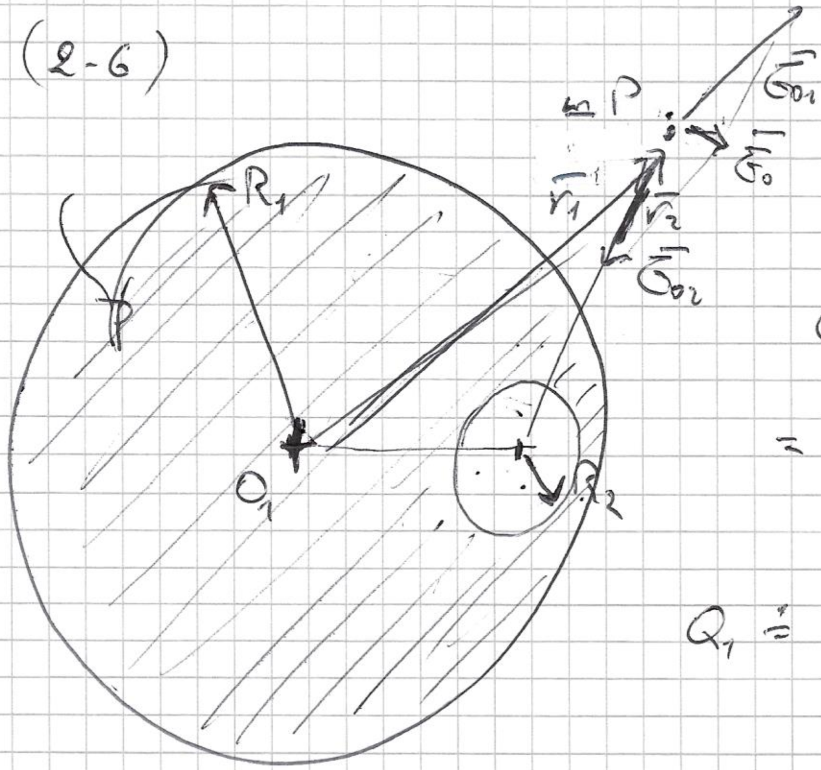
$$\overline{E}_0 = \overline{E}_{01} + \overline{E}_{02} = \frac{\rho}{3\varepsilon_0}\left(\overline{r}_1 - \overline{r}_2\right)$$



$$\overline{E}_0 = \frac{\rho d}{3\varepsilon_0}\hat{e}_x$$



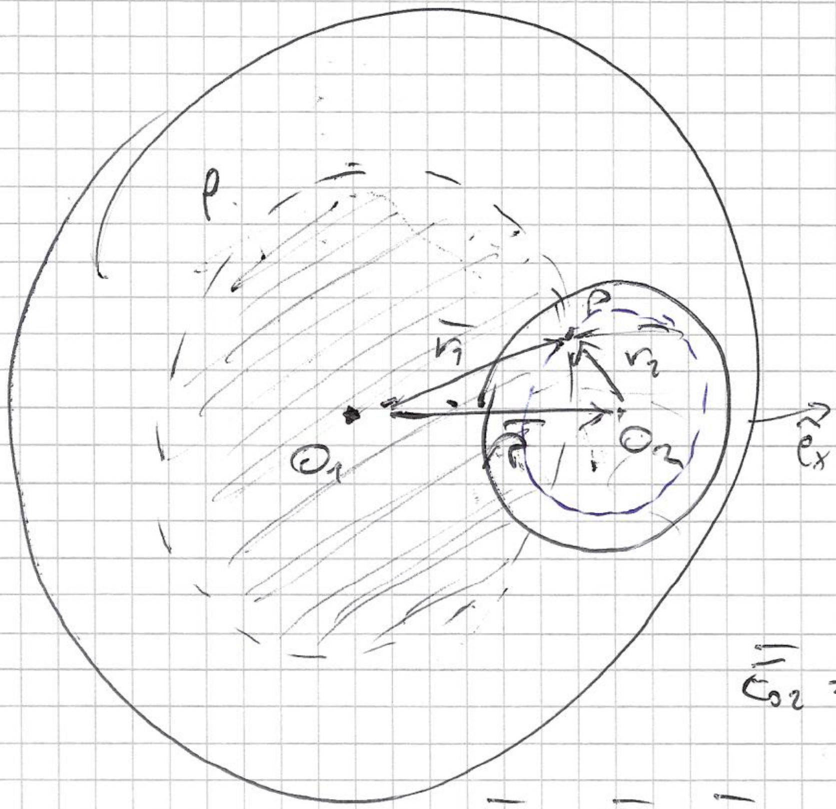
(2-6)



$$\vec{E}_0(P) = \vec{E}_{01}(P) + \vec{E}_{02}(P) =$$

$$= \frac{Q_1}{4\pi\varepsilon_0}\frac{\vec{r}_1}{r_1^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2}{r_2^3}$$

$$Q_1 = \frac{4\pi}{3}R_1^3\rho \,;\; Q_2 = -\frac{4\pi}{3}R_2^3\rho$$



$$\vec{E}_0(P) = \vec{E}_{01}(P) + \vec{E}_{02}(P)$$

Gauss su sfera r_1 per la ρ

$$\vec{E}_{01} = \frac{q_1}{4\pi\varepsilon_0}\frac{\vec{r}_1}{r_1^3}$$

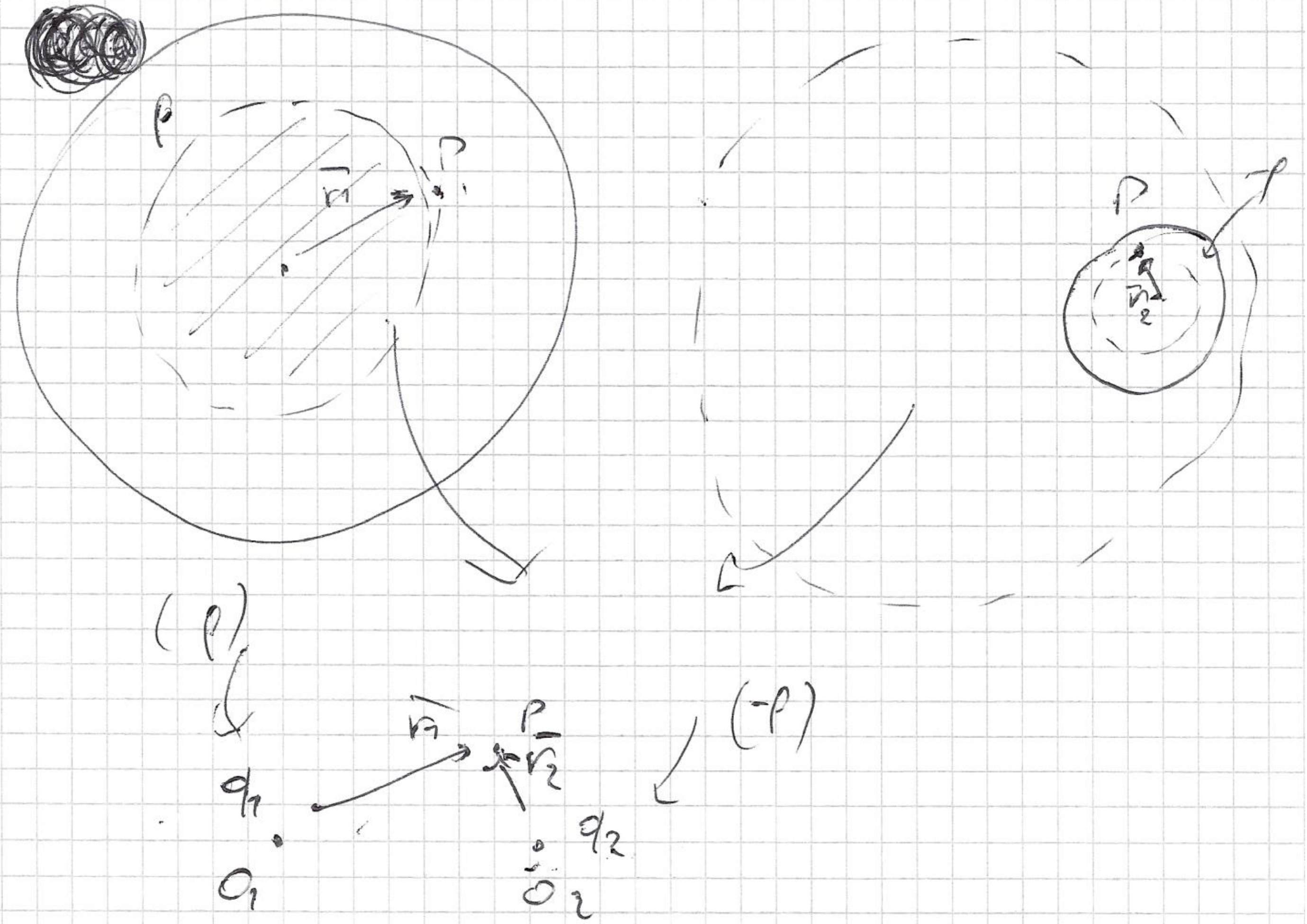
$$q_1 = \frac{4\pi}{3}r_1^3\rho$$

$$\vec{E}_{02} = \frac{q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2}{r_2^3} \qquad q_2 = -\frac{4\pi}{3}r_2^3\rho$$

$$\vec{E}_0 = \vec{E}_{01} + \vec{E}_{02} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi}{3}\rho\,(\vec{r}_1 - \vec{r}_2)$$

$$\vec{E}_0(P) = \frac{\rho}{3\varepsilon_0}(\vec{r}_1 - \vec{r}_2) = \frac{\rho}{3\varepsilon_0}\vec{R}$$

$$\vec{R} = d(O_1, O_2)\,\hat{e}_x$$

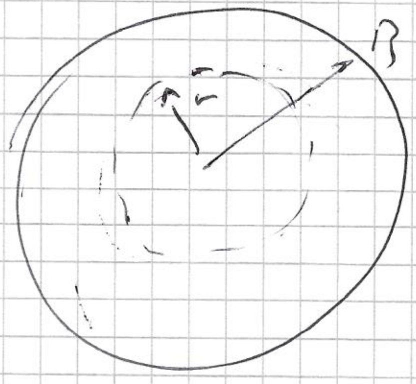


ρ
P
$\vec{r}_1$
P
−ρ
$\vec{r}_2$
(ρ)
(−ρ)
P
$\vec{r}_1$
$\vec{r}_2$
q_1
q_2
O_1
O_2

(2-7)

ρ sferica $\rho(r)$ raggio R

$r<R$ $\quad \bar{E}_0(r) =$ uniforme

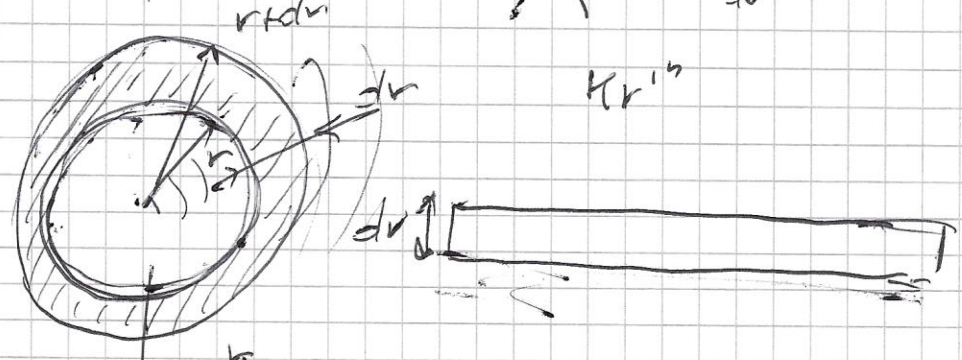


$\rho(r) = K r^n$

$$\Phi(\bar{E}_0) = \int_{sph} \bar{E}_0 \cdot d\bar{S} = \underline{E_{0r}(r)\, 4\pi r^2} =$$

$$= \frac{Q_{int}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_0^r \rho(r')\, \underbrace{4\pi r'^2 dr'}_{dV} =$$

$K r'^n$



$$= \frac{4\pi K}{\varepsilon_0}\int_0^r r'^{\,n+2} dr' = \frac{4\pi K}{\varepsilon_0}\frac{1}{n+3} r^{n+3}$$

$$\boxed{E_{0r}(r) = \frac{K}{\varepsilon_0(n+3)} r^{n+1} \quad \text{per } \rho(r) = K r^n}$$

$n+1 = 0 \longrightarrow r^{n+1} = 1$

$\Rightarrow \boxed{n = -1} \quad E_{0r}(r) = \dfrac{K}{2\varepsilon_0} \; , \; \rho = K/r$

$E_{0r} \propto r^2 \quad \rho(r) = K r^n$

$n+1 = 2 \quad \Rightarrow n = 1 \quad \rho(r) = K r$

$E_{0r} = \dfrac{K r^2}{4\varepsilon_0} \quad Q = \int_0^R \rho(r)\, 4\pi r^2 dr = 4\pi K \int_0^R r^3 dr = \pi K R^4$

$\hookrightarrow \frac{R^4}{4}$

$K = Q / \pi R^4$

$$\rho(r) = \frac{Q}{\pi R^4} \, r \, ;$$

$$E_{or}(r) = \frac{Q}{4\pi\varepsilon_0 R^4} \, r^2 \hat{e}_r$$

$$r \geq R \quad E_{or}(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} \hat{e}_r$$

"La misura del mondo"

Daniel Kehlmann

Gauss

A. Von Humboldt