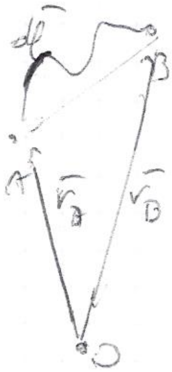
ES. #5 Potenziale elettrostatico, energia potenziale elettrostatica

28/X/2020

$q \quad \bar{E}_0 \qquad A \to B$



$$\int_A^B \bar{E}_0 \cdot d\bar{l} = \int_A^B \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \underbrace{\bar{r} \cdot d\bar{l}}_{r\,dr} = \frac{q}{4\pi\varepsilon_0} \int_A^B \frac{dr}{r^2} =$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right) \doteq V_0(A) - V_0(B)$$

In gen. $\quad \bar{E}_0 \cdot d\bar{l} = -dV_0$

$$df = \text{grad} f \cdot d\bar{l} = \vec{\nabla} f \cdot d\bar{l}$$

$$\text{grad} f = \frac{df}{dl} \hat{dl} \qquad \bar{E}_0 = -\vec{\nabla} V_0$$

$$V_0(P) - V_0(A) = -\int_A^P \bar{E}_0 \cdot d\bar{l}$$

$$V_0(P) = -\int_A^P \bar{E}_0 \cdot d\bar{l} + V_0(A)$$

$$V_0(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} + C \to V_0(\text{pt}A)$$

$$V_0(r \to \infty) = \emptyset$$

ok $\forall$ sorgente limitata nello spazio

$$V_0(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{|\bar{r} - \bar{r}_c|}$$

$\bar{E}_0$ sovrapposizione

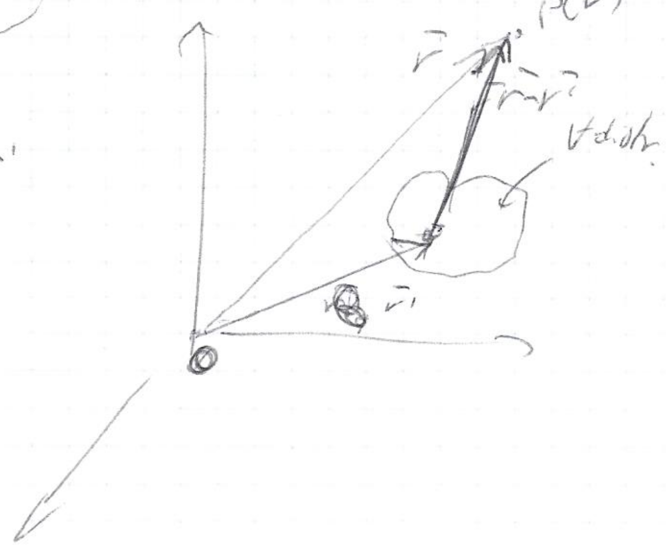
V_0 additività

$$V_0(\bar{r}) = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{Q_i}{|\bar{r}-\bar{r}_i|} \qquad Q_i\ (\bar{r}_i)$$

$$V_0(\bar{r}) = \frac{1}{4\pi\varepsilon} \int_l \frac{\lambda(\bar{r}')}{|\bar{r}-\bar{r}'|} d\bar{l}' \rightarrow dq(\bar{r}')$$

$$V_0(\bar{r}) = \frac{1}{4\pi\varepsilon} \int_S \frac{\sigma(\bar{r}')}{|\bar{r}-\bar{r}'|} dS'$$

$$V_0(\bar{r}) = \frac{1}{4\pi\varepsilon} \int_\tau \frac{\rho(\bar{r}')}{|\bar{r}-\bar{r}'|} d\tau'$$



(a, b, c $\rightarrow$ lab 3

Energia

$\bar{F}_c \qquad q \qquad d\bar{l} \qquad dW = \bar{F}_c \cdot d\bar{l}$

$A \rightarrow B$

$$\mathcal{L}_{AB} = W_{AB} = \int_A^B \bar{F}_c \cdot d\bar{l} = \int_A^B q\,\bar{E}_0 \cdot d\bar{l} = q\,(V_{0A} - V_{0B}) = -q\Delta V_0 =$$

$$= qV_{0A} - qV_{0B} = U_{0A} - U_{0B} = -\Delta U_0 \qquad \Delta f = f_{fin} - f_{iniz}$$

$U_0 \doteq qV_0$ energia elettrostatica (potenziale).

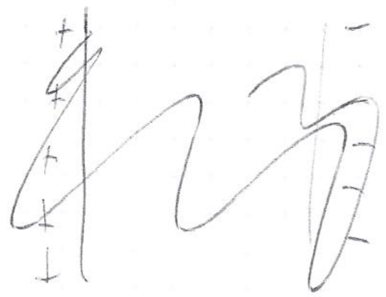
$$W_{AB} = -q\Delta V_{0\,A,B} = -\Delta U_{0\,A,B}$$

$$\Delta E_k = \underbrace{\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2} = W_{AB} = -\Delta U_0 = U_{0A} - U_{0B} =$$

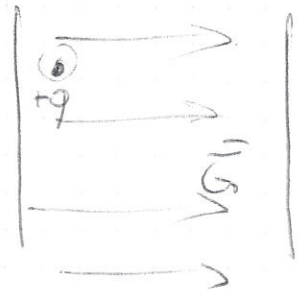
$$= \underline{q V_{0A} - q V_{0B}}$$

$$q V_{0A} + \frac{1}{2} m v_A^2 = q V_{0B} + \frac{1}{2} m v_B^2$$

$$\boxed{E_{tot} = E_k + U_0 = \frac{1}{2} m v^2 + q V_0 = \text{costante}}$$

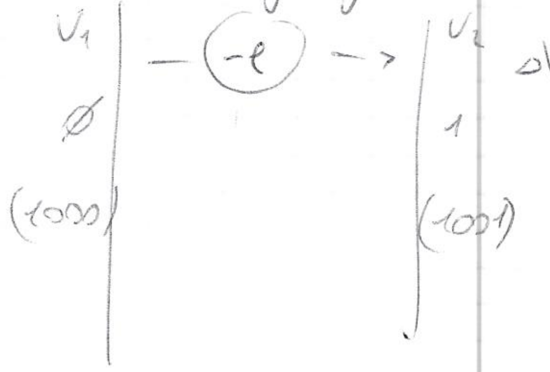


$V_1 > V_2$



eV elettronvolt

E_k guadagnato da un e^- $\Delta V = 1\ V$

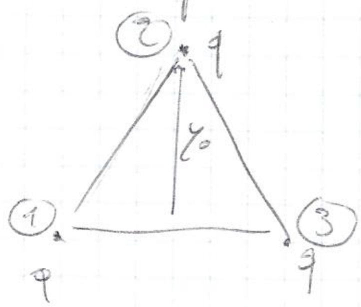


$\Delta V = V_2 - V_1$

$$\Delta U^{(e^-)} = -e \Delta V = -e \cdot 1\ V$$

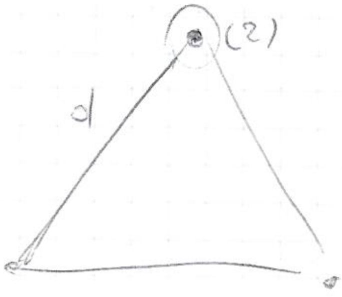
$$\Delta E_k = 1\ eV = 1.602 \ldots \cdot 10^{-19}\ J$$

(1-3) Repulsey



$$\mathcal{L} = \int \bar{F}_c \cdot d\bar{l} = \int_{y_0}^{+\infty} F_{cy}\, dy$$

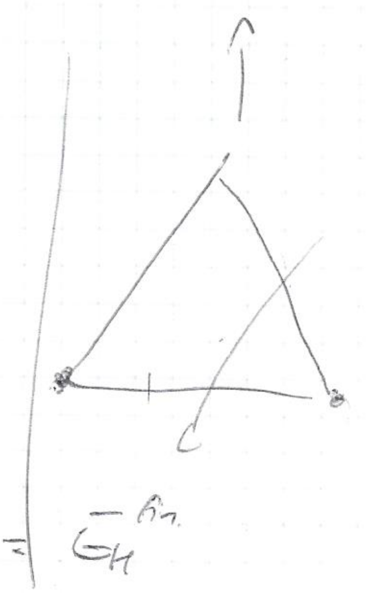
INIZ.



Ferma

$\bar{E}_k^{iniz} = \phi$

$\bar{E}_{tot} = U_0^{iniz} = qV_0$



$E_k^f = \frac{1}{2} m v_\infty^2$

$U_0^{fin} = \phi$

$$U_0^{iniz} = \bar{E}_k^{fin}$$

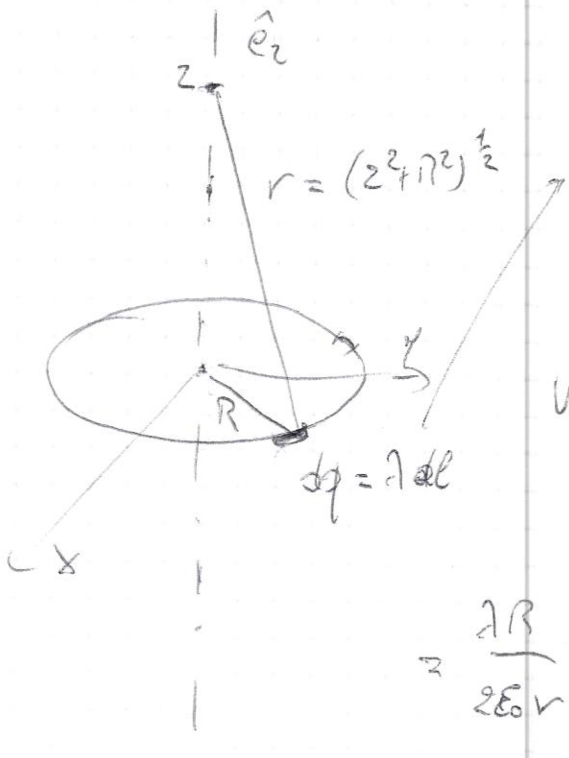
$$V_{021} = \frac{1}{4\pi\varepsilon_0} \frac{q}{d} \qquad V_{023} = \frac{1}{4\pi\varepsilon_0} \frac{q}{d}$$

$$V_{02} = V_{021} + V_{023} = \frac{1}{2\pi\varepsilon_0} \frac{q}{d} \qquad U_{02}^{iniz} = qV_{02} = \frac{1}{2\pi\varepsilon_0} \frac{q^2}{d}$$

$$U_{02}^{iniz} = 0.18 \text{ J} \rightarrow = \frac{1}{2} m v_\infty^2$$

$$v_\infty = 18.97 \text{ m/s}$$

(3-2) λ uniforme $\mathcal{C}(R)$ vacuum



$$dV_o(z) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{r}$$

$$V_o(z) = \int dV_o = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\underbrace{\int_{\mathcal{C}} dl}_{2\pi R} =$$

$$= \frac{\lambda R}{2\varepsilon_0 r} = \frac{\lambda}{2\varepsilon_0}\frac{R}{(z^2+R^2)^{\frac{1}{2}}} = \frac{q}{4\pi\varepsilon_0}\frac{1}{(z^2+R^2)^{\frac{1}{2}}}$$

$q = 2\pi R\lambda$

$z \to \infty$ $\quad z \gg R$

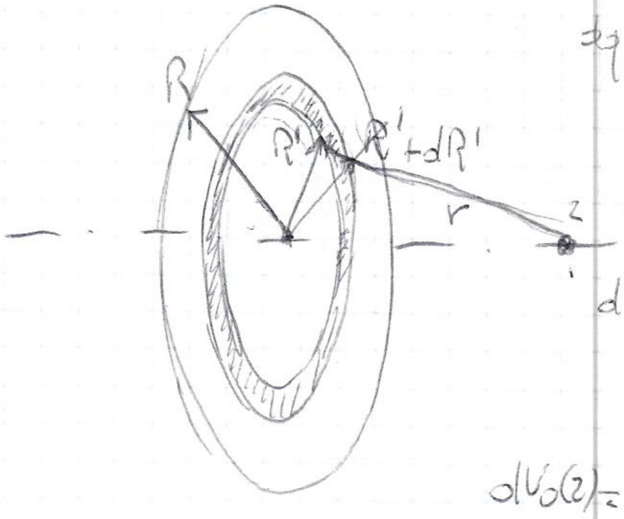
$$V_o(z) = \frac{q}{4\pi\varepsilon_0}\frac{1}{|z|}$$

$$\bar{E}_o = -\bar{\nabla} V_o \qquad E_{ox}, E_{oy}$$

$$E_{ox}\Big|_{\substack{x=0\\ y=0}} = -\frac{\partial V_o}{\partial x}\Big|_{\substack{x=0,\\ y=0}} = 0 \qquad E_{oy} = 0$$

$$E_z(z) = -\frac{\partial V_o}{\partial z}\Big|_{x,y=0} = \frac{q}{4\pi\varepsilon_0}\frac{z}{(z+R^2)^{3/2}}$$

(3-3) σ uniforme su $\mathcal{C}(R)$



$$dq = \sigma\, dS'$$

$$dS' = 2\pi R'dR'$$

$$dV_0(z) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi R'dR'}{(z^2+R'^2)^{1/2}}$$

$$dV_0(z) = \frac{\sigma}{2\varepsilon_0}\frac{R'dR'}{(z^2+R'^2)^{1/2}}$$

$$V_0(z) = \int_0^R \frac{\sigma}{2\varepsilon_0}\frac{R'dR'}{(z^2+R'^2)^{1/2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R'^2}\right]_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{z^2+R^2}-|z|\right]$$

$\forall z$

grande dist.: $z \to +\infty$

$$\sqrt{z^2+R^2} = z\sqrt{1+(R/z)^2} = z\cdot\sqrt{1+y^2} = z\cdot\underbrace{\left(1+\frac{1}{2}y^2\right)}_{\text{Taylor II ordine}}$$

$y \doteq R/z$ se $z\to\infty \Rightarrow y\to 0$

$$\sqrt{z^2+R^2} = z\cdot\left(1+\frac{1}{2}\underbrace{\frac{R^2}{z^2}}_{y^2}\right) = z + R^2/2z$$

Per $z\to\infty$

$$V_0(z) = \frac{\sigma}{2\varepsilon_0}\left(\cancel{z}+\frac{R^2}{2z}-\cancel{z}\right) = \frac{\sigma R^2}{2\varepsilon_0}\frac{1}{2z} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z}$$

$$q \doteq \sigma\pi R^2$$

$$E_{0z} = -\frac{\partial V_0}{\partial z}\bigg|_{x,y=0} = \text{sign}(z)\frac{\sigma}{2\varepsilon_0}\left[1-\frac{|z|}{\sqrt{z^2+R^2}}\right]\hat{e}_z$$

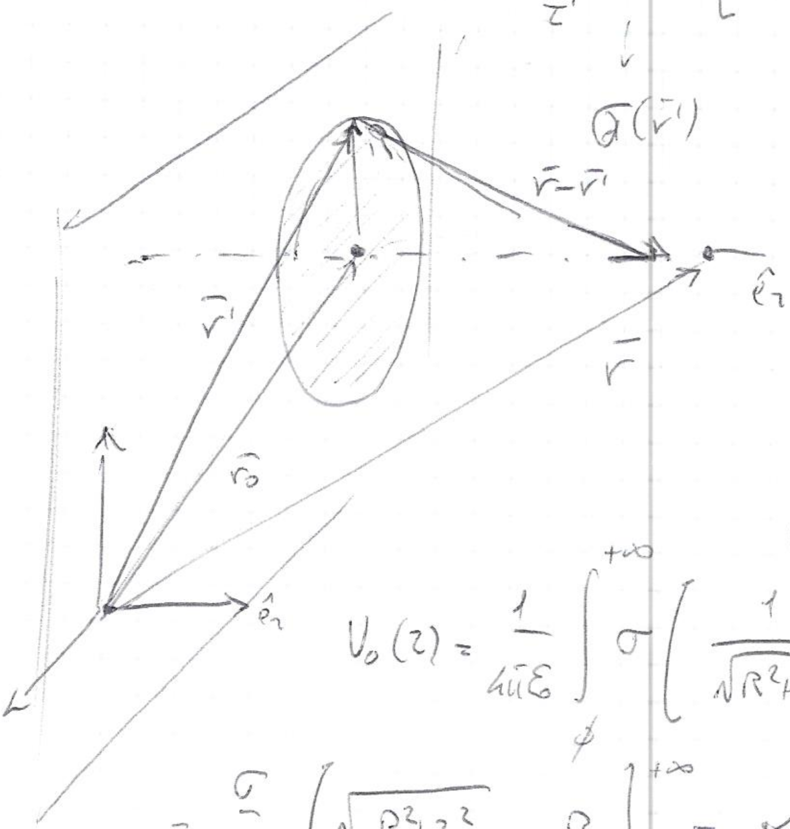
~~(3-4)~~ (3-4) σ uniforme (piano)

$V_0(\bar r) \longrightarrow V_0(\bar r) - V_0(\bar r_0)$

$$V_0(\bar r) = \frac{1}{4\pi\varepsilon_0}\int_{\tau'} \rho(\bar r')\cdot\left[\frac{1}{|\bar r-\bar r'|} - \frac{1}{|\bar r_0-\bar r'|}\right]d\tau'$$

CORRETTO!

$\rho \to \sigma(\bar r')$, $d\tau' \to dS'$



$V_0(\bar r_0) = \emptyset$

$\bar r_0$ centro distr.

$|\bar r - \bar r'| = \sqrt{R^2+z^2}$

$|\bar r_0 - \bar r'| = R$

$$V_0(z) = \frac{1}{4\pi\varepsilon_0}\int_{\emptyset}^{+\infty}\sigma\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{R}\right)2\pi R\,dR = $$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - R\right]_{\emptyset}^{+\infty} = *$$

$$\sqrt{R^2+z^2} = R\sqrt{1+(x/R)^2} \underset{\uparrow}{=} R\sqrt{1+y^2} = R\left(1+\frac{1}{2}\frac{z^2}{R^2}\right) = R + \frac{z^2}{2R}$$

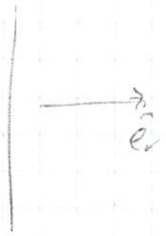
$y \doteq x/R$

$$* = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - R\right)_{R\to+\infty} - \frac{\sigma|z|}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}\left(\cancel{R} + \cancel{\frac{z^2}{2R}} - \cancel{R}\right]_{R\to+\infty} - \frac{\sigma|z|}{2\varepsilon_0} \Rightarrow$$

$$V_0(z) = -\frac{\sigma}{2\varepsilon_0}|z| \qquad \forall z$$

$$\bar E_0 = -\frac{\partial V_0}{\partial z} = \mathrm{sign}(z)\frac{\sigma}{2\varepsilon_0}\hat e_z$$

$$\bar{E}_0(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{e}_r$$

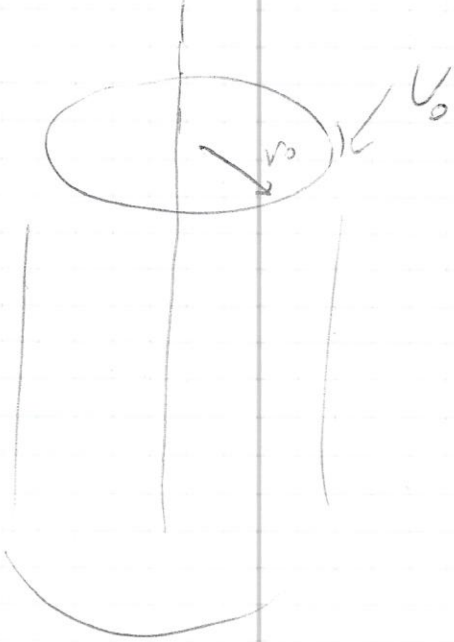


$$V_0(r) = \int \bar{E}_0 \cdot d\bar{r} + C =$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\log r + C$$

$V_0(r_0)$

$$V_0(r) - V_0(r_0) = \int_r^{r_0} \vec{E}_0(\vec{r}')\,dr' = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r}{r_0}\right)$$



(3-5) ρ uniforme, vacuum sph (R)

(a) $V_0(r)$ $\forall r \in \mathbb{R}^+$

$\boxed{r>R}$ $$\bar{E}^{(1)}_{0r}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \underline{\underline{\frac{1}{3\varepsilon_0}\frac{\rho R^3}{r^2}}}$$

$$Q = \frac{4\pi}{3}R^3\rho$$

$\boxed{r<R}$ $$\bar{E}^{(2)}_{0r}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q^{(r)}}{r^2} = \underline{\underline{\frac{1}{3\varepsilon_0}\rho r}}$$

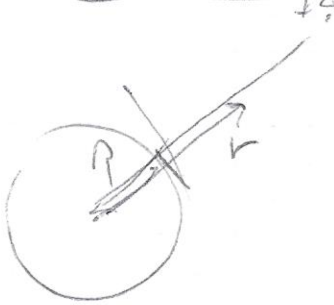
$$q^{(r)} = \frac{4\pi}{3}r^3\rho$$

$$V_0(r) - V_0(r_0) = \int_r^{r_0}\bar{E}\cdot d\bar{\ell}$$ decidere r_0, $V_0(r_0)$

$r_0 = +\infty$

$V_0(r_0) = V_0(\infty) = 0$

$$\boxed{V_0(r) - \underbrace{\cancel{V_0(\infty)}}_{0} = \int_r^{+\infty}\bar{E}_{0r}(r)dr}$$



$\boxed{r>R}$ $$V_0(r) = \int_r^{+\infty}\overbrace{\frac{\rho R^3}{3\varepsilon_0}\frac{1}{r'^2}}^{\bar{E}_0^{(1)}(r')}dr' = \frac{\rho R^3}{3\varepsilon_0}\left[-\frac{1}{r'}\right]_r^{+\infty} = \underline{\underline{\frac{\rho R^3}{3\varepsilon_0}\frac{1}{r}}}$$

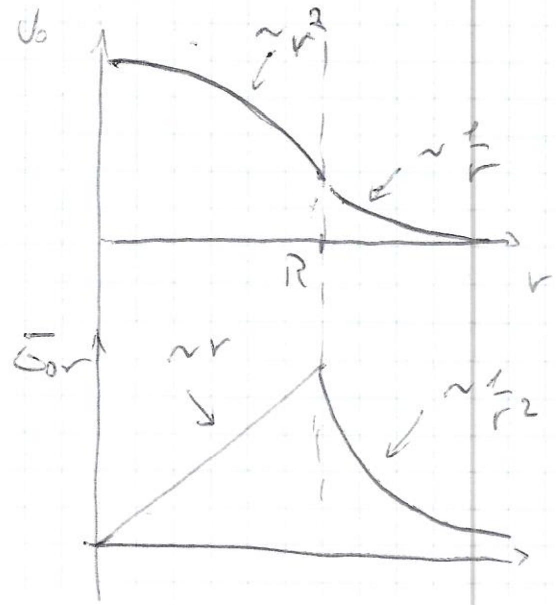
$$V_0(R) = \rho R^2/3\varepsilon_0$$

$\boxed{r<R}$

$$V_0(r) - V_0(\infty) = \int_r^R \bar{E}^{(2)}_{0r}(r')dr' + \underbrace{\int_R^{+\infty}\bar{E}^{(1)}_{0r}(r')dr'}_{\to V_0(R)} =$$

$$= \int_r^R \frac{\rho}{3\varepsilon_0}r'dr' + V_0(R) = \frac{\rho}{3\varepsilon_0}\left[r'^2\right]_r^R + V_0(R) = \frac{\rho}{6\varepsilon_0}\left[R^2 - r^2\right] + V_0(R)$$

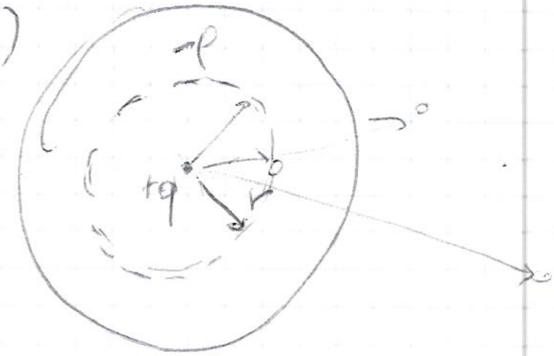
$$V_0(r) = \frac{\rho}{2\varepsilon_0}\left[R^2 - \frac{r^2}{3}\right] \qquad \boxed{r<R}$$



$$V_0(r) = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r} \qquad \boxed{r<R}$$

$$E_{0r} = -\frac{\partial V_0}{\partial r}$$

(b)



Pos. equilibrio per carica test

$$+q \neq \frac{4\pi}{3}\rho R^3$$

attr. sol. Coulomb : $\exists$ equ. $\forall r > R$

$$Q = \frac{4\pi}{3}R^3\rho > q$$

$r<R$

$$E_{0r}^{pt}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \; ; \; E_{0r}^{\rho} = -\frac{1}{4\pi\varepsilon_0}\frac{Q'(R)}{r^2} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$$

$$Q'(R) = \frac{4\pi}{3}r^3\rho = Q\frac{r^3}{R^3}$$

$$E_{0r}^{pt} + E_{0r}^{\rho} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r^2} - \frac{Qr}{R^3}\right] = 0$$

$$\hookrightarrow r = R(q/Q)^{1/3}$$

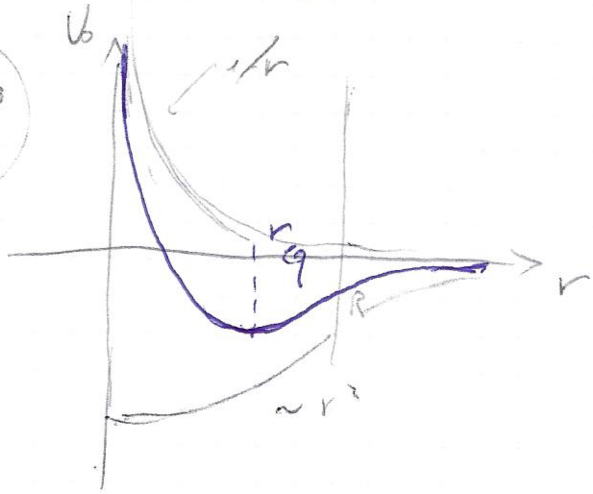
Equi.: pt stazionario di U_0 potenziale

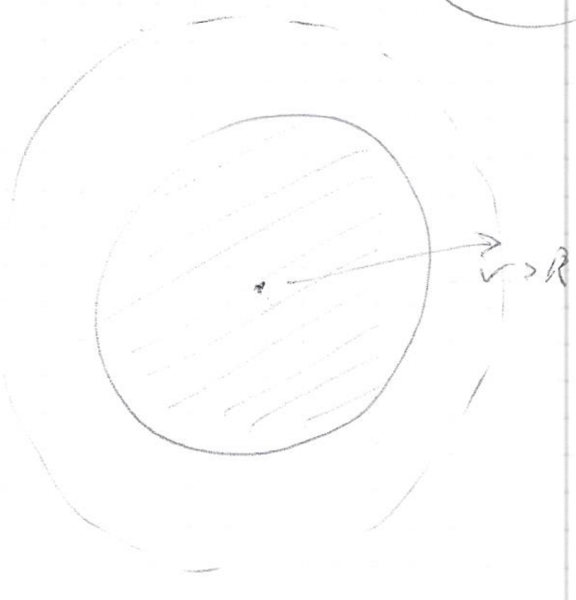
$$U_0(r) = \underbrace{-\frac{\rho}{2\varepsilon_0}\left[R^2 - \frac{r^2}{3}\right]}_{U_0^{\rho}(r)} + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q}{r}}_{U_0^{pr}(r)}$$

$$\frac{\partial U_0}{\partial r} = \phi \quad \sim \quad E_{0r} = \phi$$

$$\frac{\partial U_0}{\partial r} = \frac{\rho r}{3\varepsilon_0} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \longrightarrow \quad r_{eq} = R\left(q/Q\right)^{\frac{1}{3}}$$

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$





$q + Q \neq \phi$

$q + Q = \phi$