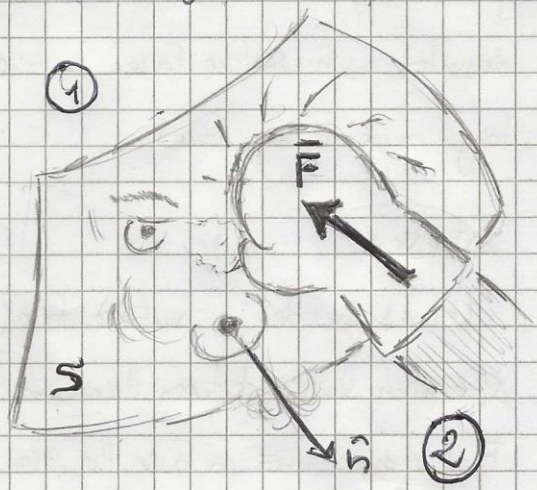
Stresses and stress tensor

Forces acting on a continuous medium

Let us consider a surface $\mathcal{S}$ of infinitesimal diameter (linear size) that separates two parts (1 and 2) of a continuum, or two different, but adjacent continua. The surface $\mathcal{S}$ has area $d\mathcal{A}$ and normal unit vector $\hat{n}$ (being infinitesimal, $\mathcal{S}$ is flat and $\hat{n}$ can be located on any of its points). What kind of forces are exchanged between 1 and 2?

Let us consider forces exerted by 2 on 1; then our convention will be taking $\hat{n}$ oriented from 1 towards 2 (see figure →).



When ① receives a well-placed punch (force $\bar{F}$) from ②, ① spits out, with a feeble wail, the unit normal vector $\hat{n}$ through the ①-② interface $\mathcal{S}$.

What kind of forces shall we take into account?

Volume forces: These can be exchanged between elements not in contact with each other, and they are proportional to the volume of the region subjected to the force.

An obvious example is the gravitational force $\bar{F}_g$; indeed $\bar{F}_g$ ∝ mass = ρV. When considering two elements of the continuum, volume interactions are negligible, since they are weak with respect to other forces at play. We should rather consider the gravitational force exerted on a continuum element by external sources, like the Earth or objects with huge mass, or by the continuum itself when it extends over enormous scales, e.g. an astrophysical continuum (i.e. a self-gravitating object like a star, etc.). Other volume forces could arise in the case of electrically-charged fluids (where volume density distribution of charges are present).

Surface (or contact) forces: These are only exchanged between adjacent, contacting parts and are proportional to the area of the contact surface between the two contiguous elements. This is due to the nature of these forces, that is intermolecular forces like those in solids and liquids (e.g., Van der Waals force between liquid molecules) or collisions

(i.e. the typical interaction between molecules in a gas). The former act on the range of intermolecular distances, that is approximately at the nanometer scale; the latter, for gases in normal conditions can go one or two orders up in range, which is still negligible with respect to the scales of interest of our investigation, if we want to keep using the continuum approximation (= macroscopic system comprising a very high number of microscopic constituent elements or particles). Therefore the interaction between two regions of the continuum separated by an ideal surface is taking place over a thickness close to zero, thus well described as a surface force. In brief, all these forces are close-range forces, as opposed to long-range volume forces.

Stresses

Let us consider a surface force $\bar{F}$ exerted by region 2 onto region 1, where S is the separation surface between them and dA is the infinitesimal area of S. By definition, $\bar{F}$ as surface area is over the area dA we can reckon an infinitesimal force $d\bar{F}$, or better, $d\bar{F}(\bar{x}, \hat{n}, dA)$, i.e. a surface force applied at position $\bar{x} \in dA$ ($\forall \bar{x} \in dA$ is fine, since dA is infinitesimal) of the surface S with local normal unit vector $\hat{n}$ oriented from 1 towards 2. As a surface force it reads

$$d\bar{F}(\bar{x}, \hat{n}, dA) = \bar{S}(\bar{x}, \hat{n})\, dA$$

and we will call $\bar{S}(\bar{x}, n)$ the STRESS exerted by region 2 on region 1 in $\bar{x}$ across a surface with normal unit vector $\hat{n}$. Equivalently, we can define

$$\bar{S}(\bar{x}, \hat{n}) = \underbrace{\frac{d\bar{F}(\bar{x}, \hat{n}, dA)}{dA}}_{\downarrow} = \lim_{A \to 0} \frac{\bar{F}(\hat{n}, A)}{A} \quad \text{with } \bar{x} \text{ always} \in A \ (A \text{ finite and plain}),$$

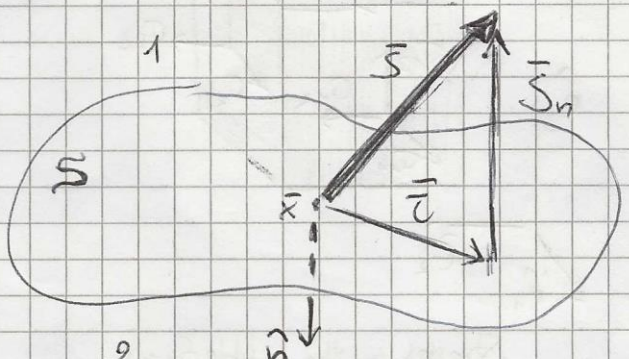
this writing does not express a derivative, but the limit of the average stress $\langle \bar{S}(\bar{x}, \hat{n}) \rangle = \bar{F}(\bar{x}, \hat{n})/A$ obtaining by bringing A to an infinitesimal area around $\bar{x}$.

In terms of dimensions, $[S] = [F][L^{-2}] = [M][L^{-1}][T^{-2}]$, just like pressure p. Pressure is indeed a stress, i.e. a force per unit surface (but it is not the only one). Its units of measure are Newton/m^2 = Pa pascal; for atmosphere measurements, bar = 10^5 Pa is often used, or atmosphere atm = 101325 Pa $\simeq$ 1 bar (= p at sea level).

Normal and shear stresses - Stress tensor

The stress on a surface S is a vector that can take any direction with respect to the surface's normal vector $\hat{n}$. We call

$$\bar{S}_n = \left(\bar{S}(\bar{x},\hat{n})\cdot\hat{n}\right)\hat{n}$$ NORMAL STRESS



and what is left of $\bar{S} - \bar{S}_n$ obviously lies in the plane containing the surface S (= projection of $\bar{S}$ onto S):

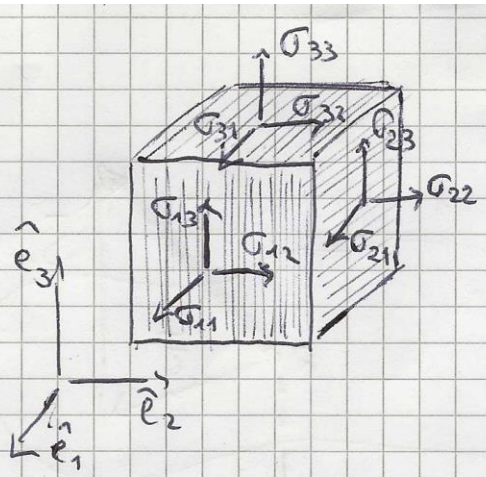
$$\bar{\tau} = \bar{S}(\bar{x},\hat{n}) - \bar{S}_n \quad \left[\text{also: } \bar{\tau} = \hat{n}\times(\bar{S}\times\hat{n}) \text{ indeed } = (\hat{n}\cdot\hat{n})\bar{S} - (\hat{n}\cdot\bar{S})\hat{n} = \bar{S} - \bar{S}_n\right]$$

SHEAR STRESS

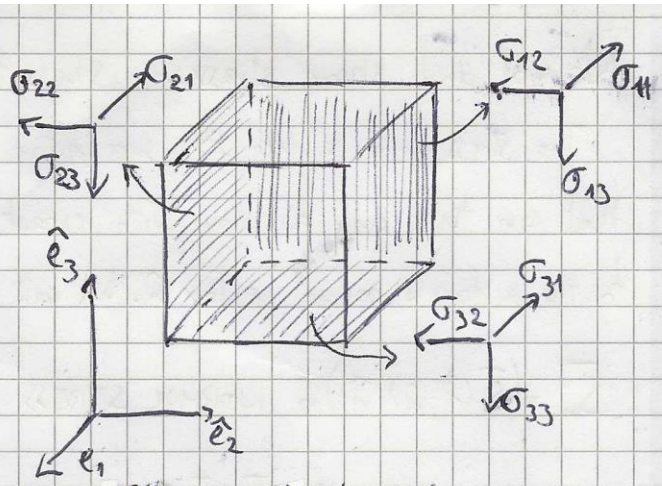
Quite intuitively we identify the normal stress as the PRESSURE.

Note that using the third law of motion,
if the stress from 2 to 1 (and $\hat{n}$ points from 1 to 2) is $\bar{S}(\bar{x},\hat{n})$,
$\Rightarrow$ 1 exerts a stress $-\bar{S}(\bar{x},\hat{n})$ on 2.
By its definition, the stress from 1 to 2 is $\bar{S}(\bar{x},-\hat{n}) \Rightarrow \underline{\bar{S}(\bar{x},-\hat{n}) = -\bar{S}(\bar{x},\hat{n})}$ (*).

On a surface in a 3-dimensional space we can decompose the stress vector into three components (one normal and two shear components); what do we need then in order to have stresses in a continuum completely identified? Let us consider an infinitesimal cubic volume of the continuum so that we can observe what is going on around the position $\bar{x}$; the cube's faces are oriented in such a way that their normal vectors are parallel to a Cartesian cs $\{\hat{e}_1,\hat{e}_2,\hat{e}_3\}$. On each face of the cube, let us say with unit normal vector $\hat{e}_i$, there is a stress vector with 3 components in the 3 $j=1,2,3$ direction; we shall then call such components σ_{ij} (j-th component of stress on i-th face). For instance, on the face with normal $i=1$ ($\parallel\hat{e}_1$) we shall have σ_{11} (normal stress) and σ_{12}, σ_{13} (shear stresses). As the cubic volume is infinitesimal around $\bar{x}$, the equality (*) $\bar{S}(\bar{x},-\hat{n}) = -\bar{S}(\bar{x},\hat{n})$ requires stresses on opposite faces to be equal in magnitude and in opposing orientations (see figure on the next page). Thus only 3 components $\times$ 3 faces = 9 elements σ_{ij} are required to fully describe the stress behaviour around the position $\bar{x}$.



Stresses on the visible faces



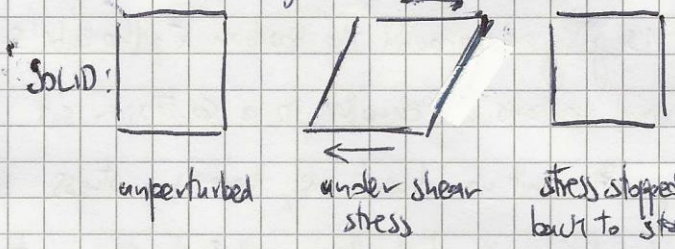
Stresses on the back faces

9 σ_{ij} elements with $i,j = 1,2,3$ calls for a 3×3 matrix, or better, we shall show that this is a 2nd-order tensor $\underline{\underline{\sigma}}$ called STRESS TENSOR (once again: σ_{ij} is a component where the first index defines the orientation of the surface where the stress is applied, and the second one defines the orientation of such stress component).

Do we always need as much as 9 components, or do situations exist where σ_{ij} takes a simpler form? Let us recall the definition of fluid:

> A fluid is a continuous medium that cannot stay in equilibrium when subjected to a shear stress.

In this respect, a fluid is fundamentally different from an elastic solid: The latter will undergo a deformation when a shear stress is applied, but will go back to (or at least towards) the original state as the stress is halted (see figure below).



In a fluid, even if the shear stress is infinitesimal, fluid particles acquire a velocity and the former equilibrium state can no longer be attained.

So while for a solid as an effect of a stress tensor σ_{ij} we can define a deformation tensor (or better, a strain rate tensor describing the velocity of deformation), for a fluid we better define a velocity gradient tensor Π_{ij} instead. Conclusion: for a fluid in equilibrium (FLUID STATICS) only normal stresses σ_{ii} are acceptable and $\Rightarrow \underline{\underline{\sigma}}$ must be diagonal. In the following we shall demostrate that then $\underline{\underline{\sigma}}$ is not only diagonal but also ISOTROPIC (σ_{ii} are all equal $\forall i$), which is equivalent to say that pressure is not a

vector but a <u>scalar</u> and takes the same value in any direction of observation at a position $\bar{x}$ within the fluid.

An important note: σ_{ij} is isotropic in yet another case, i.e. for an IDEAL FLUID (even in its dynamics), whose definition is '<u>a fluid where all phaenomena are reversible</u>'. Reversibility requires indeed

* Absence of shear stresses, since they cause friction;
* Absence of heat transport;

both phenomena would cause irreversibility and are thus forbidden in an ideal fluid. Therefore, since there are no shear stresses, in an ideal fluid the stress tensor $\underline{\underline{\sigma}}$ is always diagonal (and isotropic, we shall see) both in the static and dynamic state.

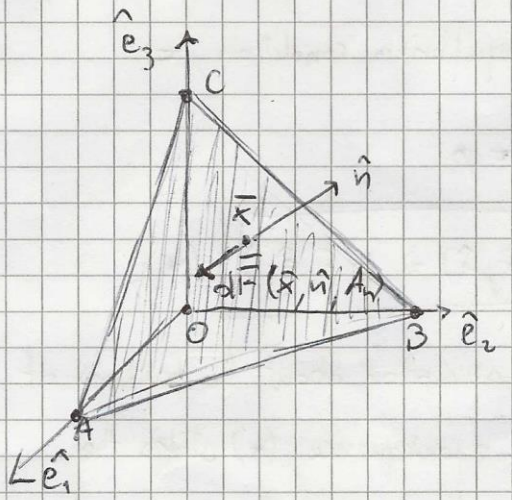
The tensor $\underline{\underline{\sigma}}$ takes on a general form that can be written as

$$\sigma_{ij} = \underbrace{\sigma'_{ij}}_{\text{viscous or deviatoric term}} \underbrace{- p\delta_{ij}}_{\text{pressure term (acting against the outwardly-pointing normal vector, since fluids can only exert compression, but no tensile forces)}}$$

Fluid statics (or hydrostatics)

Isotropy of pressure in a fluid at equilibrium (or in an ideal fluid)

Let us consider a fluid at rest, and thus subjected to normal stresses only. We shall take an element of such fluid of infinitesimal diameter ℓ and tetrahedral shape. Three of the tetrahedron faces shall have normal vectors along the cs axes, while the fourth one shall have normal unit vector $\hat{n}$ and include the point $\bar{x}$, as shown in the figure.



In brief we can summarize

Surface	Area	Normal unit vector
S_1 (BCO)	A_1	$-\hat{e}_1$
S_2 (ACO)	A_2	$-\hat{e}_2$
S_3 (ABO)	A_3	$-\hat{e}_3$
S_n (ABC)	A_n	$\hat{n}$

Let us identify the forces acting on this fluid element and then require that the sum (resultant) is zero (first condition for static equilibrium).

There may be volume forces, expressed as $\bar{h}(x)V$ (where $\bar{h}(\bar{x})$ is a force per unit volume), such as gravity; and then there will be surface forces, i.e. the pressure on the tetrahedron's faces. The application of forces on different positions in the element (the various forces for pressure stresses; the center of mass for the resultant of the volume forces), and therefore with displacements of order $O(\ell)$, results in errors $\sim O(\ell)$; when multiplied by surface force, i.e. a finite value (p) times a surface $\sim O(\ell^2)$, the overall error is $\sim O(\ell^3)$ for surface forces (let us consider the center of mass as reference position). Therefore the first equilibrium equation reads

$$\underbrace{-p(\bar{x},\hat{n})A_n\hat{n} + \sum_{i=1}^{3} p(\bar{x},\hat{e}_i)A_i\hat{e}_i}_{\sim O(\ell^2)\text{ terms}} + \underbrace{\bar{h}(\bar{x})V + O(\ell^3)}_{\sim O(\ell^3)\text{ terms}} = \phi$$

Geometry tells us that $A_i = A_n \hat{n}\cdot\hat{e}_i$, =>

$$\left[-p(\bar{x},\hat{n})\hat{n} + \sum_{i=1}^{3} p(\bar{x},\hat{e}_i)(\hat{n}\cdot\hat{e}_i)\hat{e}_i\right] A_n + \bar{h}(\bar{x})V + O(\ell^3) = \phi$$

Since we have $O(\ell^2)$ and $O(\ell^3)$ terms in the sum, requiring the total to be zero actually imposes separate cancellation for terms of different order:

$$\bar{h}(\bar{x})V + O(\ell^3) = \phi$$

$$-p(\bar{x})\hat{n}\hat{n} + \sum_{i=1}^{3} p(\bar{x},\hat{e}_i)(\hat{n}\cdot\hat{e}_i)\hat{e}_i = \phi$$

Now let us rewrite $\hat{n}$ as vector sum of its projections on the coordinate axes, $\hat{n} = \sum_{i=1}^{3}(\hat{n}\cdot\hat{e}_i)\hat{e}_i$ and plug it into the equilibrium condition, $\Rightarrow$

$$\sum_{i=1}^{3}\left[-p(\bar{x},\hat{n}) + p(\bar{x},\hat{e}_i)\right](\hat{n}\cdot\hat{e}_i)\hat{e}_i = \phi$$

that is, for each component $i = 1, 2, 3$ $\quad \boxed{p(\bar{x},\hat{n}) = p(\bar{x},\hat{e}_i)}$ (*)

where $\hat{n}$ is arbitrary, since we made no specific assumption about the positions of A, B, C on the axes determining the tetrahedron. As a consequence, (*) states the ISOTROPY OF PRESSURE in a fluid at rest; the result also holds for an ideal fluid in its dynamics, since by definition the ideal fluid is exempt from shear stresses.

This condition translates into a simplified forms of the stress tensor σ_{ij}:

$$\sigma_{ij} = -p\delta_{ij} = \begin{pmatrix} -p & \phi & \phi \\ \phi & -p & \phi \\ \phi & \phi & -p \end{pmatrix} = -p\,\underline{\underline{\mathbb{1}}}$$

Surface tension

(τ) SURFACE TENSION between two substances or phases (one of which can be vacuum) is defined as the work per unit area performed to increase the contact area between the two phases isothermally and reversibly.

In terms of dimensions, $[W] = [\tau][A]$

$$[F][L] = [\tau][L^2] \Rightarrow [\tau] = [F]/[L]$$ force per unit length

and it is also worth recalling that, by definition of $F = U - TS$ Helmholtz free energy and the first law of thermodynamics, the maximum work one can obtain from a system in contact with an infinite thermal reservoir is the opposite of the variation of F ($dW^{max} = -dF$), therefore surface tension is the free energy per unit surface contact: $dF = \tau\, dA$, and the equilibrium state of the isothermal system is the one that minimizes F (if other forces → potentials occur, e.g. gravity, equilibrium will be = minimum of all those).

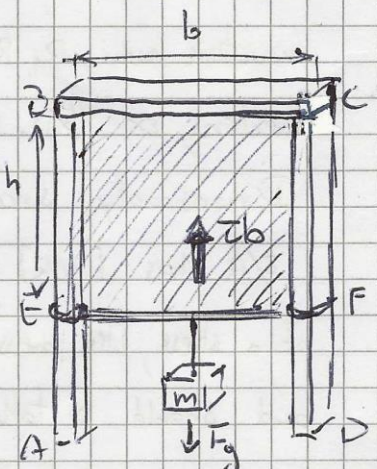
So in practical terms surface tension tells us that: if we draw a line on an interface, there will be a force per unit length τ acting in a direction normal to the line and parallel to the interface, which tries to minimize the surface area (⇒ the Helmholtz free energy).*

The origin of surface tension lies in the intermolecular cohesive forces. For a particle inside the continuum, these will be isotropic, while when the particle is closer to the interface than the range of cohesive forces, there will be a net action due to asymmetry.

Let us convince ourselves of some properties of surface tension in a more heuristic fashion. We shall start from a "home experiment" involving a flat interface.

⊙ Flat interface

Consider a rectangular frame, placed vertically, whose lower side can slide freely and has negligible mass and friction (EF). Let us immerse it in soapy water and extract it; the surface BCFE will be covered by a thin fluid film (notice that we actually have two interfaces: air - soapy water - air); we



* = well, for liquid-liquid and liquid-solid interfaces, depending on their nature it may also be the opposite; we can always say that the surface area is "extremized" at equilibrium.

can observe that the movable side $\overline{EF}$ is attracted upwards, and that is because surface tension tries to make the surface minimum ($\Rightarrow$ hence here the surface tends to be flat as it minimizes the area enclosed in $B(FE)$. To stop $\overline{EF}$ we must place a weight so that $F_f = F_g = mg$, and we observe experimentally that the upward force F_f is proportional to $b = \overline{EF}$ length of the moving frame side, so we can write

$$F_f = 2\tau b$$ where the factor 2 takes into account the two interfaces

$\Rightarrow$ tension is defined as $\tau = F_f/2b$; indeed, this force descends from a work

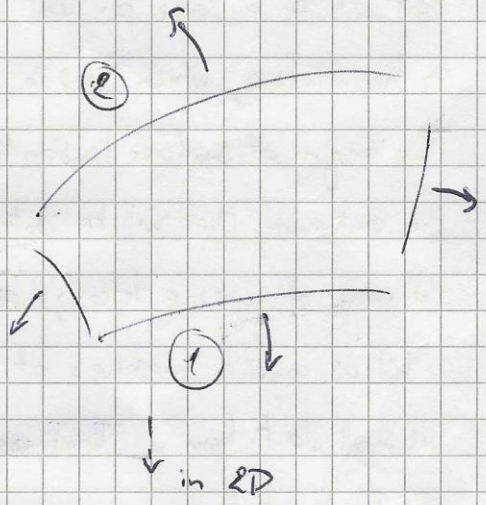
$$dW = F_f dh,$$

while also $dW = (2)\tau\, dA = (2)\tau\, b\, dh \Rightarrow F_f dh = (2)\tau b dh \Rightarrow \tau = F_f/(2)b$

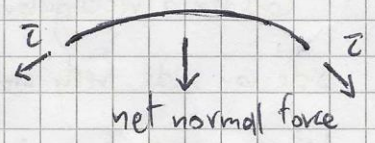
Notice that here we have a flat system, and two interfaces. A soap bubble will have two interfaces, too (it is full of air) but a curved surface. A liquid drop will have a curved surface and just one air-liquid interface. A curvature radius in the interface, in particular, implies out-of-plane forces.

⊙ Curved interface

With a curved contact surface it is intuitive to see the existence of a force component normal to the surface; it is also intuitive that this normal component is $\propto 1/R$ curvature radius, or, in a three-dimensional system, where we can define two principal curvature radii R_1, R_2 with respect to two orthogonal directions, $\propto (1/R_1 + 1/R_2)$.

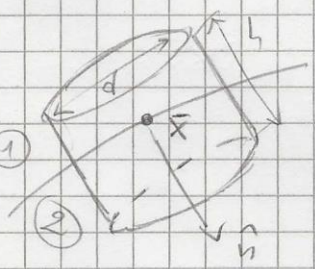


Dimensionally, since $[\tau] = [\text{Force}]/[L^2]$, the effect τ/R has $[\tau/R] = [\text{Force}]/[\text{Area}] \Rightarrow$ dimensions of a stress, like pressure. Indeed this normal force acts along the direction of pressure and should be taken into account when considering the equilibrium at the interface; one can demonstrate that across the interface, $\Delta p = \text{constant} \times (1/R_1 + 1/R_2)$ (see, e.g., Landau's chapter 7). If surface tension is negligible, we can consider pressure continuous across the interface.



Continuity of pressure at interfaces

Let us consider two fluids in equilibrium (or two ideal fluids) and let us take them as immiscible (they cannot mix intimately to the point they constitute a single phase). Then a surface that separates them exists (INTERFACE) and pressure is continuous across such interface, provided we can neglect surface tension.



In order to prove it, let us take an infinitesimal cylinder across the interface between fluids ① and ②; the height h of the cylinder is an infinitesimal of higher order with respect to d diameter of the cylinder bases.

Let us consider the equilibrium of such volume; The resultant of forces in $\bar{x}$, interface position around which we constructed the infinitesimal volume, must be zero.

Volume forces are proportional to the volume $\sim d^2 h \Rightarrow O(V)$. For instance, the most likely volume force that is gravity is $m\bar{g} = \rho d^2 h \bar{g}$.

Surface forces on the bases are evaluated in $\bar{x} \pm h/2 \Rightarrow$ the error on p is $\sim O(h)$ and the error on the force $= pd^2$ is $\sim O(h)\,O(d^2) = O(V)$ ending up being of the same order of volume forces. We also have surface forces on the lateral surface, $\sim O(dh)$.

Let us now add τ surface tension: this, for a curved surface, has a component along $\hat{n}$ (surface normal) $\sim \tau/R$, with R curvature radius.

The resulting condition is

$$p^{(1)}(\bar{x})\,d^2\hat{n} - p^{(2)}(\bar{x})\,d^2\hat{n} + \underbrace{O(d\cdot h)\hat{t}}_{\text{the pressure on the lateral surface will yield something directed along some direction } \hat{t}\perp\hat{n}\text{, plus here} \Rightarrow \text{not interesting here}} \pm \underbrace{\frac{c\tau}{R}\,d^2\hat{n}}_{\downarrow\, c \text{ is some coefficient: the sign depends on the concavity/convexity of the surface (here +), plus the properties of 1,2}} + \rho d^2 h \bar{g} + O(V) = \phi$$

Since terms of different infinitesimal order are present, they must be zero separately; terms $\sim O(d^2)$ along the direction $\hat{n}$ yield

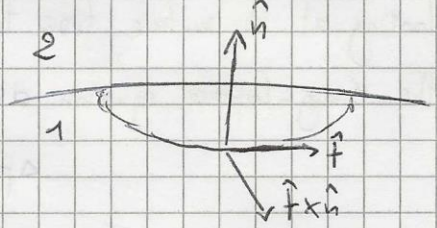
$$p^{(1)}(\bar{x}) - p^{(2)}(\bar{x}) \pm c\tau/R = \phi$$

and if surface tension τ is negligible,

$$\boxed{p^{(1)}(\bar{x}) = p^{(2)}(\bar{x})}$$ i.e. pressure is continuous at the interface.

Appendix: Young-Laplace equation (surface tension at interfaces)

At equilibrium, at the interface between two immiscible fluids pressure is continuous if we can neglect surface tension. But let us see rigorously what happens if there is a non-negligible surface tension $\bar{\tau}$, by considering an infinitesimal portion S of the interface between phases 1 and 2, enclosed by a boundary C. If $\hat{t}$ is the unit vector along C and $\hat{n}$ the unit vector normal to the surface, $\bar{\tau}$ will act along the surface pulling it outwardly in the direction $\hat{t}\times\hat{n}$ (see figure →). Equilibrium requires the resultant $\bar{F}$ of all forces to be zero, so



$$\bar{F} = \int_S (p_1 - p_2)\,\hat{n}\,dS + \tau \oint_{C=\partial S} \hat{t}\times\hat{n}\,dl = \phi \qquad (*)$$

(where errors in the application of forces in the center/on the border of S will yield infinitesimal terms of higher order ⇒ neglected). To progress with the solution we need to manipulate the line integral using the Stokes' theorem and some vector identities. Stokes' theorem states

$$\oint_{C=\partial S} \bar{P}\cdot d\bar{l} = \int_S (\mathrm{curl}\,\bar{P})\cdot d\bar{S} \quad \text{or} \quad \oint_{C=\partial S} \bar{P}\cdot\hat{t}\,dl = \int (\mathrm{curl}\,\bar{P})\cdot\hat{n}\,dS$$

Now let us have $\bar{P} = a\hat{n}\times\bar{b}$ with $\bar{b}$ constant vector; now

· ⊙ $\mathrm{curl}(\hat{n}\times\bar{b}) = \hat{n}(\mathrm{div}\,\bar{b}) - \bar{b}(\mathrm{div}\,\hat{n}) + (\bar{b}\cdot\mathrm{grad})\hat{n} - (\hat{n}\cdot\mathrm{grad})\bar{b} = (\bar{b}\cdot\mathrm{grad})\hat{n} - \bar{b}(\mathrm{div}\,\hat{n})$

($\bar{b}$ constant; $\bar{b}$ constant)

· ⊙ $(\hat{n}\times\bar{b})\cdot\hat{t} = (\hat{t}\times\hat{n})\cdot\bar{b}$

$$\Rightarrow \bar{b}\cdot\oint_C \hat{t}\times\hat{n}\,dl = \int_S \left[(\bar{b}\cdot\mathrm{grad})\hat{n} - \bar{b}(\mathrm{div}\,\hat{n})\right]\cdot\hat{n}\,dS$$

$(\mathrm{grad}\,\hat{n})\cdot\hat{n} = \frac{1}{2}\mathrm{grad}(n^2) = \phi$

$$\Rightarrow \bar{b}\cdot\oint_C \hat{t}\times\hat{n}\,dl = -\bar{b}\cdot\int_S (\mathrm{div}\,\hat{n})\,\hat{n}\,dS$$

and since $\bar{b}$ is an arbitrary vector,

$$\oint_{C=\partial S} \hat{t}\times\hat{n}\,dl = -\int_S (\mathrm{div}\,\hat{n})\,\hat{n}\,dS$$

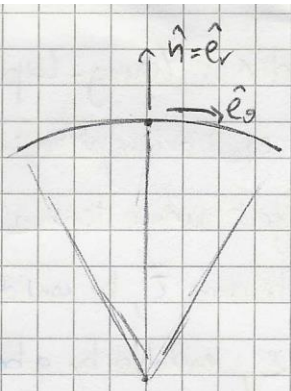
Coming back to the balance of forces (*),

$$\bar{F} = \int_S (p_1 - p_2)\,\hat{n}\,dS - \tau\int_S (\mathrm{div}\,\hat{n})\,\hat{n}\,dS = \phi \quad \text{and for the arbitrarity of } S,$$

$$\boxed{\Delta p = \tau\,\mathrm{div}\,\hat{n}} \quad \text{Young-Laplace equation}$$

Notice that $\mathrm{div}\,\hat{n}$ is linked to the local curvature of the surface ($\Delta p = \phi$ for flat surface). To figure it out, let us start from a purely two-dimensional picture; in a polar coordinate system

(r, θ) centered in the center of curvature, then $\hat{n} = \hat{e}_r$;



in polar coordinates $\operatorname{div} \vec{F} = \frac{1}{r} \frac{\partial}{\partial r}(r F_r) + \frac{1}{r} \frac{\partial}{\partial \theta} F_\theta$

so $\operatorname{div} \hat{n} = \frac{1}{r} \frac{\partial}{\partial r}(r \cdot 1) = \frac{1}{r}$ with r : curvature radius.

In three dimensions we can describe the local curvature by using two PRINCIPAL RADII OF CURVATURE R_1 and R_2, i.e. curvature radii seen when looking at the surface on two orthogonal cross-sectional planes. One can hence rewrite the Young-Laplace equation as

$$\Delta p = \tau \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

In the case of a spherical surface of radius R, with $\hat{n} = \hat{e}_r$ again,

$$\operatorname{div} \vec{F} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(F_\theta \sin\theta) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} F_\varphi$$

so $\operatorname{div} \hat{n} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \cdot 1) = \frac{2}{r}$

$$\Rightarrow \quad \Delta p = 2\tau / R$$

So, the smaller a droplet or bubble is, the higher is the pressure inside.