ES. #15 Correzione della prova di autovalutazione 15/1/2021

EX. 1

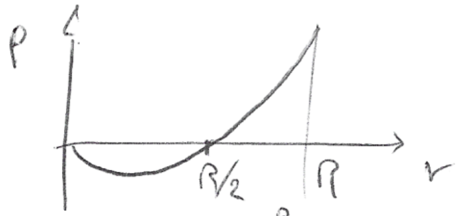
1

a) $\rho(r) = ar + br^2$

$a = -10^{20}\ C/m^4$

$b = 10^{30}\ C/m^5$

$R = -2a/b = 0.2\ nm$



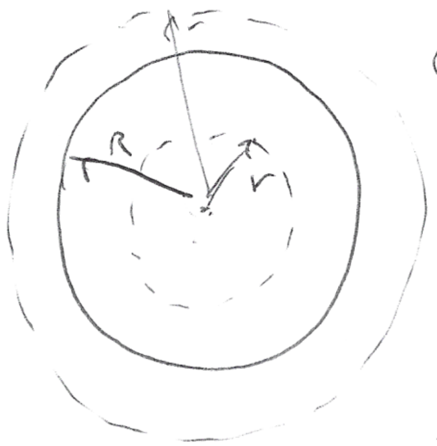
$$Q = \int_0^R \rho(r) \underbrace{4\pi r^2 dr}_{d\tau} = 4\pi \int_0^R ar^3 + br^4 dr = 4\pi \left[\frac{a}{4} r^4 + \frac{b}{5} r^5 \right]_0^R =$$

$$= 4\pi \left[\frac{a}{4} R^4 + \frac{b}{5} R^5 \right] = 4\pi R^4 \left(\frac{a}{4} + \frac{b}{5} R \right) \underset{R=-2a/b}{=} 4\pi \frac{16a^4}{b^4} \left[\frac{a}{4} - \frac{2a}{b} \frac{b}{5} \right] =$$

$\frac{1}{4} - \frac{2}{5}$

$$\Rightarrow Q = -\frac{48\pi}{5} \frac{a^5}{b^4} = 3.01 \cdot 10^{-19} C$$

b)



① $\boxed{r<R}$ $\int \bar{E}(r) \cdot d\bar{S} = \frac{1}{\varepsilon_0} \int_0^r \rho(r') 4\pi r'^2 dr'$

$$E_r^{(1)}(r) 4\pi r^2 = \frac{4\pi r^4}{\varepsilon_0} \left[\frac{a}{4} + \frac{b}{5} r \right]$$

$$\Rightarrow \boxed{E_r^{(1)}(r) = \frac{1}{\varepsilon_0} \left[\frac{a}{4} r^2 + \frac{b}{5} r^3 \right]}$$

② $\boxed{r>R}$

$$E_r^{(2)} 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E_r^{(2)}(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} =$$

$$= -\frac{12a^5}{5\varepsilon_0 b^4} \frac{1}{r^2}$$

c) ② $\boxed{r>R}$ ②

$$V(r)-V(\infty)=\int_r^{+\infty}E_r^{(2)}(r')\,dr' = \int_r^{+\infty}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}\,dr' =$$

$V(\infty)=\phi$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r'}\Big|_r^{+\infty} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = V(r)$$

① $\boxed{r<R}$

$$V(r)-V(\infty)=V(r)-V(R)+\underbrace{V(R)-\cancel{V(\infty)}}=$$

$$=\int_r^R E_r^{(1)}(r')\,dr' + \int_R^{+\infty}E_r^{(2)}(r')\,dr' =$$

$$= -\frac{1}{\varepsilon_0}\left[\frac{a}{12}r^3+\frac{b}{20}r^4\right]$$? controllare

d) $q=-1.6\cdot10^{-19}\,C = -e$ elettrone $\infty \to R$

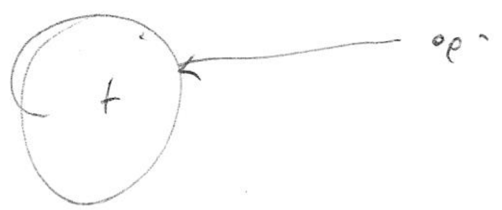
$$L_{ext}=\Delta U = U_f-U_i = q\,\Delta V = U(R)-U(\infty)=$$

$$= q\,V(R)-q(V_\infty) = -e\,V(R) = -e\,\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}=$$

ϕ

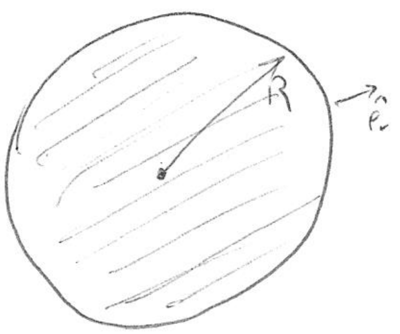
$$= -e\,\frac{1}{4\pi\varepsilon_0}\left(-\frac{48\pi a^3}{5b^4}\right)\cdot\left(-\frac{b}{2a}\right) = -e\,\frac{6a^2}{5\varepsilon_0 b^3} = -2.17\cdot10^{-18}\,J$$

~~Q=~~ $L_{ext}=-2.17\cdot10^{-18}\,J = -13.6\,eV$



a → 3
b → 4
c → 4
d → 4

Ex. 2



$R = 20\,cm$

(3)

$\bar{P} = K\bar{r}$ $\qquad \bar{r} = r\hat{e}_r$

$K = 0.0885 \cdot 10^{-6}\ C/m^3$

$P = (P_r, \cancel{\theta}, \cancel{\phi})$

a) $\rho_p(r) = -\bar{\nabla}\cdot\bar{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 P_r\right) + \dots =$

$= -\frac{1}{r^2}\frac{d}{dr}\left(r^2 P_r\right) = -\frac{1}{r^2} 3Kr^2 = -3K$

$\rightarrow Kr$

$\rho_p = -3K = -2.66 \cdot 10^{-7}\ C/m^3$

$Q_{pv} = \frac{4\pi}{3}R^3 \rho_p = \underline{-4\pi R^3 K} = -8.90 \cdot 10^{-9}C$

$\sigma_p(R) = \bar{P}(R)\cdot\hat{e}_r = KR = 1.77 \cdot 10^{-8}\ C/m^2$

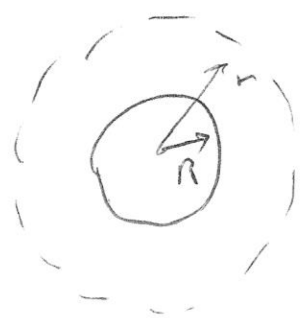
$Q_{ps} = \sigma_p \cdot 4\pi R^2 = \underline{4\pi R^3 K} = 8.90 \cdot 10^{-9} C$

$Q_p = Q_{pv} + Q_{ps} = \phi$

b) $\bar{E}(\bar{r}) = E_r(r)\,\hat{e}_r$

$\boxed{r < R}$ $\quad E_r(r)\, 4\pi r^2 = \frac{1}{\varepsilon_0}\int_\phi^r \rho_p(r')\, 4\pi r'^2 dr' = -\frac{1}{\varepsilon_0} 4\pi r^3 K$

$$\boxed{E_r(r) = -\frac{Kr}{\varepsilon_0}}$$



$\boxed{r > R}$ $\quad E_r(r)\,4\pi r^2 = \frac{1}{\varepsilon_0}\cancelto{\phi}{Q_{tot}} = \phi$

$\Rightarrow \boxed{E_r(r) = \phi}$

c) $V(\infty) = \phi$

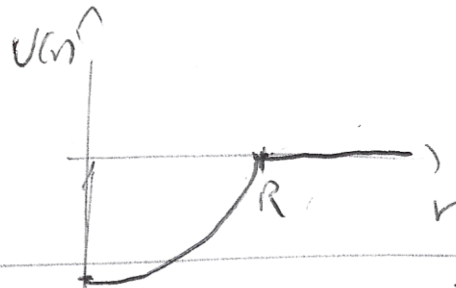
$\boxed{r > R}$ $\quad E_r = \phi \Rightarrow V$ costante

$\Rightarrow V(r > R) = \phi$

$\boxed{r < R}$ $\quad V(r) - \underbrace{V(R)}_{\phi} = \int_r^R E(r')\,dr' =$

$$= -\frac{k}{\varepsilon_0}\int_r^R r'\,dr' = -\frac{k}{2\varepsilon_0} r'^2\Big|_r^R = \frac{k}{2\varepsilon_0}(r^2 - R^2)$$

$$V(r) = \frac{k}{2\varepsilon_0}(r^2 - R^2)$$



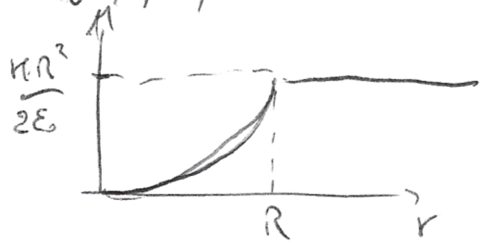
$$\boxed{E_r(R^+) - E_r(R^-) = \phi - \left(-\frac{kR}{\varepsilon_0}\right) = \frac{kR}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{kR}{\varepsilon_0}}$$

d) $* \begin{cases} V(\phi) = -\dfrac{kR^2}{2\varepsilon_0} \\ V(R) = \phi \end{cases}$ $\quad V(\phi) - V(R) = -\dfrac{kR^2}{2\varepsilon_0} = -200\,V$

e) $** \begin{cases} V(\phi) \\ V(R') \end{cases}$ $\Rightarrow$ uguale $(V(R') - V(R) = \phi)$

$R' = 30$ cm $> R$ $\quad V(\phi) - V(R') = -200\,V$

Se invece $V(\phi) = \phi$ (diversa solo per costante di integrazione)



a 4
b 3
c 4
d 2
e 2