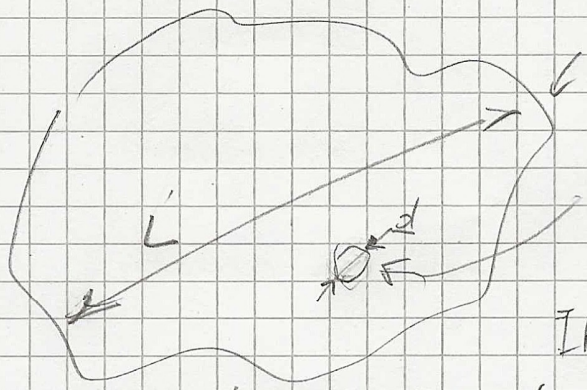
General notions on continuous media

Matter is made of discrete particles (atoms, molecule) in a lot of empty space

→ continuum approximation : a question of scale comparison



scale of the whole system : L

scale of a "physical particle" : d

If $d \ll L$ (d infinitesimal)

and in a volume of diameter d we count a statistically significant number of microscopic constituants (atoms)

⇒ a continuum approximation holds.

Why useful ? Because then all physical properties are continuous functions = vary little by little moving from point to point.

Examples : ① A tank or a river → L ~ meters (m)

→ d ~ micrometer (μm)

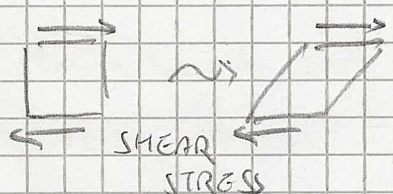
A cube of edge d contains 10^9 particles (at distances ~ nm)

⇒ ok to treat as continuum.

② A microscopic droplet → L ~ μm

→ d ~ ? nm ? Not a continuum !

Types of continuum



Fluids	Solids
(do not recover shape even after shear stress ceases)	(elastic solid : bounces back when stress ceases)
↓	↓
Study of dynamics : velocity field	deformation field

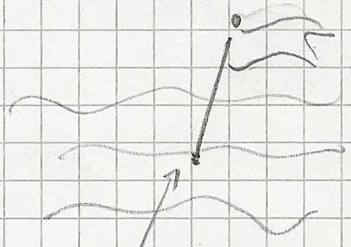
Full description = thermodynamic description

($\vec{v}$ or $\vec{E}$ + heat / Temperature field)

Heat transfer: conduction, convection, (radiation)

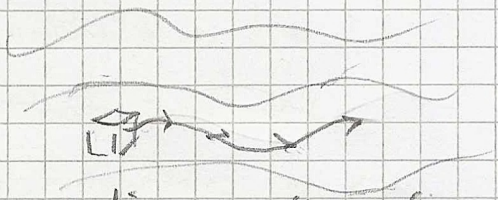
Approaches to fluid dynamics → Eulerian
→ Lagrangian

Eulerian approach



Record time variations of properties (e.g. $\vec{v}$, ρ, T) sitting at a fixed point

Lagrangian approach



Record time variations of properties sitting on a moving physical particle in its flow

↳ = continuum (fluid) particle

Consequence of Lagrangian approach: any property $F(\vec{x},t)$ is actually $F(\vec{x}(t),t)$

implicit dependence on time

explicit dependence on time

Implicit dependence arises because $\vec{x}$ position of fluid particle changes!

⇒ time derivative of $F(\vec{x}(t),t)$ must take everything into account:

TOTAL / MATERIAL / ADVECTIVE / SUBSTANTIAL DERIVATIVE

$$\frac{D}{Dt} F(\vec{x}(t),t) = \underbrace{\frac{\partial F}{\partial t}(\vec{x}(t),t)}_{\text{explicit time variation}} + \underbrace{\vec{v}(\vec{x}(t),t) \cdot \text{grad}\, F(\vec{x}(t),t)}_{\text{implicit dependence (advective term)}}$$

It is the "real" derivative $\frac{DF}{Dt} = \lim_{dt \to 0} \frac{1}{dt} \left[F(\vec{x}(t+dt), t+dt) - F(\vec{x}(t),t) \right]$